A Three Species Ecological Model with a Prey, Predator and a Competitor to both the Prey and Predator

A.V. Papa Rao, K. Lakshmi Narayan and S. Bathul

***Abstract*—The** present paper is devoted to an analytical investigation of three species ecological model with a Prey (N_1), a predator (N_2) and a competitor (N_3) to the both prey (N_1) and Predator (N_2). In addition to that, the species are provided with alternative food. The model is characterized by a set of first order non-linear ordinary differential equations. All the eight equilibrium points of the model are identified and the local and global stability of the interior equilibrium state is discussed. Further exact solutions of linearized system of equations have been derived. The stability analysis is supported by Numerical simulation using Matlab.

***Index Terms*—Prey, Predator, Competitor, Equilibrium points, Normal Steady State, Local stability, Global stability.**

***MSC 2010 Codes* — 93A30, 00A71.**

I. Introduction

Ecology relates to the study of living beings in relation to their living styles. Research in the area of theoretical ecology was initiated by Lotka [1] and by Volterra [2].Since then many mathematicians and ecologists contributed to the growth of this area of knowledge as reported in the treatises of Paul Colinvaux [3], Freedman [4], Kapur [5], [6] etc. Recently Archana Reddy [7] discussed the stability analysis of two interacting species with harvesting of both species.

Lakshmi Narayan and Pattabhiramacharyulu [8], [9] and Shiva Reddy [10], [11] et al., Ravindra Reddy [12],[13],[14] et al. Shanker [15] et al and Papa Rao [16] have discussed different prey-predator models in detail. Inspired from that, we discussed a more general three species model. The model is characterized by a set of first order ordinary differential equations. All the eight equilibrium points of the model are identified and stability criteria for interior equilibrium states are discussed.

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II. Basic Equations

The model equations for a three species prey-predator and competitor to the prey and predator system is given by the following system of first order ordinary differential equations employing the following notation:

$$\begin{aligned}
\dot{N}_1 &= a_1N_1 - \alpha_{11}N_1^2 - \alpha_{12}N_1N_2 - \alpha_{13}N_1N_3 \\
\dot{N}_2 &= a_2N_2 - \alpha_{22}N^2 + \alpha_{21}N_1N_2 - \alpha_{23}N_3N_2 \\
\dot{N}_3 &= a_3N_3 - \alpha_{33}N_3^2 - \alpha_{31}N_1N_3 - \alpha_{32}N_2N_3
\end{aligned} \tag{2.1}$$

where N_1, N_2 and N_3 are the population of the prey, predator and competitor to the prey and predator with the natural growth rates a_1, a_2 and a_3 respectively. Here, α_{11} is rate of decrease of the prey due to insufficient food and inter species competition, α_{12} is rate of decrease of the prey due to inhibition by the predator, α_{21} is rate of increase of the predator due to successful attacks on the prey, α_{22} is rate of decrease of the predator due to insufficient food other than the prey and inter species competition, α_{23} is rate of decrease of the predator due to the competition with the third species, α_{33} is rate of decrease of the Competitor to the both prey and predator due to insufficient food and inter species competition, α_{32} is rate of decrease of the competitor due to the competition with the predator, α_{13} is rate of decrease of the prey due to the competition with the both prey and predator and α_{31} is rate of decrease of the competitor due to the competition with the prey .

III. Equilibrium States

The system under investigation has 8 equilibrium states. They are:

E_1 : The extinct state

$$\bar{N}_1 = 0, \quad \bar{N}_2 = 0, \quad \bar{N}_3 = 0 \tag{3.1}$$

E_2 : The state in which only the predator survives and the prey and competitor to the prey and predator are extinct

$$\bar{N}_1 = 0, \quad \bar{N}_2 = \frac{a_2}{\alpha_{22}}, \quad \bar{N}_3 = 0 \tag{3.2}$$

E_3 : The state in which both the prey and the predators are extinct and competitor to the prey and predator survive

$$\bar{N}_1 = 0, \quad \bar{N}_2 = 0, \quad \bar{N}_3 = \frac{a_3}{\alpha_{33}} \tag{3.3}$$

