An Interval Solid Transportation Problem with Vehicle Cost, Fixed Charge and Budget

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***Abstract*—In this paper, we made an investigation on interval solid transportation problem (STP).A capacitated, interval valued solid transportation problem with interval nature of resources, demands, capacity of conveyance and costs. Here transportation cost inversely varies with the quantity-to be transported from plants to warehouses in addition to a fixed per unit cost and a small vehicle cost. We develop this paper under unit transportation cost, resources, demands, conveyances capacities etc. as interval numbers. To erect this manuscript, an additional constraint is imposed on the total budget at each goal. We employ two interval approaches as Hu and Wang's and Mahato and Bhunia's approaches. The interval transportation model is converted into its crisp equivalent by using Weighted Tchebycheff method and also the model is illustrated with numerical example using the LINGO.13 optimization software.**

***Index Terms*—STP, Hu and Wang's approaches, Mahato and Bhunia's approaches, vehicle cost, fixed charge, weighted Tchebycheff method.**

***MSC 2010 Codes* – 90C05, 90B06**

I. Introduction

As real numbers have an associated arithmetic and mathematical analysis, interval numbers have a dissimilar interval arithmetic and interval analysis investigated by Jaulin et. al., Kearfott et. al., Moore et. al. [7], [8], [9], [10]. A new order relation and its application to vague information have been given by Hu and Wang [3]. The intervals are diagrammatically described by Kulpa [13], [14], and, applying this notion, he explained not only the interval arithmetic but also introduced a new pattern for learning the interval order relations. It has been studied that the definitions given in [9] are not applicable for all pairs of intervals. In comparison to that defined in [9], Ishibuchi and Tanaka [6] proposed some improved definitions of interval order relations. Chanas et al. [11] tried to generalize the work of Ishibuchi and Tanaka with the concept of t_0,t_1-cut of the intervals and Hu and Wang [3] proposed a modified version of intervals order relation. According to the decision makers' point of view a customized version of Ishibuchi and Tanaka's [6] was proposed by Mahato and Bhunia [12] on interval ranking definition. Hitchcock [5] was originally developed the transportation problem (TP). Solid transportation problem (STP) was stated by Schell [4] which is the extension of TP. The fixed charge problem was initialized by Hirsch and Dantzig [16] in 1968, and it has been widely applied in decision-making and optimization problems now and Ojha et. al.[15] solve a solid transportation problem with fixed charge and vehicle cost. Recently A. Baidya et. al. [1], [2] solves two problems on safety factor in solid transportation problem. Here using 'Hu and Wang' and 'Mahato and Bhunia' approaches in order relation of interval we investigate a solution of interval valued solid transportation problem with vehicle cost, fixed charge and budget at each destination. In this manuscript we consider all the transportation parameters as unit transportation cost, supplies, demands and conveyances capacities are interval in nature. The multi objective transportation problem are reduced to single objective transportation problem using weighted Tchebycheff method. Also applying gradient based optimization-generalized reduced gradient (GRG) method we obtain the optimal solution of our respective model. The model is illustrated with specific numerical data and two approaches are compared.

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II. Some Preliminaries

In this section, some definitions and preliminary results are given which will be used in this paper.

Definition 2.1 An interval number A is defined as

$$A = [a_L, a_R] = \{x : a_L \leq x \leq a_R, x \in \Re\}$$

Here $a_L, a_R \in \Re$ are the lower and upper bounds of the interval A, respectively. □

Definition 2.2 Let, $A = [a_L, a_R]$ and $B = [b_L, b_R]$ are two intervals. Here, we shall give basic formulas of interval mathematics as follows:

Addition: $A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R]$

Subtraction: $A - B = [a_L, a_R] - [b_L, b_R] = [a_L - b_R, a_R - b_L]$

Scalar Multiplication:

$$kA = k[a_L, a_R] = \begin{cases} [ka_L, ka_R] & \text{if} \quad k \geq 0 \\ [ka_R, ka_L] & \text{if} \quad k \leq 0 \end{cases}$$

