Flow of an Incompressible Micropolar Fluid Through a Channel Bounded By a Permeable Bed

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***Abstract*—The flow of a micropolar fluid in a channel bounded below by a permeable bed is investigated. The flow is analyzed by four parameters; they are viscous cross-flow Reynolds number, micropolar Reynolds number, microinertia parameter and micropolar parameter. The governing equations are solved to determine the expressions for velocity and microrotation fields. The effects of various parameters on the velocity and microrotation fields are discussed. Several graphs of physical interest are displayed and discussed.**

***Index Terms*—Micropolar fluid, Non-Newtonian fluid, Micropolar Reynolds number, Microrotation, Permeable bed.**
***MSC 2010 Codes* –58D30**

I. INTRODUCTION

Viscous flow though and past porous media is a subject of prime interest to Scientists and Engineers because of its occurrence in various physical and physiological situations such as oil wells and arterial system. The fluid that exists in the above systems can't be treated always as Newtonian. The rheological properties of individual red cells become very important in determining the flow resistance in small blood vessels. In view of this blood flow in a blood vesse,l most of the times is considered and behaves like a non - Newtonian fluid.

The model of micropolar fluid introduced by Eringen [1] represents fluids consisting of rigid, randomly oriented or spherical particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids exhibit some microscopic effects arising from the local structure and micro motion of the fluid elements. Ariman, et. al., [2] gave a detailed survey of microcontinum fluid mechanics with several applications in physiological fluid flows.

Some interesting aspects of theory and applications of micropolar fluids are dealt in a recent book of Lukasazewicz [3], Philip and Chandra [4] have investigated the peristaltic transport of simple microfluid which accounts for microrotation and microstrectching of the particles contained in a small volume elements, using long wave length approximation. Giriji Devi and Devanathan [5] have considered the peristaltic transport of the micropolar fluid in a laboratory frame of reference. Flow of micropolar fluid through a channel with injection is studied by Hiremath [6].Vajravelu et.al [7] studied the Peristaltiv transport of a micropolar fluid in channel with permeable wall. R.Hemadri Reddy et. al [8] discussed Peristaltic pumping of a micro polar fluid in an inclined channel. A.Kavitha et. al [9] studied Peristaltic flow of a micro polar fluid in a vertical channel with long wavelength approximation.

In view of several physical and industrial applications, the flow of a micropolar fluid in a channel bounded below by a permeable bed is investigated. The expressions for velocity and microrotation are obtained. The effects of various parameters on the velocity and micro-rotation fields are discussed.

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II. NOMENCLATURE

Q	- Velocity vector
$\overline{v}$	- Microrotation
ρ	- Density of the fluid
j	- Microinteria of the fluid
R_1	- Viscous cross- flow Reynolds number
R_2	- Micropolar Reynolds number
R_3	- Non-dimensional micro inertia parameter
R_4	- Micropolr Parameter
P	- Pressure
$\mu, k, \alpha, \beta, \gamma$	- The material constants
H	- Gapwidth of the channel
x-axis	- Lower permeable bed
y-axis	- Perpendicular to it

III. MATHEMATICAL FORMULATION OF THE PROBLEM

The governing equations of the flow of an incompressible micropolar fluid, in the absence of body force and body couple, are

$$\nabla.\overline{q} = 0 \tag{1}$$

$$\rho\frac{D\overline{q}}{Dt} = -\nabla p + (\mu + k)\nabla^2\overline{q} + k\overline{\nu} \tag{2}$$

$$\rho J\frac{D\overline{v}}{Dt} = (\alpha + \beta + \gamma)\nabla(\nabla.\overline{v}) - \gamma\nabla\times(\nabla\times v) + K\nabla\times\overline{q} - 2k\overline{v} \tag{3}$$

Here $\vec{q}$ and $\overline{v}$ are velocity and microrotatiton vectors, ρ and j are the density and micro-inertia of the fluid, p is the pressure and μ, k, α, β and γ are the material constants.