E_4 : The state in which both the predator and competitor to the prey and predator are extinct and prey alone survive

$$\bar{N}_1 = \frac{a_1}{\alpha_{11}}, \quad \bar{N}_2 = 0, \quad \bar{N}_3 = 0 \tag{3.4}$$

E_5 : The state in which both the prey and the predators exist and competitors to the prey predator are extinct.

$$\bar{N}_1 = \frac{(a_1\alpha_{22} - a_2\alpha_{12})}{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}}, \quad \bar{N}_2 = \frac{(a_2\alpha_{11} + a_1\alpha_{21})}{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}}, \quad \bar{N}_3 = 0 \tag{3.5}$$

This case arises only when

$$a_1\alpha_{22} > a_2\alpha_{12} \tag{3.6}$$

E_6 : The state in which both prey and competitor to the prey and predator exist and predator extinct

$$\bar{N}_1 = \frac{(a_1\alpha_{33} - a_3\alpha_{13})}{(\alpha_{11}\alpha_{33} - \alpha_{13}\alpha_{31})}, \quad \bar{N}_2 = 0, \quad \bar{N}_3 = \frac{(a_3\alpha_{11} - a_1\alpha_{31})}{(\alpha_{11}\alpha_{33} - \alpha_{13}\alpha_{31})} \tag{3.7}$$

The equilibrium state exist only when

$$a_1\alpha_{33} > a_3\alpha_{13}, \quad a_3\alpha_{11} > a_1\alpha_{31} \tag{3.8}$$

E_7 : The state in which both predator and competitor to the prey and predator exist and prey extinct

$$\bar{N}_1 = 0, \quad \bar{N}_2 = \left(\frac{a_2\alpha_{33} - a_3\alpha_{23}}{\alpha_{22}\alpha_{33} - \alpha_{23}\alpha_{32}}\right), \quad \bar{N}_3 = \left(\frac{a_3\alpha_{22} - a_2\alpha_{32}}{\alpha_{22}\alpha_{33} - \alpha_{23}\alpha_{32}}\right) \tag{3.9}$$

The equilibrium state exist only when

$$a_2\alpha_{33} > a_3\alpha_{23}, a_3\alpha_{22} > a_2\alpha_{32}, \alpha_{22}\alpha_{33} > \alpha_{23}\alpha_{32} \tag{3.10}$$

E_8 : The state in which the prey, predator and competitor to the prey and predator exist

$$\bar{N}_1 = \frac{A}{D}, \quad \bar{N}_2 = \frac{B}{D}, \quad \bar{N}_3 = \frac{C}{D} \tag{3.11}$$

where

$$D = \alpha_{11}(\alpha_{22}\alpha_{33} - \alpha_{23}\alpha_{32}) + \alpha_{12}(\alpha_{21}\alpha_{33} + \alpha_{31}\alpha_{23}) - \alpha_{13}(\alpha_{21}\alpha_{32} + \alpha_{31}\alpha_{22})$$

$$A = a_1(\alpha_{22}\alpha_{33} - \alpha_{23}\alpha_{32}) - \alpha_{12}(a_2\alpha_{33} - a_3\alpha_{23}) + \alpha_{13}(a_2\alpha_{32} - a_3\alpha_{22})$$

$$B = (a_2\alpha_{11}\alpha_{33} + a_1\alpha_{21}\alpha_{33} + a_1\alpha_{31}\alpha_{23}) - (a_3\alpha_{11}\alpha_{23} + a_3\alpha_{21}\alpha_{13} + a_2\alpha_{13}\alpha_{31})$$

$$C = (a_3\alpha_{11}\alpha_{22} + a_3\alpha_{21}\alpha_{12} + a_2\alpha_{12}\alpha_{31}) - (a_2\alpha_{11}\alpha_{32} + a_1\alpha_{21}\alpha_{32} + a_1\alpha_{31}\alpha_{22})$$