□

Order relation of intervals: Let $A = [a_L, a_R]$ and $B = [b_L, b_R]$ be a pair of arbitrary interval. These can be classified as follows:

Type-I: Non-overlapping intervals;

Type-II: Partially overlapping intervals;

Type-III: Completely overlapping intervals;

General Interval Ranking Definitions:

Hu and Wang's Approach: Initiating new approaches, they have tried to fulfill the shortcomings of the previous definitions. Their interval ranking relation " $\prec=$ " is defined as follows:

Definition 2.3 For any two intervals $A = [a_L, a_R] = \langle a_M, a_W\rangle$ and $B = [b_L, b_R] = \langle b_M, b_W\rangle$,

$$A \prec= B \text{ if an only if } \begin{cases} a_M < b_M & \text{for} \quad a_M \neq b_M \\ a_W \geq b_W & \text{for} \quad a_W \neq b_W \end{cases}$$

Furthermore $A \prec B$ if and only if $A \prec= B$ and $A \neq B$. □

Mahato and Bhunia's Approach: Let $A = [a_L, a_R] = \langle a_M, a_W\rangle$ and $B = [b_L, b_R] = \langle b_M, b_W\rangle$ be two intervals costs/times for the minimization problems.

Optimistic decision-making:

Definition 2.4 For the minimization problems, the order relation " $\leq_{omin}$ " between the intervals A and B is

$$A \leq_{omin} B \text{ if and only if } a_L \leq b_L,$$

$$A <_{omin} B \text{ if and only if} A <_{omin} B \text{ and } A \neq B.$$

□

Pessimistic decision-making: In this case, the decision maker determines the minimum cost/time for minimization problems according to the principle "Less uncertainty is better than more uncertainty".

Definition 2.5 For minimization problems, the order relation $<_{pmin}$ between the interval $A = [a_L, a_R] = \langle a_M, a_W\rangle$ and $B = [b_L, b_R] = \langle b_M, b_W\rangle$ for pessimistic decision making are

- $A <_{pmin} B$ if and only if $a_M < b_M$, for Type-I and Type-II intervals,
- $A <_{pmin} B$ if and only if $a_M \leq b_M$ and $a_W < b_W$, for Type-III intervals.

□

However, for Type-III intervals with $a_M < b_M$ and $a_W > b_W$, pessimistic decisions cannot be determined. In this case, the optimistic decision is to be considered.

Definition 2.6 Let $X = \Re$ and suppose that $\xi = \{\xi_1, \xi_2 \dots \xi_n\}$ and $\eta = \{\eta_1, \eta_2 \dots \eta_n\}$ be any two points in $\Re^n$. Define the mapping $d_p : X \times X \to \Re^n$ and $d_\infty : X \times X \to \Re^n$ as follows:

$$d_p(\xi, \eta) = \left\{ \sum_{i=1}^{n} |\xi_i - \eta_i|^p \right\}^{\frac{1}{2}}$$

where $1 \leq p < \infty$ and $d_\infty(\xi, \eta) = \text{Max}_{1 \leq i \leq n}\{|\xi_i - \eta_i|\}$ Then d_p, d_∞ are metrics on the same set $X = \Re^n$.□

Definition 2.7 Let $X = l_p, 1 \leq p < \infty$, be the set of all sequences $\xi = \{\xi_i\}$ of real scalars such that $\sum_{i=1}^{n} |\xi_i|^p < \infty$. Define the mapping $d : X \times X \to \Re$ by

$$d(\xi, \eta) = \left\{ \sum_{i=1}^{n} |\xi_i - \eta_i|^p \right\}^{\frac{1}{p}}$$

where $\xi = \{\xi_i\}$ and $\eta = \{\eta_i\}$ are in l_p.

