Group Symmetry and Similarity Solutions for MHD Mixed-Convection Flow of Power-Law Fluid over a Non-Linear Stretching Surface

R. M. Darji and M. G. Timol

***Abstract*—Using deductive group symmetry analysis, similarity solutions are derived for the magnetohydrodynamic (MHD) boundary layer flow of power-law fluid over a non-linear stretching surface. The system of partial differential equations governing the problem is transformed into the system of ordinary differential equations by deriving similarity transformation. Similarity solutions of transformed boundary layer equations are produced numerically using classical Runge-kutta fourth order method for wide range of flow parameters. The effect of magnetic field is studied on various relevant physical quantities for a typical non-Newtonian power-law fluid.**

***Index Terms*—Group symmetry, Power-law fluid, similarity solution, boundary layer.**

***MSC 2010 Codes* – 76A05, 76M55, 54H15**

I. Introduction

THE formulation of the group-theoretic method, also called symmetry analysis, is contained in the general theories of continuous transformation groups that were introduced and treated extensively by Lie about 130 years ago. Symmetry groups or simply symmetries are invariant transformations which do not alter the structural form of the equation under investigation [1]. A special form of Lie group of transformations, known as deductive group (See [5], [6], [7], [8]), at most general approach to derive the similarity transformation. In the present paper we derive possible similarity solutions of the non-linear partial differential equations under consideration.

The boundary layer flow of Newtonian fluids past stretching sheeting sheet was first discussed by Crane [9]. Later on same problem was extended by several researchers for different physical situations. All these work is restricted to Newtonian fluids and in literature rare work found regarding Non-Newtonian fluids. Many types of Non-Newtonian fluids have found in literature but the most useful type is the power law fluid. The rheological model for power-law fluid s given by [12]

$$\tau_{ij} = -P\delta_{ij} + K\left|\sum\sum e_{lm}e_{lm}\right|^{\frac{n-1}{2}} e_{ij}$$

where *P* is the pressure, δ_{ij} is the Kronecker delta and *K* and *n* are consistency and flow behaviour indices of the fluid. When $n > 1$ the fluid is described as dilatant, $n < 1$ pseudoplastic and when $n = 1$ it is simply the Newtonian fluid. Recently couple different analysis have been carried out by Prasad et al. [13], [14] for power law fluid.

In virtue of these works, we apply the deductive group symmetry analysis to analyse the particular boundary layer problem of power-law fluid.

II. Problem Formulation

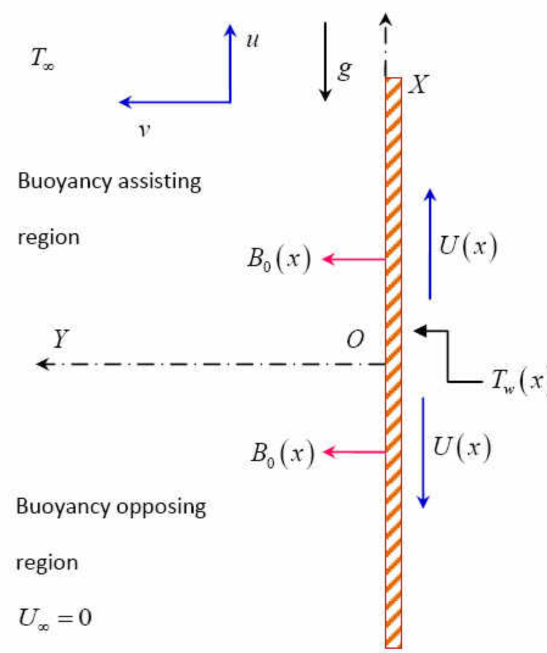

Fig. 1. Flow geometry of the problem

Consider two-dimensional steady state, mixed convection boundary layer flow of an electrically conducting power-law fluid under the effect of a transverse magnetic field $B_0(x)$ due to a stretching vertical heated sheet, as shown in Fig. 1. Let the temperature of the ambient fluid be T_∞. The continuous stretching sheet is assumed velocity of the form $U(x) = bx^m$ and temperature of the form $T_w(x)$ where b is the stretching constant, x is the distance from the slot and L is the reference length. It is also assumed that the magnetic Reynolds number Re_m is very small; i.e. $\mathrm{Re}_m = \mu_0\sigma bL \ll 1$ where μ_0 is the magnetic permeability and σ is the electric conductivity. We

R. M. Darji is with the Department of Mathematics, Sarvajanik College of Engineering and Technology, Surat, Gujarat, 395001 INDIA e-mail: rmdarji@gmail.com