The equilibrium state exists only when

$$A > 0,\ B > 0,\ C > 0 \text{ and } D > 0 \tag{3.12}$$

IV. The Stability of The Equilibrium States

Let

$$N = (N_1, N_2, N_3)^T = \bar{N} + U \tag{4.1}$$

where $U = (u_1, u_2, u_3)^T$ is the perturbation over the equilibrium state $\bar{N} = (\bar{N_1}, \bar{N_2}, \bar{N_3})^T$. The basic equations (2.1) are linearized to obtain the equations for the perturbed state.

$$\dot{U} = AU \tag{4.2}$$

where

$$A = \begin{bmatrix} A_{11} & -\alpha_{12}N_1 & -\alpha_{13}N_1 \\ \alpha_{21}N_2 & A_{22} & -\alpha_{23}N_2 \\ -\alpha_{31}N_3 & -\alpha_{32}N_3 & A_{33} \end{bmatrix}$$

with

$$A_{11} = a_1 - 2\alpha_{11}N_1 - \alpha_{12}N_2 - \alpha_{13}N_3$$

$$A_{22} = a_2 - 2\alpha_{22}N_2 + \alpha_{21}N_1 - \alpha_{23}N_3$$

$$A_{33} = a_3 - 2\alpha_{33}N_3 - \alpha_{31}N_1 - \alpha_{32}N_2$$

The characteristic equation for the system is

$$\det(A - \lambda I) = 0 \tag{4.3}$$

The equilibrium state is stable, if three roots of the equation (4.3) are negative in case they are real of the roots have negative real parts if they are complex.

A. *Stability of the Equilibrium State* E_8

First, we consider the local stability of the equilibrium E_8.

The variational matrix of the system (2.1) at interior equilibrium state E_8 is

$$A=\begin{bmatrix} -\alpha_{11}N_1 & -\alpha_{12}N_{11} & -\alpha_{13}N_1 \\ \alpha_{21}N_2 & -\alpha_{22}N_2 & -\alpha_{23}\overline{N_2} \\ -\alpha_{31}N_1 & -\alpha_{32}N_3 & -\alpha_{33}N_3 \end{bmatrix} \tag{4.1.1}$$

The characteristic equation of interior equilibrium state $E_8\left(\bar{N}_1,\bar{N}_2,\bar{N}_3\right)$ is

$$\lambda^3+b_1\lambda^2+b_2\lambda+b_3=0 \tag{4.1.2}$$

where

$$b_1=\alpha_{11}\bar{N}_1+\alpha_{22}\bar{N}_2+\alpha_{33}\bar{N}_3$$

$$\begin{aligned} b_2 &= \left(\alpha_{11}\alpha_{33}-\alpha_{13}\alpha_{31}\right)\overline{N_1}\,\overline{N_3} \\ &\quad +\left(\alpha_{11}\alpha_{22}+\alpha_{12}\alpha_{21}\right)\overline{N_1}\,\overline{N_2} \\ &\quad +\left(\alpha_{33}\alpha_{22}-\alpha_{23}\alpha_{32}\right)\overline{N_2}\,\overline{N_3} \end{aligned}$$

$$b_3=\left[\begin{pmatrix} \alpha_{11}\alpha_{33}\alpha_{22}+\alpha_{21}\alpha_{12}\alpha_{33}+\alpha_{23}\alpha_{12}\alpha_{31})- \\ (\alpha_{13}\alpha_{31}\alpha_{22}+\alpha_{11}\alpha_{23}\alpha_{32}+\alpha_{21}\alpha_{32}\alpha_{13} \end{pmatrix}\right] \times \overline{N_1}\,\overline{N_2}\,\overline{N_3}$$

By Routh-Hurwitz criterion, all eigen values of the above characteristic equation have negative real parts if and only if

$b_1>0,\ \left(b_1b_2-b_3\right)>0$ and $b_3\left(b_1b_2-b_3\right)>0$

Clearly, $b_1>0$ and $b_3>0$ if

$$(\alpha_{11}\alpha_{33}\alpha_{22}+\alpha_{21}\alpha_{12}\alpha_{33}+\alpha_{23}\alpha_{12}\alpha_{31}) > (\alpha_{13}\alpha_{31}\alpha_{22}+\alpha_{11}\alpha_{23}\alpha_{32}+\alpha_{21}\alpha_{32}\alpha_{13})$$

And based on certain algebraic deductions applicable in this case, it can be verified that

$$b_3(b_1b_2-b_3)>0$$

Therefore, the roots of (4.1.1) are real and negative or complex conjugates having negative real parts.