III. Weighted Tchebycheff Method

When $p \to \infty$, and $w_i \geq 0$, the d_p metric is called a Tchebycheff metric and the corresponding problem (called weighted Tchebycheff problem) with interval objectives is of the form:

$$\text{Minimize } \text{Max}_{i=1,2,\dots k}(w_i|A_i(x) - Z_i^*|)$$

$$\text{Subject to} \quad \sum_{i=1}^{k} w_i = 1 \text{ and } x \in S.$$

IV. Assumption and Notations

- ⋆ $[C_{ijkL}, C_{ijkR}]$= interval valued unit transportation cost from i-th origin to j-th destination by k-th conveyance.
- ⋆ $[Z_L, Z_R]$= interval valued objective function.
- ⋆ x_{ijk}= unknown quantity which is to be transported form i-th origin to j-th destination by k-th conveyance.
- ⋆ $a_i = [a_{iL}, a_{iR}]$= interval availability at i-th origin.
- ⋆ $b_j = [b_{jL}, b_{jR}]$= interval requirement at the j-th destination.
- ⋆ $e_k = [e_{kL}, e_{kR}]$= interval conveyances capacity of the k-th conveyance.
- ⋆ $B_j = [B_{jL}, B_{jR}]$= interval budget at the j-th destination.
- ⋆ $[f_{Lijk}, f_{Rijk}]$= the interval fixed charge for transportation of any amount from i-th source to j-th destination via k-th conveyance.

V. Model Formulation

5.1 Model: Formulation of interval Valued STP with budget, vehicle cost, fixed charge and interval unit transportation costs, demands, availabilities and conveyance capacities:

$$\begin{aligned}
\text{Min } Z &= \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}[C_{ijkL}, C_{ijkR}]x_{ijk} + [F_L(x_{ijk}), F_R(x_{ijk})] \\
&\quad + [f_{Lijk}, f_{Rijk}] \\
&= \Bigg[\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(C_{ijkL}x_{ijk} + F_L(x_{ijk}) + f_{Lijk}), \\
&\quad \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(C_{ijkR}x_{ijk} + F_R(x_{ijk}) + f_{Rijk})\Bigg] \\
&= [Z_L, Z_R]
\end{aligned}$$

Subject to the constraints,

$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} = a_i = [a_{iL}, a_{iR}], i = 1, 2, \dots, M; \quad (1)$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} = b_j = [b_{jL}, b_{jR}], j = 1, 2, \dots, N; \quad (2)$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} = e_k = [e_{kL}, e_{kR}], k = 1, 2, \dots, K; \quad (3)$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K}[c_{ijkL}, c_{ijkR}]x_{ijk} \leq B_j = [B_{jL}, B_{jR}], j = 1, 2, \ldots, N; \quad (4)$$

$x_{ijk} \geq 0, i = 1, 2, \ldots, M; j = 1, 2, \ldots, N; k = 1, 2, \ldots, K.$
Where, $[F_L(x_{ijk}), F_R(x_{ijk})]$ be the interval vehicle carrying cost for the quantity x_{ijk} from i-th source O_i to j-th destination D_j via k-th conveyance is defined as

$$F(x_{ijk}) = \begin{cases} m.v & \text{if } m.v_c = x_{ijk} \\ (m+1).v & \text{otherwise} \end{cases}$$

$m = \frac{x_{ijk}}{v_c}, v_c$ =Interval Vehicle Capacity and v=Interval Vehicle cost.
The problem is feasible if and only if $A \cap B \cap E \neq \phi$, where

$$A = \sum_{i=1}^{M} a_i = \left[\sum_{i=1}^{M} a_{iL}, \sum_{i=1}^{M} a_{iR}\right]$$

$$B = \sum_{j=1}^{N} b_j = \left[\sum_{j=1}^{N} b_{jL}, \sum_{j=1}^{N} b_{jR}\right]$$

$$E = \sum_{k=1}^{K} e_k = \left[\sum_{k=1}^{K} e_{kL}, \sum_{k=1}^{K} a_{kR}\right]$$

5.2 Crisp Transformation of the objective function of the above model: Since it is desired to obtain an optimal interval value for the above model, we may minimize two characteristic of objective function, the left objective interval, Z_L, and its right, Z_R.