Consider a two dimensional steady flow of an incompressible micropolar fluid through the channel bounded below by a permeable bed and above by a rigid plate. The fluid is injected with a velocity v from the permeable bed. The flow in the channel is governed by Eringen's micropolar fluid model. The flow in the permeable bed is according to Darcy's law. The gap width 'h' between the lower permeable bed and upper plate in small compared to the length and breadth of the plate and permeable bed so that the edge effects are assumed to be negligible. As h is small, the injected fluid impinges an the upper plate and flows in the interspace between permeable bed and the plates. The x-axis is chosen along the lower permeable bed and y-axis up ward perpendicular to it. The origin is taken at the centre of the lower plate. We consider the slow motion approximation which is effected by neglecting the interial terms in the governing equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

$$0=-\frac{\partial p}{\partial x}+(\mu+k)\nabla^2 u+k\frac{\partial \phi}{\partial y} \tag{5}$$

$$0=-\frac{\partial p}{\partial y}+(\mu+k)\nabla^2 v-k\frac{\partial \phi}{\partial x} \tag{6}$$

$$0=\gamma\nabla^2-k\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)-2k \tag{7}$$

IV. Non-dimensionalization of the Flow Quantities

We introduce the following non dimensional quantities to make basic equations and boundary conditions dimensions

$$x^*=\frac{x}{h},\ \ y^*=\frac{y}{h},\ \ u^*=\frac{u}{v},\ \ v^*=\frac{v}{h},\ \ p^*=\frac{p}{\rho v^2},\ \ q^*=\frac{\phi h}{V} \tag{8}$$

In the view of the above non dimensional quantities the basic equations (1) to (3) take the following form after dropping asterisks.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

$$0=-\frac{\partial p}{\partial x}+\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\nabla^2 u+\frac{1}{R_2}\frac{\partial \phi}{\partial y} \tag{10}$$

$$0=\frac{\partial p}{\partial y}+\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\nabla^2 v-\frac{1}{R_2}\frac{\partial \phi}{\partial x} \tag{11}$$

$$0=\frac{1}{R_3}\nabla^2\phi-\frac{R_4}{R_3}\left(\frac{\partial u}{\partial y}-\frac{\partial u}{\partial x}\right)-\frac{2R_4}{R_3}\phi \tag{12}$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

The non-dimensional parameters appearing in the above equations are defined by

$$R_1=\frac{\rho Vh}{\mu},\ \ R_2=\frac{\rho Vh}{K},\ \ R_3=\frac{\rho jvh}{\gamma},\ \ R_4=\frac{kh^2}{\gamma} \tag{13}$$

The boundary conditions are

$$y=0,\ \ u=\frac{\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y},\ \ \phi=\frac{\sqrt{Da}}{\alpha}\frac{\partial \phi}{\partial y},\ \ v=1 \tag{14}$$

$$y=1,\ \ u=0,\ \ v=0,\ \ \phi=0 \tag{15}$$

The expressions for the velocity and microrotation components are obtained, in the following section, by solving the equations (9) to (12) subjected to the boundary conditions (14 & 15).

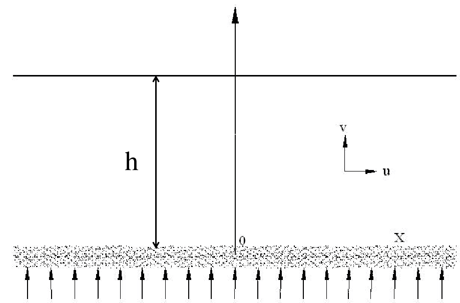

Fig. 1. Physical Model

V. Solution of the Problem

The elimination of pressure between the equations (10) and (11) yields

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\nabla^2\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)+\frac{1}{R_2}\nabla^2\phi=0 \tag{16}$$

The boundary conditions (14 & 15) and the equations (9), (12) and (16) admit solutions of the form (following Hiremath, 1983).