M. G. Timol is with Department of Mathematics, Veer Narmad South Gujarat University, Surat, Gujarat, 395007 INDIA e-mail: mgtimol@gmail.com

neglect the induced magnetic field. The governing equations of the problem are given by, [16]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\nu\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n-\frac{\sigma B_0^2}{\rho}u\pm g\,\beta\,(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components in the x and y direction, respectively, ν is the kinematic viscosity of the fluid, n is the power law index, ρ is the fluid density, g is the acceleration due to gravity, β is the coefficient of thermal expansion. Let T be the temperature of the fluid and α be the thermal diffusivity of the fluid.
The corresponding boundary conditions are:

$$\left.\begin{array}{l} u(x,0)=U(x)\quad [=bx^m] \\ v(x,0)=0 \\ T(x,0)=T_w(x) \\ u(x,y)\to 0,\quad T(x,y)\to T_\infty \quad \text{as} \quad y\to\infty \end{array}\right\} \tag{4}$$

III. Non-dimensional Governing Equations

In the above equations setting $T-T_\infty=\theta$, they can made dimensionless using the following quantities,

$$\left.\begin{array}{l} \bar{x}=\frac{Gr_x}{L}x,\quad \bar{y}=(\mathrm{Re}_x\cdot Gr_x)^{1/(n+1)}\frac{y}{L}, \\ \bar{u}=\frac{u}{U},\qquad \bar{v}=(\mathrm{Re}_x/Gr_x)^{1/(n+1)}\frac{v}{U}, \\ \bar{\tau}_{\bar{y}\,\bar{x}}=(\mathrm{Re}_x/Gr_x)^{-1/(n+1)}\frac{\tau_{y\,x}}{\rho U^2}, \\ \bar{\theta}=\frac{\theta}{T_w-T_\infty},\quad \bar{\theta}_{\bar{w}}=\frac{T_w}{T_w-T_\infty},\quad \mathrm{Pr}=\frac{\alpha}{\nu}, \\ \mathrm{Re}_x=\frac{U^{2-n}L^n}{\nu},\quad Gr_x=\frac{L^3}{\nu^2}g\beta\,(T_w-T_\infty) \end{array}\right\} \tag{5}$$

where L is reference length, ν is kinematic viscosity, Re_x is local Reynolds number, Gr_x is local Grashof number, T_∞ ambient temperature, T_w is temperature of plate, U is velocity (main stream velocity in X-direction), P_r is Prandtl number.
Substitute the values in equations (1) to (4) and dropping the asterisks (for simplicity), we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=n\left(-\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial^2 u}{\partial y^2}-S(x)\,u+\lambda\theta \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{3\,\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2} \tag{8}$$

Where $S(x)=\frac{\sigma B_0^2(x)}{\rho}$ is the magnetic field strength and $\lambda=\pm Gr_x/\mathrm{Re}_x$ is the buoyancy or mixed convection parameter. Introducing non-dimensional stream function ψ such that $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ which satisfies equation (6) identically.
Equations (6) to (8) become

$$\frac{\partial\psi}{\partial y}\,\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\,\frac{\partial^2\psi}{\partial y^2}=n\left(-\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\frac{\partial^3\psi}{\partial y^3}-S(x)\,\frac{\partial\psi}{\partial y}+\lambda\theta \tag{9}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\,\frac{\partial\theta}{\partial y}=\frac{1}{3\,\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2} \tag{10}$$

Together with boundary conditions:

$$\left.\begin{array}{l} \frac{\partial\psi}{\partial y}(x,0)=cx^m,\quad c=b\left(\frac{L}{Gr}\right)^m \\ \frac{\partial\psi}{\partial x}(x,0)=0 \\ \theta(x,0)=\theta_w(x) \\ \frac{\partial\psi}{\partial y}(x,y)\to 0,\quad \theta(x,y)\to 0 \quad as \quad y\to\infty \end{array}\right\} \tag{11}$$

IV. Application of Deductive Group Symmetry Method

A. The Group Systematic Formulation

Consider a one parametric group in one-parameter say $'a'$, C_G of the form:

$$C_G:\quad \bar{Q}=\aleph^Q(a)\,s+\Re^Q(a) \tag{12}$$

Where Q stands for x,y,ψ,θ,S whereas $\aleph's$ and $\Re's$ are real-valued and are at least differentiable in the real argument a. To transform the differential equation, transformations of the derivatives of ψ are obtained from C_G via chain-rule operations:

$$\left.\begin{array}{l} \bar{Q}_{\,\bar{i}}=\left(\frac{\aleph^Q}{\aleph^i}\right)Q_{\,i} \\ \bar{Q}_{\,\bar{i}\,\bar{j}}=\left(\frac{\aleph^Q}{\aleph^i\aleph^j}\right)Q_{\,i\,j} \end{array}\right\};\quad Q=\psi,\theta,S;\quad i,j=x,y \tag{13}$$