$$Z_L = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(C_{ijkL}x_{ijk} + F_L(x_{ijk}) + f_{Lijk}$$

$$Z_R = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(C_{ijkR}x_{ijk} + F_R(x_{ijk}) + f_{Rijk}$$

5.3 Crisp transformation of the constraints of the above model using Hu and Wang's approach on interval order relation: The crisp conversion of the constraints (1), (2), (3) and (4) of the above model using Hu and Wang's approach we have is as follows:

$$a_{iL} \leq \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq a_{iR} \quad (5)$$

$$b_{jL} \leq \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \leq b_{jR} \quad (6)$$

$$e_{kL} \leq \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq e_{kR} \quad (7)$$

$$\text{and } \sum_{i=1}^{M}\sum_{k=1}^{K} \frac{(C_{ijkR} + C_{ijkL})}{2} x_{ijk} \leq \frac{(B_{jR} + B_{jL})}{2} \quad (8)$$

respectively.
5.4 Crisp transformation of the constraints of the above model using Mahato and Bhunia's Approach on interval order relation: Using Mahato and Bhunia's Approach we have the following crisp conversion of the constraints (1), (2), (3) and (4) of the above model:
(5), (6), (7) and

$$\sum_{i=1}^{M}\sum_{k=1}^{K} C_{ijkL}x_{ijk} \leq B_{jL} \quad (9)$$

respectively.

VI. Numerical Illustration

Identical product is produced in two industries and sent to two different warehouses using two different modes of conveyances. The unit transportation cost, fixed charge, supply, demand, conveyance capacity, budget at destination and vehicle cost are all interval in nature and are given below:
Input Data:
Interval available source:

$$a_1 = [a_{1L}, a_{1R}] = [32, 48]; a_2 = [a_{2L}, a_{2R}] = [28, 44];$$

Interval required demand:

$$b_1 = [b_{1L}, b_{1R}] = [28, 45]; b_2 = [b_{2L}, b_{2R}] = [31, 45];$$

Interval conveyance capacities:

$$e_1 = [e_{1L}, e_{1R}] = [27, 44]; e_2 = [e_{2L}, e_{2R}] = [39, 47];$$

Interval budget for destination:

$$B_1 = [B_{1L}, B_{1R}] = [490, 497], B_2 = [B_{2L}, B_{2R}] = [501, 515];$$

Interval unit transportation cost:

$$[C_{111L}, C_{111R}] = [12, 13]; [C_{211L}, C_{211R}] = [13, 17];$$
$$[C_{121L}, C_{121R}] = [15, 19]; [C_{221L}, C_{221R}] = [14, 17];$$
$$[C_{112L}, C_{112R}] = [11, 13]; [C_{212L}, C_{212R}] = [12, 14];$$
$$[C_{122L}, C_{122R}] = [13, 15]; [C_{222L}, C_{222R}] = [15, 17];$$

Interval fixed charge for transportation:

$$[f_{111L}, f_{111R}] = [1.2, 1.3]; [f_{211L}, f_{211R}] = [1.3, 1.7];$$
$$[f_{121L}, f_{121R}] = [1.5, 1.9]; [f_{221L}, f_{221R}] = [1.4, 1.7];$$
$$[f_{112L}, f_{112R}] = [1.1, 1.3]; [f_{212L}, f_{212R}] = [1.2, 1.4];$$
$$[f_{122L}, f_{122R}] = [1.3, 1.5]; [f_{222L}, f_{222R}] = [1.5, 1.7];$$

Input for vehicle:

$$v_c = 7, v = 5.$$

We consider a practical example of a transport company where the possible values of the parameters such as the unit transportation costs, supplies, demands, conveyance capacities and budget precisely determined. These linguistic data can be transferred into interval numbers. With the above input data we have the following form of our given model:

$$Z_L = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}(C_{ijkL}x_{ijk} + F_L(x_{ijk}) + f_{Lijk})$$

$$Z_R = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}(C_{ijkR}x_{ijk} + F_R(x_{ijk}) + f_{Rijk})$$

