# An Effective Algorithm to Solve Assignment Problems: Opportunity Cost Approach

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***Abstract*—The purpose of this paper is to propose an effective algorithm for finding the optimal solution of an Assignment Problem to reduce computational cost. We also provide examples to illustrate the proposed algorithm.**

***Index Terms*—Assignment Problem, Hungarian Algorithm, Linear Programming.**

***MSC 2010 Codes* —90C08, 90C10, 90C05.**

## I. INTRODUCTION

THE optimization problem of minimizing total cost in assigning (giving) available jobs to different men/machines in an organization/manufacturing units under the condition that one job is given to one machine and one machine has to take only one job, is generally referred to as an Assignment Problem.

Several important problems such as shortest path, weighted matching, transportation and minimum cost flow and problems in Network Flow Theory can be reduced to Assignment Problem (in short A.P.) and A.P. is a suitable testing ground for new computational ideas in solving many more general problems. The first specialized method to solve assignment problem is Kuhn's Hungarian Method (see [1]). For detailed description of Hungarian Method, one can refer some of the standard textbooks on Operations Research (For instance, see [2]). In [3], Hadi Basirzadeh introduced a new approach to solve A.P. namely, Ones Assignment Method for solving a wide range of such problems. The basic idea of this method is to create some ones in the assignment matrix and then by finding a complete assignment to these ones. The main drawback of Ones Assignment Method is that it cannot be applied to A.P. with assignment cost entry is zero and in certain cases the method does not yield optimal solution (see [4]). Subsequently, Ones Assignment Method was improved and revised by some authors to overcome the existing drawbacks (see [5], [6], and [7]). Though, this revised Ones Assignment methods are substantially different from that of Kuhn's Hungarian method, the number of computations to find optimal solution to an A.P. is not really reduced. Several other alternative and innovative methods to solve A.P. have been proposed in recent years, few of them are simple to implement compared with Hungarian Method (see [8] [9] and [10]). The purpose of this paper is to propose an effective algorithm to find the optimal solution of an assignment problem aiming to reduce computational cost in the existing methods.

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## II. MATHEMATICAL PRELIMINARIES AND THE PROPOSED ALGORITHM

In this section, we recall few basic concepts of A.P. and we propose an algorithm to find the optimal solution of an Assignment problem.

Let $C_{ij}$ be the Assignment cost associated with $i^{th}$ job to $j^{th}$ machine and

$$x_{ij} = \begin{cases} 1 \text{ if } ith \text{ job is given to } jth \text{ machine} \\ 0 \text{ otherwise} \end{cases}$$

Then one can represent the given A.P. in the form of a matrix with entries $C_{ij}$ (also known as Cost Effectiveness Matrix (in short CEM)) in which $i^{th}$ row corresponds to $i^{th}$ job and $j^{th}$ column to $j^{th}$ machine. Further, we say that A.P. is balanced if number of rows in CEM is equal to number of columns, otherwise it is called unbalanced. The unbalanced A.P. can be converted into balanced one by introducing the sufficient number of dummy jobs or machines with zero assignment cost in those rows and columns of CEM.

**The Proposed Algorithm**

**Step1:** Construct CEM for a given Assignment Problem. First we balance CEM if it is not. Let the balanced CEM be A with n rows and n columns.

**Step2:** If the given AP is a maximization problem, then we first convert it into a minimization problem by subtracting all the cost values of the cost matrix from the highest cost value in that matrix.

**Step3:** Find the difference between the smallest and next smallest value in each row (we call this difference, the Row Opportunity Cost (in short ROC)) and write them against the corresponding row.

**Step4:** Identify the row with largest opportunity cost. The tie in largest opportunity cost can be resolved by choosing a row with least cost. Now, choose the cell with smallest cost in that selected row as an assigned cell. If $C_{ij}$ be the assigned cell in this step, then delete $i^{th}$ row and $j^{th}$ column so that the new CEM is obtained with (n-r) rows and (n-r) columns after $r^{th}$ iteration (we call this new CEM, the Reduced CEM).

**Step5:** Repeat Step3 and Step4 to the Reduced CEM obtained in the previous iteration until all jobs are assigned uniquely.

## III. Numerical Illustrations

In this section, we provide numerical examples to illustrate the proposed algorithm.