$$u=xf^1(y),\ v=-f(y),\ \phi=xg(y) \tag{17}$$

Using the equations (17) in the equations (16) and (12) we obtained

$$(R_1+R_2)f^{IV}+R_1g''=0 \tag{18}$$

$$g''-R_4(f''+2g)=0 \tag{19}$$

According the boundary conditions (14) and (15) become

$$f'(0)=\frac{\sqrt{Da}}{\alpha}f''(0),\ \ f(0)=-1,\ \ g(0)=\frac{\sqrt{Da}}{\alpha}g'(0) \tag{20}$$

$$f'(1)=0,\ \ f(1)=0,\ \ g(1)=0 \tag{21}$$

Velocity and microrotation fields

The solutions are equations (18) and (19) subject to the boundary conditions (20) and (21) are given by

$$\begin{aligned}f(y)=\tfrac{1}{R_4}\Big\{&\left(1-\tfrac{2R_4}{n^2}\right)I_1e^{ny}+\left(1-\tfrac{2R_4}{n^2}\right)I_2e^{-ny}\\&-\tfrac{R_4}{n^2}I_3\tfrac{y^3}{3}-\tfrac{R_4}{n^2}I_4y^2\Big\}+I_5y+I_6\end{aligned} \tag{22}$$

$$g(y)=\frac{I_1}{n^2}e^{ny}+\frac{I_2}{n^2}e^{-ny}+I_3y+I_4 \tag{23}$$

Where

$$I_1 = \left[\left(1+\frac{m}{n}\right)e^{-n} + 1 - \frac{m}{n}\right]/H$$
$$I_2 = -\left[\left(1-\frac{m}{n}\right)e^{n} + 1 + \frac{m}{n}\right]/H$$
$$I_3 = (2-m)\left[\left(e^{n}-1\right)I_1 + \left(e^{-n}-1\right)I_2\right]$$
$$I_4 = (2-m)\left(I_1 + I_2\right)$$
$$I_5 = m\left(I_1 - I_2\right)/n$$
$$I_6 = \frac{m}{n^2}\left(I_1 + I_2\right) - 1$$
$$H = -\frac{m}{n} + \frac{m}{n}e^{-n}\left(2+2k\right)e^{-n} - km + \frac{2k+1}{k+1}\left(1 - e^{-n} - km\right)$$

Pressure distribution

Using equations (17), (22) and (23) in the equations (10) and (11) and integrating the pressure distribution is obtained as

$$p - p_0 = \frac{I_4}{R_1}\left(1 + \frac{R_1}{R_2}\right)\left(y^2 - x^2 - y\right) \tag{24}$$

where p_0 is the pressure at the origin.

Shear stress

The shear stress are given by

$$\tau_{xy} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) - mx\left(I_1 e^{ny} + I_2 e^{-ny}\right) + \frac{1}{2-m}\left(2I_3 y + I_4\right) \tag{25}$$

$$\tau_{yx} = \frac{1}{R_1} - mx\left(I_1 e^{ny} + I_2 e^{-ny}\right) + \frac{1}{2-m}\left(2I_3 y + I_4\right) \tag{26}$$

Couple stress

The couple stress are given by

$$m_{yz} = \frac{1}{R_2} x g'(y)$$
$$m_{yz} = \frac{1}{R_3} x\left(nI_1 e^{ny} - nI_2 e^{-ny} - \frac{I_3}{(2-m)}\right)$$
$$m_{xz} = \frac{1}{R_3} g(y)$$
$$m_{xy} = \frac{1}{R_3}\left(I_1 e^{ny} + I_2 e^{-ny} - \frac{1}{(2-m)}\left(I_3 y + I_4\right)\right)$$

VI. Deductions and Discussions

To have a better understanding of the flow characteristics, numerical computations are performed with Reynolds number $R_1 = 0.5$ and with micro polar parameters R_2, R_3, R_4, taking various values.

It is observed that the micro polar parameters do not alter the velocity profiles appreciably. The microrotation profiles are skew symmetric, and are appreciably influenced by R_4 only. As R_4 increases, the magnitude of microrotation increases.

We observe that as $Da \to o$, the velocity, the microrotation fields or shear stress reduce to the results of Hiremath (1983) for the flow of a micropolar fluid through a channel with injection at the lower plate.

The variation of velocity with y is calculated from Equation (23) and is shown in fig.(2). For a different values of R_4 with $R_1 = 0.5$, $R_2 = 0.5$, it is observed that the velocity component u decreases with the increasing micropolar Parameter R_4.