Subject to the constraints

$$a_{1L} \leq x_{111} + x_{121} + x_{112} + x_{122} \leq a_{1R},$$

$$a_{2L} \leq x_{211} + x_{221} + x_{212} + x_{222} \leq a_{2R},$$

$$b_{1L} \leq x_{111} + x_{112} + x_{211} + x_{212} \leq b_{1R},$$

$$b_{2L} \leq x_{121} + x_{122} + x_{221} + x_{222} \leq b_{2R},$$

$$e_{1L} \leq x_{111} + x_{121} + x_{211} + x_{221} \leq e_{1R},$$

$$e_{2L} \leq x_{112} + x_{122} + x_{212} + x_{222} \leq e_{2R},$$

Using **Hu and Wang's approaches** we have the following crisp form of the budget constraint:

$$\frac{(C_{111R} + C_{111L})}{2} x_{111} + \frac{(C_{112R} + C_{112L})}{2} x_{112} + \frac{(C_{211R} + C_{211L})}{2} x_{211} + \frac{(C_{212R} + C_{212L})}{2} x_{212} \leq \frac{(B_{1R} + B_{1L})}{2},$$

$$\frac{(C_{121R} + C_{121L})}{2} x_{121} + \frac{(C_{122R} + C_{122L})}{2} x_{122} + \frac{(C_{221R} + C_{221L})}{2} x_{221} + \frac{(C_{222R} + C_{222L})}{2} x_{222} \leq \frac{(B_{2R} + B_{2L})}{2}.$$

Using **Mahato and Bhunia's approaches** we have the following crisp form of the budget constraint:

$$C_{111L} x_{111} + C_{112L} x_{112} + C_{211L} x_{211} + C_{212L} x_{212} \leq B_{1L},$$

$$C_{121L} x_{121} + C_{122L} x_{122} + C_{221L} x_{221} + C_{222L} x_{222} \leq B_{2L}.$$

The problem is feasible since,

$$A \cap B \cap E = [60, 92] \cap [59, 90] \cap [66, 91] = [66, 90] \neq 0.$$

VII. Results:

The above constrained optimization problems are executed using LINGO 13.0 and the results of Model are presented below:

7.1 Optimal result of the model under Hu and Wang's approach:

The optimal compromise solution of the respective model with Hu and Wang's approach using Weighted Tchebycheff Method is as follows:
$x_{111} = 30.94, x_{112} = 1.2, x_{212} = 6.97, x_{222} = 32.10$ and remaining all are zero.
The interval total transportation cost is,

$$[Z_L, Z_R] = [1001.83, 1120.71]$$

7.2 Optimal result of the model under Mahato and Bhunia's approach:

The required optimal solution of the relevant model using Weighted Tchebycheff Method is as follows:
$x_{111} = 27.1, x_{211} = 9.0, x_{112} = 6.12, x_{222} = 31.99$ and remaining all are zero.
The interval total transportation cost is,

$$[Z_L, Z_R] = [1043.69, 1190.75].$$

VIII. Discussion

In this paper, we solve a STP with interval vehicle cost, fixed charge, budget, unit transportation cost, demands, supplies, conveyance capacities. After careful investigation we see that the interval total transportation cost with Hu and Wang's approach is minimum that of Mahato and Bhunia's approach i.e., we get the optimal transportation cost of our respective model with Hu and Wang's approach. Thus we conclude that the Hu and Wang's approach is very useful that of Mahato and Bhunia's approach to solve an interval valued solid transportation problem with budget constraint, vehicle cost and fixed charge.

IX. Conclusion

In this paper, we formulate and solve a solid transportation problem with interval transportation parameters and we put a comparison between the solution obtain by two different interval approaches ('Hu and Wang's approach' and 'Mahato and Bhunia's approach') using weighted Tchebycheff method. In the present study mathematical problems were solved using LINGO 13.0 Software. In framework with genuine field problem, the technique could be used as very effectual and promising and in view of a practical significance.

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