Existence of Mild Solutions for Nonlocal Integro-Differential Equations with Measure of Noncompactness

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Abstract—**In this article, we study the existence of mild solutions for integrodifferential equations with nonlocal conditions in Banach spaces. The results are obtained under the conditions in respect of the Hausdorff's measure of noncompactness.**

Index Terms— ***Fixed point; Hausdorff measure of noncompactness, integrodifferential equation, mild solution, nonlocal conditions.***

MSC 2010 Codes **—47G20, 34K30.**

I. Introduction

IN this paper, we discuss the existence of mild solutions of the following integro-differential equation with nonlocal condition

$$u'(t) = A(t,u)u(t) + f(t, u(s),$$

$$\int_0^t a(t,s)k(s,u(s)), ds, \int_0^a b(t,s)h(s,u(s))ds)$$

$$u(0) = g(u) + u_0$$

Where f : $[0,a]$ x X x X x X $\longrightarrow$ X and A : $[0,a]$ x X $\longrightarrow$ X are continuous functions, g : $C([0,a];X)$ $\longrightarrow$ X, $u_0 \in$ X and X is a real Banach space with norm $\|.\|$.

For convenience, we denote

$$(Gu)(s) = \int_0^t a(t,s)k(s,u(s))ds,$$

$$(Hu)(s) = \int_0^t b(t,s)h(s,u(s))ds.$$

Then the above integrodifferential equation can be written as,

$$u'(t) = A(t,u)u(t) + f(t,u(s),(Gu)(s),(HHu)(s)) \qquad (1.1)$$

$$u(0) = g(u) + u_0 \qquad (1.2)$$

where $t \in (0,a)$.

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In recent years, the existence, uniqueness and some other properties of solutions to semi-linear evolution equations similar to (1.1) have been extensively studied. We can refer to [12, 13, 14, 15] and references cited there in.

The non-local condition has been introduced to extend the study of the classical initial value problems [4,7,9,18,20]. It is more precise for describing nature phenomena than the classical condition since more information is taken into account, thereby decreasing the negative effects incurred by a possibly erroneous single measurement taken at the initial time. The semi–linear nonlocal initial value problems was initiated by Byszewski, we refer to some of the papers below. Byszewski [5,6], Byszewski and Lasmikantham [8] give the existence and uniqueness of mild solutions and classical solutions when f and g satisfy Lipschitz-type conditions. Ntouyas and Tsamatos [16, 17] study the case with compactness conditions. Q. Dong, G. Li, J. Zhang [18] discussed quasilinear nonlocal integrodifferential equations in Banach spaces.

In [15] Lin and Liu discussed semi-linear integrodifferential equation under Lipschitz – type conditions. Byszewski and Acka[7] give the existence of mild solutions of a functional differential equations. Subsequently many authors are devoted to studying of nonlocal problems, see [1,2,7,8,10,11,15,21] for the references and remarks about the advantage of the nonlocal problems over the classical initial value problems.

Motivated by the above approach and using the method of Hausdorff's measure of non-compactness, we give the existence of mild solutions for integrodifferential equations with nonlocal conditions (1.1)–(1.2). Our results improve and extend some corresponding results in [3,5,6,15,18,19].

Let $(X,\|.\|)$ be a real Banach space. We Denote by C($[0,A]$;X) the space of X - valued continuous functions on $[0,a]$ with the norm $\|u\| = \sup\{\|u_t\|, t \in [0,a]\}$ and by

$L(0,a;X)$ the space of X – valued Bochner integrable functions on $[0,a]$,with the norm $\|u\| L=\int_0^a \|u(t)\| dt$.

II. PRELIMINARIES

The Hausdroff's measure of non-compactness β_Y is defined by β_Y (B) = inf { r >0, B can be covered for bounded set B in a Banach space Y }.