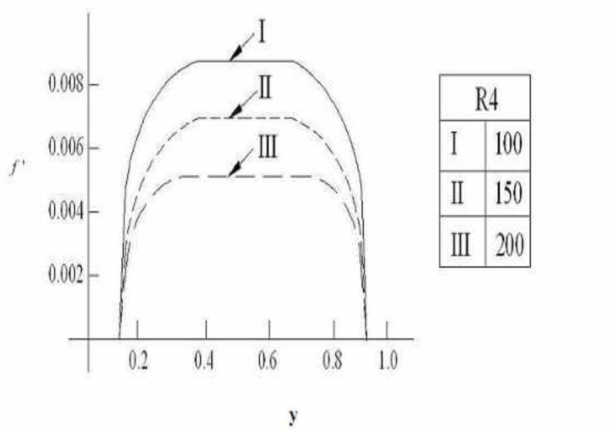

Fig. 2. The variation of velocity profile u for different R_4 values for fixed $R_1 = 0.5$, $R_2 = 0.5$, $Da = 0.0001$, $\alpha = 0.5$, $x = 1$

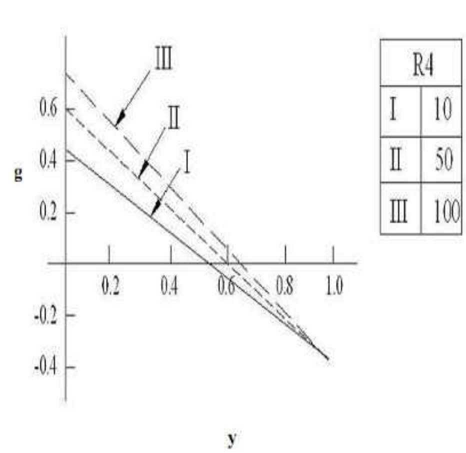

Fig. 3. The variation of microrotation for different R_4 values for fixed $R_1 = 0.5$, $R_2 = 0.5$, $Da = 0.0001$, $\alpha = 0.5$, $x = 1$

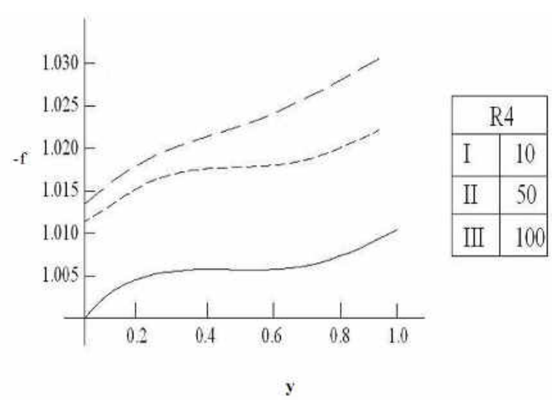

Fig. 4. Velocity profiles for different values of R_4 for fixed $R_1 = 0.5$, $R_2 = 0.5$, $Da = 0.0001$, $\alpha = 0.5$, $x = 1$

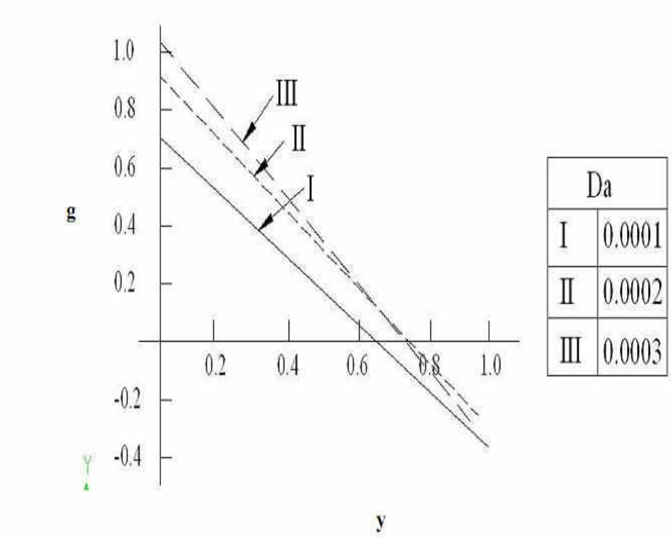

Fig. 5. Microrotation for different values of Da for fixed $R_1 = 0.5$, $R_2 = 0.5$, $R_4 = 100$, $\alpha = 0.5$, $x = 1$