Equations (9) and (10) are said to be invariantly transformed, for some functions $\xi_1(a)$ and $\xi_2(a)$ whenever,

$$\begin{array}{l} \frac{\partial\bar{\psi}}{\partial\bar{y}}\,\frac{\partial^2\bar{\psi}}{\partial\bar{y}\partial\bar{x}}-\frac{\partial\bar{\psi}}{\partial\bar{x}}\,\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}-n\left(-\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}\right)^{n-1}\frac{\partial^3\bar{\psi}}{\partial\bar{y}^3}+\bar{S}(\bar{x})\,\frac{\partial\bar{\psi}}{\partial\bar{y}}-\lambda\bar{\theta} \\ =\xi_1(a)\left[\begin{array}{l} \frac{\partial\psi}{\partial y}\,\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\,\frac{\partial^2\psi}{\partial y^2} \\ -n\left(-\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\frac{\partial^3\psi}{\partial y^3}+S(x)\,\frac{\partial\psi}{\partial y}-\lambda\theta \end{array}\right] \end{array} \tag{14}$$

$$\begin{array}{l} \frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial\bar{\theta}}{\partial\bar{x}}-\frac{\partial\bar{\psi}}{\partial\bar{x}}\,\frac{\partial\bar{\theta}}{\partial\bar{y}}-\frac{1}{3\,\mathrm{Pr}}\frac{\partial^2\bar{\theta}}{\partial\bar{y}^2} \\ =\xi_2(a)\,\left[\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\,\frac{\partial\theta}{\partial y}-\frac{1}{3\,\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}\right] \end{array} \tag{15}$$

Substituting the values from the equations (12) and (13) in above system of equations, yields

$$\begin{array}{l} \frac{(\aleph^\psi)^2}{\aleph^x(\aleph^y)^2}\left[\frac{\partial\psi}{\partial y}\,\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\,\frac{\partial^2\psi}{\partial y^2}\right] \\ -n\frac{(\aleph^\psi)^n}{(\aleph^y)^{2n+1}}\left(\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\frac{\partial^3\psi}{\partial y^3}+\left(\aleph^S S+\Re^S\right)\frac{\aleph^\psi}{\aleph^y}\frac{\partial\psi}{\partial y}-\lambda\left(\aleph^\theta\theta+\Re^\theta\right) \\ =\xi_1(a)\left[\frac{\partial\psi}{\partial y}\,\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\,\frac{\partial^2\psi}{\partial y^2}-n\left(\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\frac{\partial^3\psi}{\partial y^3}+S\frac{\partial\psi}{\partial y}-\lambda\theta\right] \end{array} \tag{16}$$

$$\begin{array}{l} \frac{\aleph^\psi\aleph^\theta}{\aleph^x\aleph^y}\left[\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\,\frac{\partial\theta}{\partial y}\right]-\frac{\aleph^\theta}{(\aleph^y)^2}\left[\frac{1}{3\,\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}\right] \\ =\xi_2(a)\,\left[\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\,\frac{\partial\theta}{\partial y}-\frac{1}{3\,\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}\right] \end{array} \tag{17}$$

The invariance of equations (16) and (17) together with boundary conditions, implies that

$$\left.\begin{array}{l} \Re^\theta=\Re^S=\Re^y=\Re^\psi=0 \\ \frac{(\aleph^\psi)^2}{\aleph^x(\aleph^y)^2}=\frac{(\aleph^\psi)^n}{(\aleph^y)^{2n+1}}=\frac{\aleph^S\aleph^\psi}{\aleph^y}=\aleph^\theta=\xi_1(a) \\ \frac{\aleph^\psi\aleph^\theta}{\aleph^x\aleph^y}=\frac{\aleph^\theta}{(\aleph^y)^2}=\xi_2(a) \end{array}\right\} \tag{18}$$

These yields,

$$\aleph^x=(\aleph^y)^3,\quad \aleph^\psi=(\aleph^y)^2,\quad \aleph^\theta=\frac{1}{\aleph^y},\quad \aleph^S=\frac{1}{(\aleph^y)^2} \tag{19}$$