Lemma 2.1 [3]. Let Y be a real Banach space and B, C $\subseteq$ Y be bounded, with the following properties:

(1) B is pre-compact if and only if β_X (B) = 0;

(2) β_Y (B) = $\beta_Y(\overline{B}) = \beta_Y$ (conv B), where $\overline{B}$ and conv B mean the closure and convex hull of B respectively;

(3) β_Y (B) $\le \beta_Y$ (C), where B $\subseteq$ C;

(4) β_Y (B + C) $\le \beta_Y$ (B) + β_Y (C), where

B + C = { x + y : x $\in B, y \in c$ };

(5) β_Y (B $\cup$ C) $\le$ max { β_Y (B), β_Y (C) };

(6) $\beta_Y(\lambda B) \le |\lambda| \beta_Y$ (B) for any $\lambda \in$ R;

(7) If the map Q : D (Q) $\subseteq$ Y $\to$ Z is Lipschitz continuous with constant k, then

β_Z (QB) $\le$ k β_Y (B) for any bounded subset B $\subseteq$ D(Q), where Z be a Banach space;

(8) β_Y (B) = inf {d_Y(B,C); C $\subseteq$ Y is precompact } = inf { d_Y(B,C); C $\subseteq$ Y is finite-valued}, where d_Y(B,C) means the nonsymmetric (or symmetric) Hausdorff distance between B and C in Y;

(9) If $\{W_n\}_{n=1}^{+\infty}$ is decreasing sequence of bounded closed nonempty subsets of Y and $\lim_{n\to\infty} \beta_Y(W_n)=0$, then $\bigcap_{n=1}^{+\infty} W_n$ is nonempty and compact in Y. ■

The map Q: $W \subseteq Y \to Y$ is said to be a β_Y *-contraction* if there exists a positive constant k<1 such that $\beta_Y(Q(B)) \le k\beta_Y(B)$ for any bounded closed subset $B \subseteq W$, where Y is a Banach space.

Lemma 2.2 (Darbo-Sadovskii [3]). If $W \subseteq Y$ is bounded closed and convex, the continuous map $Q: W \to W$ is a β_Y-contraction, then the map Q has at least one fixed point in W. ■

In this paper we denote by β the Hausdorff's measure of noncompactness of X and denote β_c by the Hausdorff's measure of noncompactness of C([a,b];X). To discuss the existence, we need the following Lemmas in this paper.

Lemma 2.3 ([3]). If $W \subseteq C([0,b];X)$ is bounded, then $\beta(W(t)) \le \beta_c(W)$ for all $t \in [0,b]$, where $W(t)=\{u(t); u \in W\} \subseteq X$. Furthermore if W is equicontinuous on [a,b], then $\beta(W(t))$ is continuous on [a,b] and

$\beta_c(W)=\sup\{\beta(W(t)), t \in [a,b]\}$. ■

Lemma 2.4 ([3]). If $\{u_n\}_{n=1}^{\infty} \subset L^1(a,b;X)$ is uniformly integrable, then the function $\beta(\{u_n(t)_{n=1}^{\infty}\})$ is measurable and

$$\beta\left(\left\{\int_0^t u_n(s)\,ds\right\}_{n=1}^{\infty}\right) \le 2\int_0^t \beta\left(\{u_n(s)\}_{n=1}^{\infty}\right) ds. \quad (2.1)$$

■

Lemma 2.5 ([3]). If $W \subseteq C([0,b];X)$ is bounded and equicontinuous, then $\beta(W(s))$ is continuous and

$$\beta\left(\int_0^t W(s)\,ds\right) \le \int_0^t \beta(W(s))\,ds. \quad ■ \qquad (2.2)$$

From [3], we know that for any fixed $u \in c([0,b];X)$ there exist a unique continuous function $U_u:[0,b]\times[0,b] \to B(X)$ defined on $[0,b]\times[0,b]$ such that

$$U_u(t,s) = I + \int_s^t A_u(\omega) U_u(\omega,s)\,d\omega, \quad (2.3)$$

where B(X) denote the Bananch space of bounded linear operators from X to X with the norm $\|Q\| = \sup\{\|Qu\| : \|u\| = 1\}$,and I stands for the identity operator on X, $A_u(t) = A(t,u(t))$. From (2.3), we have

$$U_u(t,t) = I, U_u(t,s)U_u(s,r) = U_u(t,r), (t,s,r) \in [0,b]\times[0,b]\times[0,b],$$

$$\frac{\partial U_u(t,s)}{\partial t} = A_u(t) U_u(t,s) \text{ for almost all } t \in [0,b], \forall s \in [0,b]$$