Thus, the system is locally stable for interior equilibrium point $E_8\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$.

B. *Solution of the Linearized System of Equations*

The solution of linearized system of equations (4.2) is given by

$$u_1=A_1e^{s_1t}+B_1e^{s_2t}+C_1e^{s_3t} \tag{4.2.1}$$

$$u_2=A_2e^{s_1t}+B_2e^{s_2t}+C_2e^{s_3t} \tag{4.2.2}$$

$$u_3=A_3e^{s_1t}+B_3e^{s_2t}+C_3e^{s_3t} \tag{4.2.3}$$

where s_1, s_2 and s_3 are roots of the equation (4.1.1).

$$A_1=\left[\frac{K_1-K_2}{\left(s_1-s_2\right)\left(s_1-s_3\right)}\right]$$

$$B_1=\left[\frac{K_3-K_4}{\left(s_2-s_1\right)\left(s_2-s_3\right)}\right]$$

$$C_1=\left[\frac{K_5-K_6}{\left(s_3-s_1\right)\left(s_3-s_2\right)}\right]$$

$$A_2=\left[\frac{K_7-K_8}{\left(s_1-s_2\right)\left(s_1-s_3\right)}\right]$$

$$B_2=\left[\frac{K_9-K_{10}}{\left(s_2-s_1\right)\left(s_2-s_3\right)}\right]$$

$$C_2=\left[\frac{K_{11}-K_{12}}{\left(s_3-s_1\right)\left(s_3-s_2\right)}\right]$$

$$A_3=\left[\frac{K_{13}+K_{14}}{\left(s_1-s_2\right)\left(s_1-s_3\right)}\right]$$

$$B_3=\left[\frac{K_{15}+K_{16}}{\left(s_2-s_1\right)\left(s_2-s_3\right)}\right]$$

$$C_3=\left[\frac{K_{17}+K_{18}}{\left(s_3-s_1\right)\left(s_3-s_2\right)}\right]$$

where

$$\begin{aligned} K_1 &= u_{10}(s_1+\alpha_{22}\overline{N_2})(s_1+\alpha_{33}\overline{N_3}) \\ &\quad -u_{20}\alpha_{12}\overline{N_1}(s_1+\alpha_{33}\overline{N_3}) \\ &\quad -\left[u_{30}\alpha_{13}\overline{N_1}(s_1+\alpha_{22}\overline{N_2})\right] \end{aligned}$$

$$\begin{aligned} K_2 &= u_{10}\alpha_{23}\alpha_{32}\overline{N_2}\,\overline{N_3}+u_{30}\alpha_{12}\alpha_{23}\overline{N_1}\,\overline{N_2} \\ &\quad +u_{20}\alpha_{13}\alpha_{32}\overline{N_1}\,\overline{N_3} \end{aligned}$$

$$\begin{aligned} K_3 &= u_{10}(s_2+\alpha_{22}\overline{N_2})(s_2+\alpha_{33}\overline{N_3}) \\ &\quad -u_{20}\alpha_{12}\overline{N_1}(s_2+\alpha_{33}\overline{N_3}) \\ &\quad -\left[u_{30}\alpha_{13}\overline{N_1}(s_2+\alpha_{22}\overline{N_2})\right] \end{aligned}$$

$$\begin{aligned} K_4 &= u_{10}\alpha_{23}\alpha_{32}\overline{N_2}\,\overline{N_3} \\ &\quad +u_{30}\alpha_{12}\alpha_{23}\overline{N_1}\,\overline{N_2}+u_{20}\alpha_{13}\alpha_{32}\overline{N_1}\,\overline{N_3} \end{aligned}$$