The one-parameter group G, sub-group of C_G is of the form:

$$G : \left\{ \begin{array}{l} G_S : \left\{ \begin{array}{l} \bar{x} = (\aleph^y)^3 x + \Re^x \\ \bar{y} = \aleph^y y \end{array} \right. \\ \bar{\psi} = (\aleph^y)^2 \psi \\ \bar{\theta} = \frac{1}{\aleph^y} \theta \\ \bar{S} = \frac{1}{(\aleph^y)^2} S \end{array} \right. \tag{20}$$

B. Absolute Invariants of Group

If $\eta = \eta(x, y)$ is the absolute invariant of the independent variables x and y then, the three absolute invariants for the dependent variables ψ ,θ and S are:

$$g_j(x, y, \psi, \theta, S) = F_j(\eta), \quad j = 1, 2, 3 \tag{21}$$

A function $g(x, y, \psi, \theta, S)$ is an absolute invariant of a one-parameter group if it satisfies the following first-order linear partial differential equation [20], [21]

$$\sum_{i=1}^{5} (\alpha_i Q_i + \beta_i) \frac{\partial g}{\partial Q_i} = 0, \qquad Q_i = x, y, \psi, \theta, S \tag{22}$$

Where

$$\alpha_i = \left. \frac{\partial \aleph^i}{\partial a} \right|_{a=a^0} \quad \text{and} \quad \beta_i = \left. \frac{\partial \Re^i}{\partial a} \right|_{a=a^0} \qquad i = 1, 2, 3, 4, 5 \tag{23}$$

and $'a^0{}'$ denotes the value of parameter $'a'$ which yields the identity element of the group G.
Since $\Re^\theta = \Re^S = \Re^y = \Re^\psi = 0$ implies that $\beta_2 = \beta_3 = \beta_4 = 0$ and from (23) we get $\alpha_1 = 3\alpha_2 = \frac{3}{2}\alpha_3 = -\frac{1}{3}\alpha_4 = -\frac{3}{2}\alpha_5$.
The equation (22) reduces to

$$(\alpha_1 x + \beta_1) \frac{\partial g}{\partial x} + \alpha_2 y \frac{\partial g}{\partial y} + \alpha_3 \psi \frac{\partial g}{\partial \psi} + \alpha_4 \theta \frac{\partial g}{\partial \theta} + \alpha_5 S \frac{\partial g}{\partial S} = 0. \tag{24}$$

The absolute invariant of independent variable owing the equation (24) is $\eta = \eta(x, y)$ given by first order linear partial differential equation

$$(\alpha_1 x + \beta_1) \frac{\partial \eta}{\partial t} + \alpha_2 y \frac{\partial \eta}{\partial y} = 0.$$

Using the definitions of $\alpha' s$, we get

$$(x + \kappa) \frac{\partial \eta}{\partial x} + \frac{y}{3} \frac{\partial \eta}{\partial y} = 0, \quad \text{where } \kappa = \frac{\beta_1}{\alpha_1} \tag{25}$$

Solving by variable separable method, we get

$$\eta(x, y) = y(x + \kappa)^{-1/3} \tag{26}$$

Similarly for the dependent variables we have from (24),

$$\left. \begin{array}{l} F(\eta) = \psi(x + \kappa)^{-2/3} \\ G(\eta) = \theta(x + \kappa)^{1/3} \\ H(\eta) = S(x + \kappa)^{2/3} \end{array} \right\} \tag{27}$$

Since $S(x)$ is independent of y, $H(\eta)$ mush be constant say h (known as magnetic strength).
Hence set of absolute invariants for the group G is given by,

$$\left. \begin{array}{l} \psi = (x + \kappa)^{2/3} F(\eta) \\ \theta = (x + \kappa)^{-1/3} G(\eta) \\ S = h(x + \kappa)^{-2/3} \end{array} \right\} \tag{28}$$

C. Reduction to Ordinary Differential Equation

Substituting the values of partial derivatives in equation (9) and (10), one can derive

$$n(-F'')^{n-1} F''' - \frac{1}{3}(F')^2 + \frac{2}{3} F F'' - hF' + \lambda G = 0 \tag{29}$$

$$2FG' + GF' - \frac{1}{\mathrm{Pr}} G'' = 0 \tag{30}$$

Also for the requirement of similarity solution the temperature near wall, θ_w must be proportional to $(x + \kappa)^{-1/3}$. That is it should be of the form $\theta_w = \theta_0 (x + \kappa)^{-1/3}$, θ_0 , θ_0 is some constant and the prescribe velocity is the form $U(x) = x^{1/3}$. This is the precise restriction for the existence of similarity solution.
The relevant boundary conditions become,

$$\left. \begin{array}{lll} F(\eta = 0) = 0, & F'(\eta = 0) = c, & F'(\eta \to \infty) = 0 \\ G(0) = \theta_0, & G(\eta \to \infty) = 0 & \end{array} \right\} \tag{31}$$