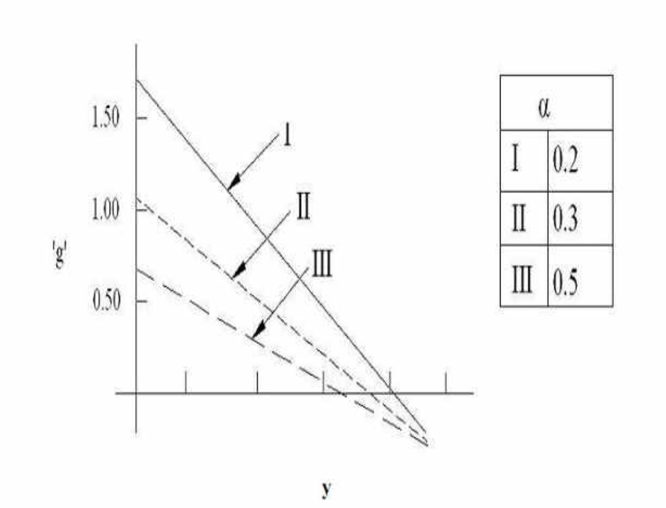

Fig. 8. The variation of microrotation with α for fixed $R_1 = 0.5$, $R_2 = 0.5$, $R_4 = 100$, $Da = 0.0001$, $x = 1$

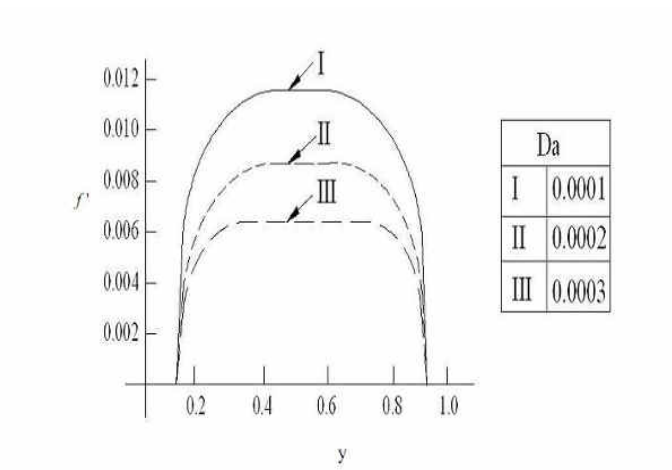

Fig. 6. Velocity profiles u for different values of Da for fixed $R_1 = 0.5$, $R_2 = 0.5$, $R_4 = 100$, $\alpha = 2$, $x = 1$

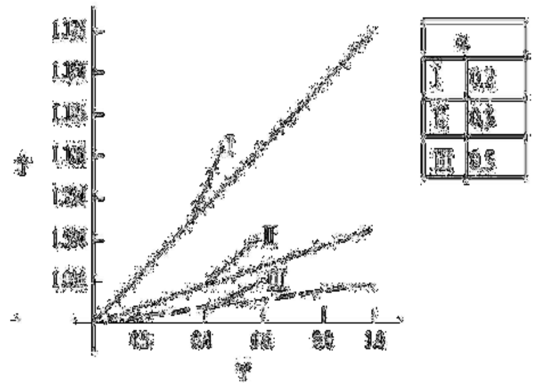

Fig. 9. The velocity profiles for different values of α for fixed $R_1 = 0.5$, $R_2 = 0.5$, $R_4 = 100$, $Da = 0.0001$, $x = 1$