$$K_5 = u_{10}(s_3+\alpha_{22}\overline{N_2})(s_3+\alpha_{33}\overline{N_3}) - u_{20}\alpha_{12}\overline{N_1}(s_3+\alpha_{33}\overline{N_3}) - \left[u_{30}\alpha_{13}\overline{N_1}(s_3+\alpha_{22}\overline{N_2})\right]$$

$$K_6 = u_{10}\alpha_{23}\alpha_{32}\overline{N_2}\,\overline{N_3} + u_{30}\alpha_{12}\alpha_{23}\overline{N_1}\,\overline{N_2} + u_{20}\alpha_{13}\alpha_{32}\overline{N_1}\,\overline{N_3}$$

$$K_7 = u_{20}(s_1+\alpha_{11}\overline{N_1})(s_1+\alpha_{33}\overline{N_3}) - u_{30}\alpha_{23}\overline{N_2}(s_1+\alpha_{11}\overline{N_1}) + u_{10}\alpha_{21}\overline{N_2}(s_1+\alpha_{33}\overline{N_3})$$

$$K_8 = u_{10}\alpha_{23}\alpha_{31}\overline{N_2}\,\overline{N_3} + u_{30}\alpha_{13}\alpha_{21}\overline{N_1}\,\overline{N_2} - u_{20}\alpha_{13}\alpha_{31}\overline{N_1}\,\overline{N_3}$$

$$K_9 = u_{20}(s_2+\alpha_{11}\overline{N_1})(s_2+\alpha_{33}\overline{N_3}) - u_{30}\alpha_{23}\overline{N_2}(s_2+\alpha_{11}\overline{N_1}) + u_{10}\alpha_{21}\overline{N_2}(s_2+\alpha_{33}\overline{N_3})$$

$$K_{10} = u_{10}\alpha_{23}\alpha_{31}\overline{N_2}\,\overline{N_3} + u_{30}\alpha_{13}\alpha_{21}\overline{N_1}\,\overline{N_2} - u_{20}\alpha_{13}\alpha_{31}\overline{N_1}\,\overline{N_3}$$

$$K_{11} = u_{20}(s_3+\alpha_{11}\overline{N_1})(s_3+\alpha_{33}\overline{N_3}) - u_{30}\alpha_{23}\overline{N_2}(s_3+\alpha_{11}\overline{N_1}) + u_{10}\alpha_{21}\overline{N_2}(s_3+\alpha_{33}\overline{N_3})$$

$$K_{12} = u_{10}\alpha_{23}\alpha_{31}\overline{N_2}\,\overline{N_3} + u_{30}\alpha_{13}\alpha_{21}\overline{N_1}\,\overline{N_2} - u_{20}\alpha_{13}\alpha_{31}\overline{N_1}\,\overline{N_3}$$

$$K_{13} = u_{30}(s_1+\alpha_{11}\overline{N_1})(s_1+\alpha_{22}\overline{N_2}) - u_{20}\alpha_{32}\overline{N_3}(s_1+\alpha_{11}\overline{N_1}) + u_{10}\alpha_{31}\overline{N_3}(s_1+\alpha_{22}\overline{N_2})$$

$$K_{14} = u_{30}\alpha_{12}\alpha_{21}\overline{N_1}\,\overline{N_2} + u_{20}\alpha_{12}\alpha_{31}\overline{N_1}\,\overline{N_3} - u_{10}\alpha_{21}\alpha_{32}\overline{N_2}\,\overline{N_3}$$

$$K_{15} = u_{30}(s_2+\alpha_{11}\overline{N_1})(s_2+\alpha_{22}\overline{N_2}) - u_{20}\alpha_{32}\overline{N_3}(s_2+\alpha_{11}\overline{N_1}) + u_{10}\alpha_{31}\overline{N_3}(s_2+\alpha_{22}\overline{N_2})$$