Further the local skin friction coefficient is given by

$$\frac{1}{2} C_f (\mathrm{Re}_x)^{1/(n+1)} = [-f''(0)]^n$$

V. Numerical Solution

The transformed non-linear system of ordinary differential (29) and (30) subject to the boundary conditions (31) is solved numerically by using the classical Runge-Kutta fourth order method. The appropriate algorithm is used in Matlab.
First, the boundary value problem reduced to system of five first order simultaneous ODE by

$$\frac{df_i}{d\eta} = F_i(\eta, f_1, f_2, ... f_n); \quad (i = 1, 2, ...5 = n) \tag{32}$$

According to this, the system of ordinary differential equations (29) and (30) will reduce to the system of first ODEs as,

$$f_1 = F, \quad f_2 = F', \quad f_3 = F', \quad f_4 = G, \quad f_5 = G'$$

And

$$\begin{array}{l} F_1 = f_2, \quad F_2 = f_3, \\ F_3 = \left(\frac{1}{3} f_2^2 - \frac{2}{3} f_1 f_3 + h f_2 - \lambda G\right) \div n(-f_3)^{n-1} \\ F_4 = f_5, \quad F_5 = \mathrm{Pr}\,(2 f_1 f_5 + f_2 f_4) \end{array}$$

The boundary conditions become,

$$\left. \begin{array}{lll} f_1(0) = 0, & f_2(0) = c, & f_4(0) = \theta_0 \\ f_2(\eta_\infty) = f_4(\eta_\infty) = 0 & & \end{array} \right\}$$

Here dashes denote differentiation with respect to η . Next, after choosing η_∞ (the numerical infinity) an initial guess for the missing initial condition is made and the uniform step length $\Delta\eta$ for all the numerical solutions is chosen for $[0, \eta_\infty]$. For the numerical infinity we have considered $\eta_\infty = 3$ and the step length $\Delta\eta = 0.25$. Further without loss of generality we take the values of c and θ_0 unity. The numerical solutions are obtained for various flow parameter like Pr, λ, h subject to different flow index n. (See Figs. 2 to 8)