**Example 3.1:** The time in minutes required by the machines M1, M2, M3, and M4 to complete the Jobs J1, J2, J3 and J4 are given in the following table.

| Machines | Jobs | | | |
|---|---|---|---|---|
| | J1 | J2 | J3 | J4 |
| M1 | 3 | 4 | 7 | 6 |
| M2 | 4 | 6 | 8 | 9 |
| M3 | 5 | 7 | 4 | 6 |
| M4 | 7 | 9 | 10 | 3 |

Given CEM is a 4 x 4 matrix. Therefore, the process stops when given four jobs are assigned uniquely. We now calculate ROC as follows:

| Machines | Jobs | | | | Row Opportunity Cost |
|---|---|---|---|---|---|
| | J1 | J2 | J3 | J4 | |
| M1 | 3 | 4 | 7 | 6 | (1) |
| M2 | 4 | 6 | 8 | 9 | (2) |
| M3 | 5 | 7 | 4 | 6 | (1) |
| M4 | 7 | 9 | 10 | 3 | (4) |

The maximum Row Opportunity Cost corresponds to fourth row and the minimum cost in this row is $C_{44}$. So, we assign machine M4 to job J4 and delete fourth row and fourth column. Repeating the Step2 and Step3 in the algorithm to the following reduced matrix

| Machines | Jobs | | | Row Opportunity Cost |
|---|---|---|---|---|
| | J1 | J2 | J3 | |
| M1 | 3 | 4 | 7 | (1) |
| M2 | 4 | 6 | 8 | (2) |
| M3 | 5 | 7 | 4 | (3) |

The maximum Row Opportunity Cost in this iteration corresponds to third row and the minimum cost in this row is $C_{33}$. So assign machine M3 to job J3 and delete the corresponding row and column. The reduce matrix is

| Machines | Jobs | | ROC |
|---|---|---|---|
| | J1 | J2 | |
| M1 | 3 | 4 | (1) |
| M2 | 4 | 6 | (2) |

The maximum Row Opportunity cost is 2 and the minimum cost cell in this row corresponds to

(M2, J1). Finally, we are left with only one possibility of assigning M1 to J2. Hence, the optimal solution is achieved and is as follows

| Machines | Jobs | Time |
|---|---|---|
| M1 | J2 | 4 |
| M2 | J1 | 4 |
| M3 | J3 | 4 |
| M4 | J4 | 3 |

The optimal time taken to complete all the jobs is 4 + 4 + 4 + 3 =15.

**Remark 3.2:** One can verify that the optimal solution obtained in Example 3.1 using the proposed method is same that of one obtained by Hungarian Method. However, the proposed method is simple and easy to implement.

**Notation:** While applying the proposed method to solve A.P we will indicate the reduced rows and columns by (--) against them and assigned cells with square brackets [ ].

The following example illustrates the proposed method to solve the unbalanced Assignment Problem.

**Example3.3:** Consider the following unbalanced A.P. with 4 x 3 CEM. The problem is to find the optimal assignment to the machines so that time taken to complete all the jobs is minimized.

| Machines | Jobs | | |
|---|---|---|---|
| | J1 | J2 | J3 |
| M1 | 21 | 14 | 7 |
| M2 | 15 | 10 | 5 |
| M3 | 15 | 10 | 5 |
| M4 | 12 | 8 | 4 |

First we balance the given AP by introducing a dummy column (Job J4) with zero cost values; thereafter we apply the proposed method as follows.

| Machines | Jobs | | | | Row Opportunity Cost | | |
|---|---|---|---|---|---|---|---|
| | J1 | J2 | J3 | J4 | 1st | 2nd | 3rd |
| M1 | 21 | 14 | 7 | [0] | (7) | (--) | (--) |
| M2 | 15 | 10 | [5] | 0 | (5) | (--) | (--) |
| M3 | 15 | [10] | 5 | 0 | (5) | (5) | (--) |
| M4 | [12] | 8 | 4 | 0 | (4) | (4) | (--) |
| Deleted Columns | | | | (--) | | | |
| | | | (--) | | | | |
| | | (--) | | | | | |

The optimal solution is as follows:

| Machines | Jobs | Time |
|---|---|---|
| M1 | J4 | 0 |
| M2 | J3 | 5 |
| M3 | J2 | 10 |
| M4 | J1 | 12 |

The optimal time taken to complete all the jobs is 5 + 10 + 12 =27. One can easily see that there is a tie in the opportunity cost of first iteration and we can select either second row or third row as the minimum assignment cost in those rows is same. If we select third row instead of second row in this iteration, then an alternative assignment plan is obtained and there will be no change in optimal time of completion.

## IV. Conclusions

Finally, we conclude our paper with the following remarks:

1. One can also find the optimal solution to find A.P. using Column-Opportunity cost instead of Row Opportunity cost approach.
2. The proposed algorithm is systematic than the existing methods introduced by Aderinto Y.O et al. and M.S. Gaglani [8, 9].
3. The method proposed by A. Thiruppathi et al. [10] demands to compute both row as well as column opportunity cost values to decide an assignment in a particular iteration, whereas the proposed method herein requires to compute either row opportunity or column opportunity values but not both.

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