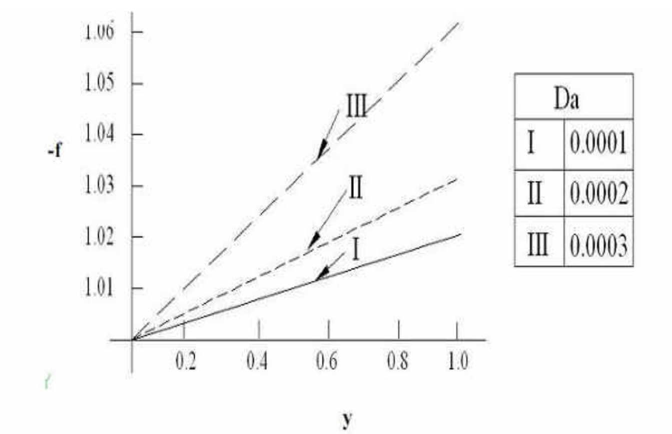

Fig. 7. Velocity profiles for viscous fluid with Da for fixed $R_1 = 0.5$, $R_2 = 0.5$, $R_4 = 100$, $\alpha = 2$, $x = 1$

The variation of velocity with y is calculated from (24) as shown in fig (3). For different values of R_4 with $R_1 = 0.5$ and $R_2 = 0.5$, it is observed that the magnitude of microrotation increases with the increase in R_4.

The variation of velocity with y is calculated from (24) as shown in figure (4). For different values of Da, with $R_1 = 0.5$, $R_2 = 0.5$, $R_4 = 100$, it is observed that the velocity component decreases with the increasing Darcy Number Da, i.e., the increase in the permeability results in an increase in the velocity in the channel.

The variation of velocity with y is calculated from (24) as shown in fig (5). For different values of R_3 with $R_1 = 0.5$, $R_2 = 0.5$. It is observed that the magnitude of microrotation increases with the increase in R_4.

The variation of velocity with y is calculated from (24) as shown in Fig (6). For different values of Da, with $R_1 = 0.5$, $R_2 = 0.5$, $R_4 = 100$, it is observed that velocity component increases with the increasing Da. i.e. that the increase in the permeability gives rise to an increase in the velocity in the

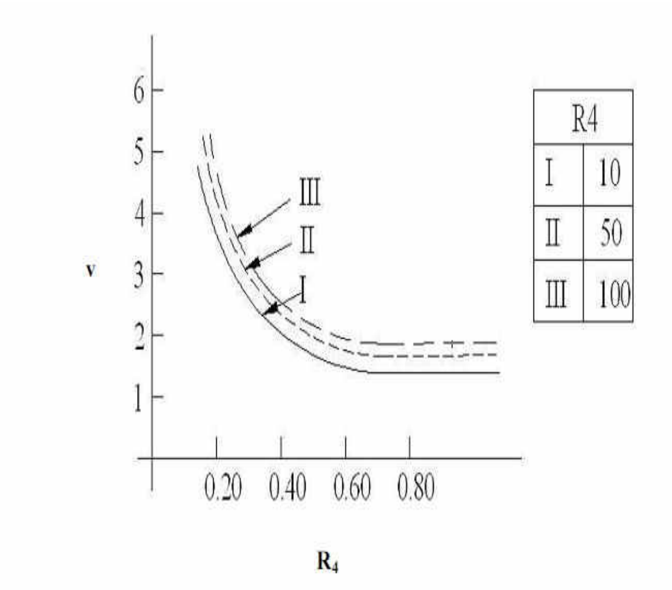

Fig. 10. The variation of wall stress difference τ_0/x with R_2 for different R_4 values for fixed $R_1 = 0.5$, $Da = 0.0001$, $\alpha = 0.5$

channel.

The variation at velocity with y is calculated from equation (25) is shown in fig (7). For different values of Da, it is observed that the magnitude of micro rotation increases with the increasing of α.

The variation of velocity with y is calculated from equation (25) as shown in figure (8). For different values of R_4 it is observed that the magnitude of microrotation increases with the increase of R_4.

The variation of wall stress with the micropolar parameters $R_2 \& R_4$ is calculated from equation (25) and is shown in figures (9, 10). It is observed that the wall stress τ_0/x decreases with R_2 for a given R_4 for a fixed R_2 the wall shear stress decreases with an increasing in R_4.

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