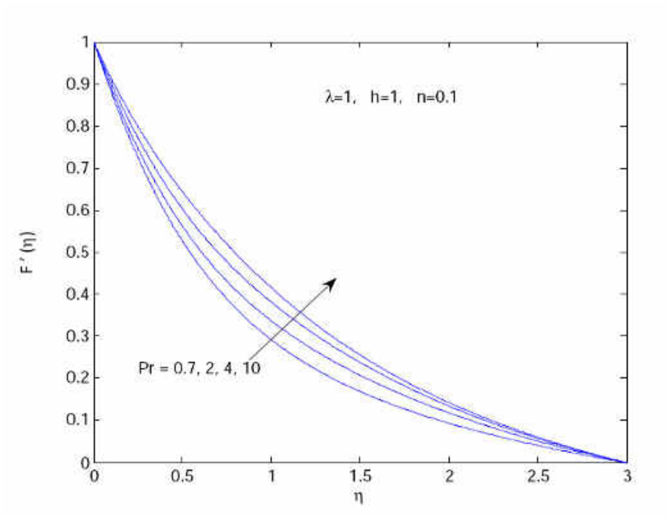

Fig. 2. Dimensionless velocity profile for various *Pr*

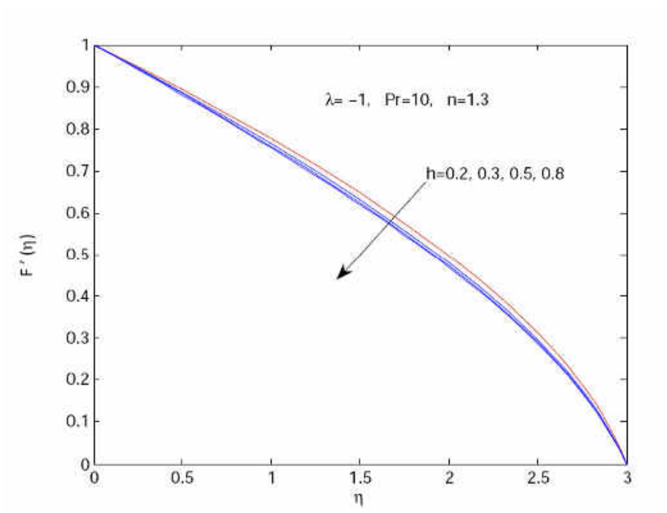

Fig. 3. Dimensionless velocity profile for various *h*

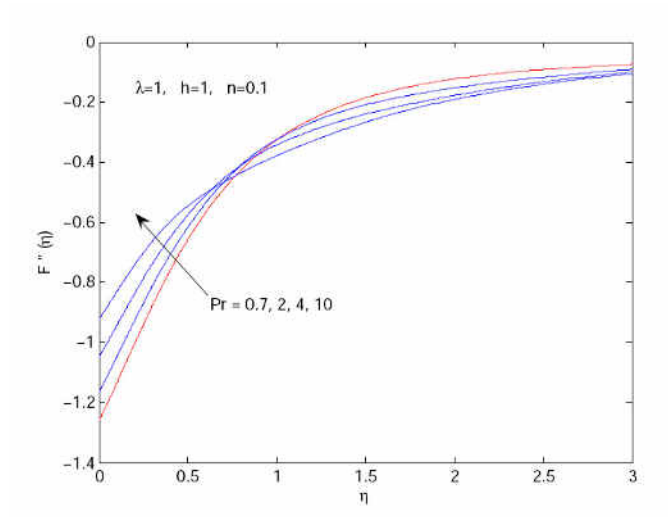

Fig. 4. Dimensionless slope of velocity profile for various *Pr*

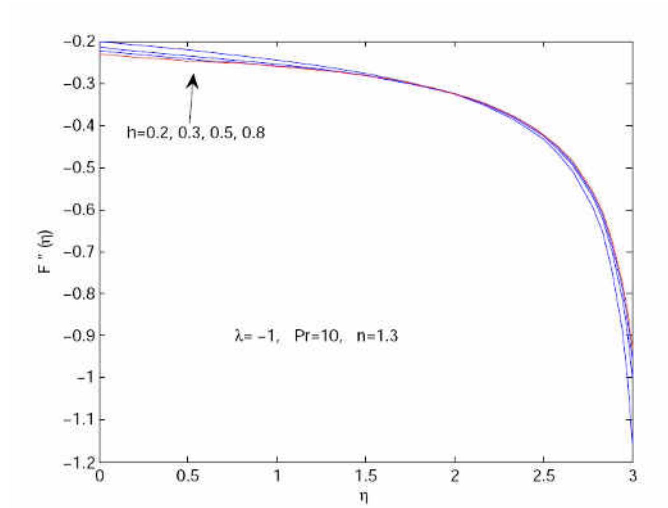

Fig. 5. Dimensionless slope of velocity profile for various *h*

VI. RESULTS ANS DISCUSSIONS

- Figs. 2, 3 are plotted for the horizontal velocity profiles $F(\eta)$ for different values of Pr and h for shear thinning fluids (n = 0.1), and shear thickening fluids (n = 1.3) respectively, controlling other fluid parameters. Both the cases show the trends to decrease the velocity under the effect of other flow parameters. Also Figs. 4,5 provide the trends for slope of horizontal velocity profile. In virtue of these profiles it is observed that the local skin friction coefficient depending $f''(0)$ increases under the effect of various flow parameters.
- In Figs. 6, 7, 8 temperature profiles $G(\eta)$ are plotted for different values of the physical parameters. Fig. 6 is plotted for the effect of Pr on temperature. Increase in Pr leads to decrease the temperature that leads to decrease in the thickness of the thermal boundary layer. This is consistent with the physical situation that thermal boundary layer thickness decreases as Pr increases in buoyancy assisting forces. Fig. 7 depicts the temperature profiles for different values of h for shear thickening fluid (dilatant). Fig 8 differed the temperature profiles for buoyancy assisting forces ($\lambda > 0$) from the buoyancy opposing forces ($\lambda > 0$).

VII. CONCLUSION

In the present paper, first time we have derived a general group of transformations using the deductive group symmetry method - a typical case of lie group method for a particular boundary layer problem. The governing system of PDEs transformed into the system of ODE subject to the similarity requirement, by employing the derived transformations. Numerical solutions are produced by Runge-Kutta fourth order scheme using Matlab computational algorithm.