$$K_{16} = u_{30}\alpha_{12}\alpha_{21}\overline{N_1}\,\overline{N_2} + u_{20}\alpha_{12}\alpha_{31}\overline{N_1}\,\overline{N_3} - u_{10}\alpha_{21}\alpha_{32}\overline{N_2}\,\overline{N_3}$$

$$K_{17} = u_{30}(s_3+\alpha_{11}\overline{N_1})(s_3+\alpha_{22}\overline{N_2}) - u_{20}\alpha_{32}\overline{N_3}(s_3+\alpha_{11}\overline{N_1}) + u_{10}\alpha_{31}\overline{N_3}(s_3+\alpha_{22}\overline{N_2})$$

$$K_{18} = u_{30}\alpha_{12}\alpha_{21}\overline{N_1}\,\overline{N_2} + u_{20}\alpha_{12}\alpha_{31}\overline{N_1}\,\overline{N_3} - u_{10}\alpha_{21}\alpha_{32}\overline{N_2}\,\overline{N_3}$$

where u_{10}, u_{20} and u_{30} are the initial strengths of u_1, u_2 and u_3 respectively.

V. GLOBAL STABILITY

Theorem: The equilibrium point $E_8(\bar{N}_1, \bar{N}_2, \bar{N}_3)$ is globally asymptotically stable.

Proof: Let us consider the following Lyapunov function

$$V(\overline{N}_1,\overline{N}_2,\overline{N}_3) = \left\{N_1 - \overline{N}_1 - \overline{N}_1 \ln\left[\frac{N_1}{\overline{N}_1}\right]\right\} + \left\{N_2 - \overline{N}_2 - \overline{N}_2 \ln\left[\frac{N_2}{\overline{N}_2}\right]\right\} + \left\{N_3 - \overline{N}_3 - \overline{N}_3 \ln\left[\frac{N_3}{\overline{N}_3}\right]\right\} \quad (5.1)$$

Differentiating V with respect to t, we get

$$\frac{dV}{dt} = \left(\frac{N_1-\overline{N}_1}{N_1}\right)\frac{dN_1}{dt} + \left(\frac{N_2-\overline{N}_2}{N_2}\right)\frac{dN_2}{dt} + \left(\frac{N_3-\overline{N}_3}{N_3}\right)\frac{dN_3}{dt} \quad (5.2)$$

$$\frac{dV}{dt} = -\left(\alpha_{11} + \frac{(\alpha_{12}-\alpha_{21}+\alpha_{13}+\alpha_{31})}{2}\right)\left[N_1-\overline{N_1}\right]^2 - \left(\alpha_{22} + \frac{(\alpha_{12}-\alpha_{21}+\alpha_{13}+\alpha_{31})}{2}\right)\left[N_2-\overline{N_2}\right]^2 - \left(\alpha_{33} + \frac{(\alpha_{12}-\alpha_{21}+\alpha_{13}+\alpha_{31})}{2}\right)\left[N_3-\overline{N_3}\right]^2$$

Then,

$$\frac{dV}{dt} < 0$$

Therefore, the interior equilibrium point E_8 is globally asymptotically stable if $\alpha_{12} \geq \alpha_{21}$.

VI. NUMERICAL EXAMPLE

Example 1. Let

a_1=1.5; a_2=2.65; a_3=3.45;

α_{11}=0.1; α_{12}=0.3; α_{13}=0.01; α_{22}=0.2;

α_{21}=0.3; α_{23}=0.2; α_{33}=0.2; α_{31}=0.01; α_{32}=0.1.