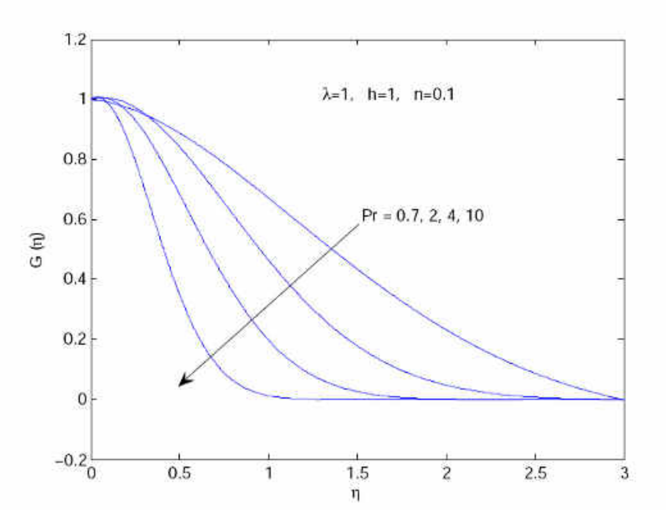

Fig. 6. Dimensionless temperature profile for various *Pr*

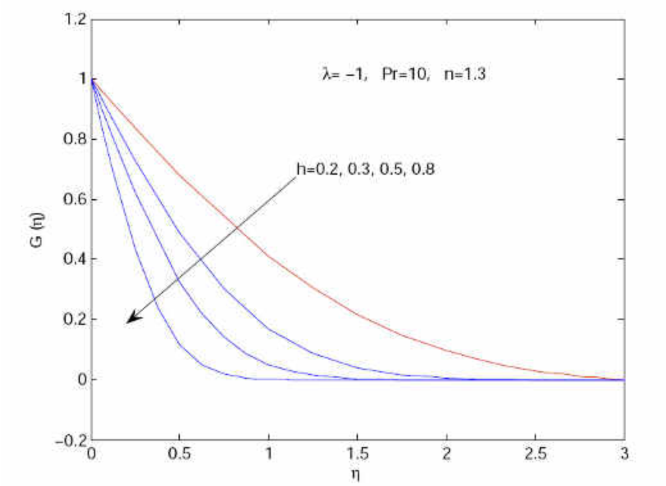

Fig. 7. Dimensionless temperature profile for various *h*

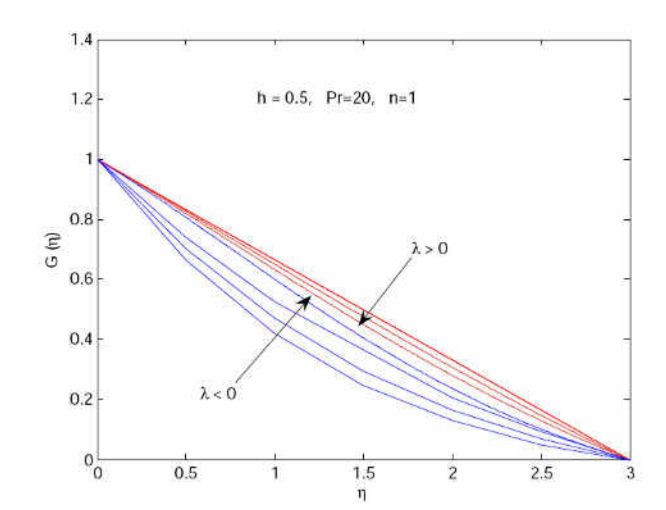

Fig. 8. Dimensionless temperature profile for various λ

REFERENCES

[1] C. G. Bluman and S. Kummai, *Symmetries and Differential Equations*, Springer-Verlag, New-York, 1989.

[2] A. D. Polyanin and A. I. Zhurov, "Exact solutions of non-linear differential-difference equations of a viscous fluid with finite relaxation time," *Int. J. of Non-linear Mech.* , vol. 57, pp. 116–122, 2013.

[3] A. G. Ramgadia and A. K. Saha, "Three-dimensional numerical study of turbulent flow and heat transfer in a wavy-walled duct," *Int. J. of Heat and Mass Transfer.* , vol. 67, pp. 98–117, 2013.

[4] I.L. Freire and M. Torrisi, "Symmetry methods in mathematical modeling of Aedes aegypti dispersal dynamics," *Nonlinear Analysis: Real World Applications*, vol. 14, no. 3, pp. 1300–1707, 2013.

[5] R. M. Darji and M. G. Timol, "Deductive group theoritic analysis for MHD flow of a sisko fluid in a porous medium," *Int. J. Appl. Math. and Mech.*, vol. 7, no. 19, pp. 49–58, 2011.

[6] R. M. Darji and M. G. Timol, "Deductive group symmetry analysis for a free convective boundary-layer flow of electrically conducting non-Newtonian fluids over a vertical porous-elastic surface," *Int. J. of Adv. in Aply. Math. and Mech.*, vo1. 1, no.1, pp. 1–16, 2013.

[7] R. M. Darji and M. G. Timol, "Invariance analysis and similarity solution of heat transfer for MHD viscous power-law fluid over a non-linear porous surface via group theory," *Int. J. of Adv. in Aply. Math. and Mech.*, vo1. 1, no.2, pp. 116–132, 2013.

[8] A. K. Al-Salihi , A. H. Hasmani and M. G. Timol, "The new systematic procedure in group theoretic methods with applications to class of boundary layer flows," *Int. J. of Adv. in Aply. Math. and Mech.*, vo1. 1, no.1, pp. 47–60, 2013.

[9] L. J. Crane, "Flow past a stretching plate," *ZAMP*, vol. 21, pp. 645–647, 1970.

[10] N.V. Chemetov and F. Cipriano, "Boundary layer problem: NavierStokes equations and Euler equations," *Nonlinear Analysis: Real World Applications*, vol. 14, no. 6, pp. 2091–2104, 2013.

[11] B. Mayil Vaganan, T. Shanmuga Priya, "Exact analytic, regular perturbation and numerical solutions of symmetry reductions of a (2+1) -dimensional KdVBurgers equation," *Nonlinear Analysis: Real World Applications*, vol. 14, no. 3, pp. 1265–1275, 2013.

[12] B. Vujanovic, A. M. Stauss and D. J. Djukiv, "A variational solution of the Rayleigh problem for power law non-Newtonian conducting fluid," *Ing-Arch.*, vol. 41, pp. 381–386, 1972.

[13] K. V. Prasad, P. S. Datti and K. Vajravelu, "Hydromagnetic flow and heat transfer of a non-Newtonian power law fluid over a vertical stretching sheet," *Int. J. of Heat and Mass Transfer*, vol. 53, pp. 879–888, 2010.

[14] K. V. Prasad, K. Vajravelu and P. S. Datti, "Mixed convection heat transfer over a non-linear stretching surface with variable fluid properties," *Int. J. of Non-Linear Mech.*, vol. 45, pp. 320–330, 2010.

[15] Z. Kebiche, C. Castelain and T. Burghelea, "Experimental investigation of the Rayleigh-Bénard convection in a yield stress fluid," *Journal of Non-Newtonian Fluid Mechanics* , vol. 203, pp. 9–23, 2014.

[16] H. I. Andersson, K. H. Bech and B. S. Dandapat, "Magnetohydrodynamic flow of a power-law fluid over a stretching sheet," *Int. J. Non-Linear Mech.*, vol. 27, pp. 929-936, 1992.

[17] R. Arora, M. J. Siddiqui and V.P. Singh, "Similarity method for imploding strong shocks in a non-ideal relaxing gas," *Int. J. of Non-linear Mech.* , vol. 57, pp. 1–9, 2013.

[18] F. Garoosi, G. Bagheri and F. Talebi, "Numerical simulation of natural convection of nanofluids in a square cavity with several pairs of heaters and coolers (HACs) inside," *Int. J. of Heat and Mass Transfer.* , vol. 67, pp. 362-376, 2013.

[19] S.A. Patel and R.P. Chhabra, Steady flow of Bingham plastic fluids past an elliptical cylinder, *Journal of Non-Newtonian Fluid Mechanics* , vol. 202, pp. 32-53, 2013.

[20] M. J. Moran and R. A. Gaggioli, "Reduction of the number of variables in systems of partial differential equations with auxiliary conditions," *Int. J. of Non-Linear SIAM J. Appl. Math.*, vol. 16, pp. 202–215, 1968.

[21] A. J. A. Morgan, "The reduction by one of the number of independent variables in some systems of partial differential equations," *Quart. J. Math.*, vol. 3, pp. 250–259, 1952.

[22] M. Narayana, S.N. Gaikwad, P. Sibanda and R.B. Malge, "Double diffusive magneto-convection in viscoelastic fluids," *Int. J. of Heat and Mass Transfer*, vol. 67, pp. 194–201, 2013.