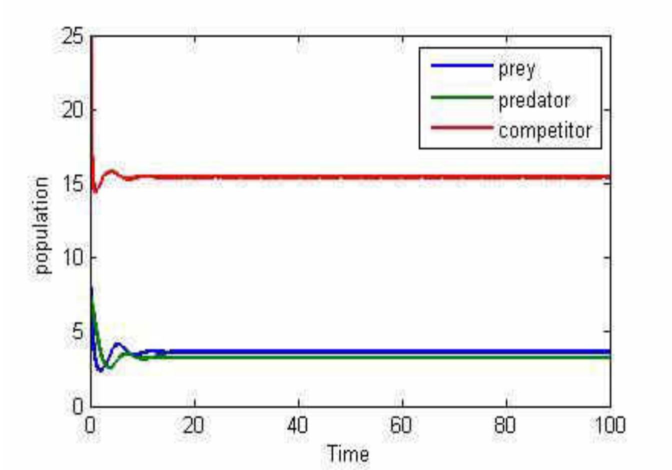

Fig 6.1.1: The Variation of N1, N2 & N3 with respective Time (t) for system of Eq (2.1)

From the above graph N1, N2 & N3 converges with diminishing amplitude as time goes on.

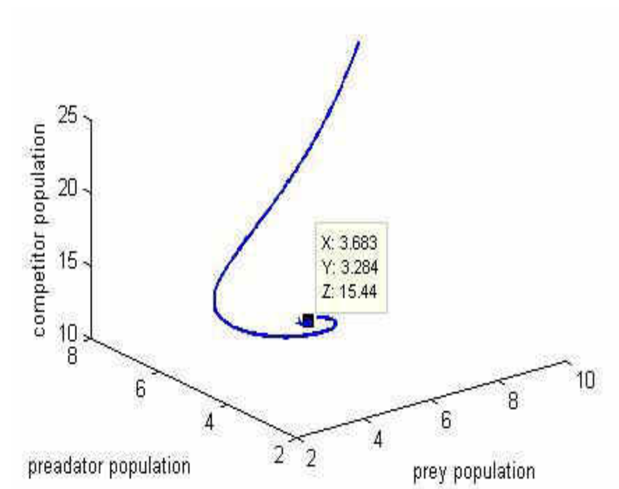

Fig 6.1.2: The Phase portrait of N1, N2, N3 for system of Eq (2.1)

The above graph shows the N1, N2 & N3 phase portrait. The curve is concentric spiral and the system of equations (2.1) for the given parametric values it is globally asymptotically stable and converges to equilibrium point

E (3.683, 3.284, 15.44).

The above graph shows the variation with initial strengths 10, 8, 25 of prey, predator and competitor populations respectively.

Example 2. Let

a_1=1; a_2=1; a_3=1.5; α_{11}=0.2; α_{12}=0.1; α_{13}=0.1; α_{22}=0.2; α_{21}=0.1; α_{23}=0.1; α_{33}=0.2; α_{31}=0.1; α_{32}=0.2

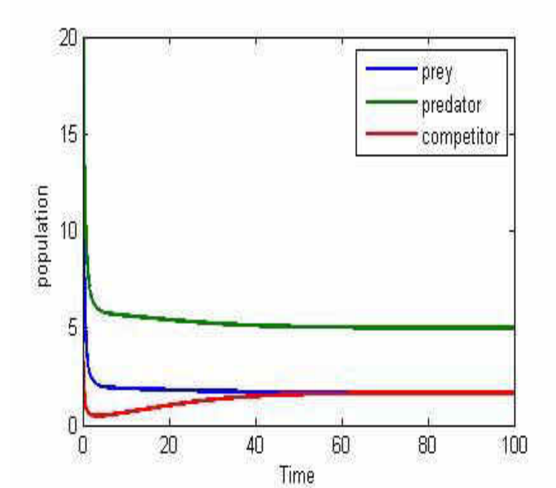

Fig 6.2.1: The Variation of N1, N2 & N3 with respective Time (t) for system of Eq (2.1)

From the above graph N1, N2 & N3 converges. As time goes on increases the population sizes tends to equilibrium points.

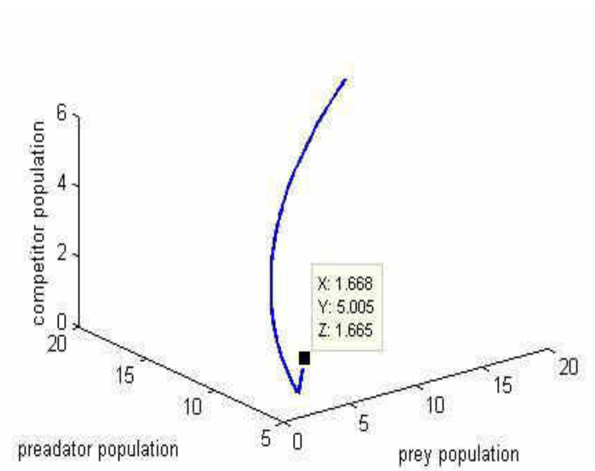

Fig 6.2.2: The Phase portrait of N1, N2, N3 for system of Eq (2.1)

The above graph shows the N1, N2 & N3 phase portrait. The curve is globally asymptotically stable to equilibrium point E (1.668, 5.005, 1.665), for the system of equations (2.1) for the given parametric values.

The above graph shows the variation with initial strengths 20, 20, 5 of prey, predator and competitor populations respectively.

For the linearized system of equations exact solutions have been derived and given from equations 4.2.1, 4.2.2 and 4.2.3. The Trajectories for the solutions with the following parametric values are shown below.

Example 3. Let a_1=1.5; a_2=2.65; a_3=3.45; α_{11}=0.1; α_{12}=0.3; α_{13}=0.01; α_{22}=0.2; α_{21}=0.3; α_{23}=0.2; α_{33}=0.2; α_{31}=0.01; α_{32}=0.2

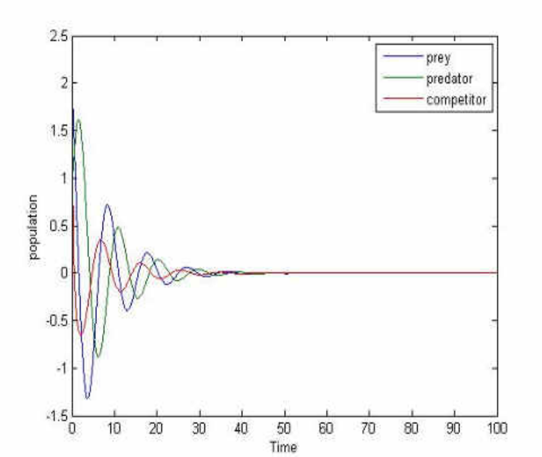

Fig 6.3.1: The Variation of u_1, u_2 & u_3 with respective Time (t) for system of Eq (4.2)

From the above graph u_1, u_2 & u_3 converges with diminishing amplitude. As time goes on increases the population sizes tends to equilibrium point.

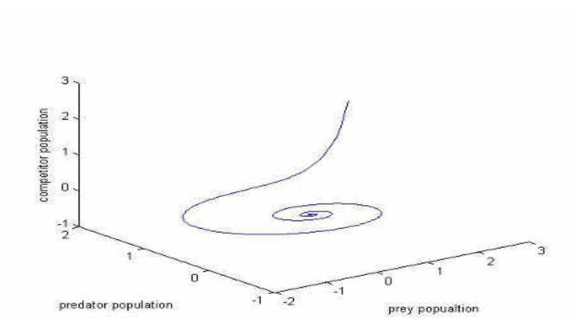

Fig 6.3.2: The Phage portrait of u_1, u_2 & u_3 for system of Eq (4.2)

The above graphs show the variation with initial strengths 10, 8, 25 of prey, predator and competitor populations respectively.

VII. CONCLUSIONS

In this paper, we made an attempt to study a three species ecological model with a prey, predator and competitor to both the prey and predator. All possible equilibrium points are identified and stability of the interior equilibrium point is discussed using Routh-Hurwitz criteria for local stability and using Lyapunov function for global stability. The solutions of linearized equations for interior equilibrium point are determined and are represented graphically with a suitable example. Further the non-linear systems of equations are examined for global stability with the help of suitable examples by using Matlab. The analytical discussion and numerical examples clearly shows that the system is globally asymptotically stable.

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