Definition 2.6. A continuous function $u(t)\in C([0,b];X)$ such that

$$u(t)=U_u(t,0)u_0+U_u(t,0)g(u)+\int_0^t U_u(t,s)\int_0^s f(s,u(\tau),(Gu)(\tau),(Hu)(\tau))dsd\tau \tag{2.4}$$

and $u(0)=g(u)+u_0$ is called a mild solution of (1.1)-(1.2). ■

The evolution family $\{U_u(t,s)\}_{0\le s\le t\le b}$ is said to be ***equicontinuous*** if $(t,s)\to\{U_u(t,s)x:x\in B\}$ is equicontinuous for t>0 and for all bounded subset B in X.

The following Lemma is obvious.

Lemma 2.7. If the evolution family $\{U_u(t,s)\}_{0\le s\le t\le b}$ is equicontinuous and $\eta\in L(0,b;R^+)$, then the set

$$\left\{\int_0^t U_u(t-s,s)u(s)ds, \| u(s)\|\le\eta(s) \text{ for a.e. } s\in[0,b]\right\}$$

is equicontinuous for $t\in[0,b]$. ■

In section III, we give some existence results when g is compact and f satisfies the conditions with respect to Hausdorff's measure of noncompactness. In section 4, we use the different method to discuss the case when g is Lipschitz continuous and f satisfies the conditions with the Hausdorff's measure of noncompactness.

In this paper, we denote $M=\sup\{\| U_u(t,s)\|:(t,s)\in[0,b]\times[0,b]\}$ for all $u\in X$. Without loss of generality, we let $u_0=0$.

III. The Existence of Results for Compact g

In this section, by using the usual techniques of the Hansdorff's measure of noncompactness and its applications in differential equations is Banach spaces.

Here we list the following hypothesis:

(H1) The evolution family $\{U_u(t,s)\}_{0\le s\le t\le b}$ generated by A(t,u) is equicontinuous, and $\| U_u(t,s)\|\le M$ for almost all $t,s\in[0,b]$.

(H2) $g:C([0,b];X)\to X$ is continuous and compact and there exist N>0 such that $\| g(u)\|\le N$ for all $u\in C([0,b];X)$.

(H3) $f:[0,b]\times[0,b]\times X\to X$ satisfies the Caratheodory-type condition i.e., f(.,.,u) is measurable for all $u\in X$ and $f(t,s,\cdot)$ is continuous for a.e. $t,s\in[a,b]$;

(H4) There exist a function $h:[0,a]\times\mathbb{R}^+\to\mathbb{R}^+$ such that $h(\cdot,r)\in L(0,b;\mathbb{R}^+)$ for every $r\ge 0, h(t,\cdot)$ is continuous and increasing and $\| f(t,u,(Gu),(Hu))\|\le h(t,\| u\|,\| Gu\|,\| Hu\|)$ for a.e. $t\in[0,a]$, and all $u\in C([0,a];X)$, and for all positive constants K_1, K_2, the scalar equation

$$m(t)=K_1+K_2\int_0^t h(s,m(s),n(s),q(s))ds,\quad t\in[0,a] \tag{3.1}$$

has at least one solution;

(H5) There exist $\eta\in L(0,a;\mathbb{R}^+),\varsigma\in L(0,a;\mathbb{R}^+)$ such that

$$\beta(f(t,D_1,D_2,D_3))\le\eta_1(t)\beta(D_1)+\eta_2(t)\beta(D_2)+\eta_3(t)\beta(D_3)$$

for a.e. $t,s\in[0,a]$, and for any bounded subset $D\subset C([0,a],X)$. Here we let

$$\int_0^t[\eta_1(s)+k_1\eta_2(s)+k_2\eta_3(s)]ds\le K$$

Theorem 3.1. Assume the hypotheses (H1)-(H5) are satisfied, then the nonlocal initial value problem (1.1)-(1.2) has at least one mild solution.

Proof. Let m(t) be a solution of the scalar equation

$$m(t)=MN+M\int_0^t h(s,m(s),n(s),q(s))ds \tag{3.2}$$

Defined a map $Q:C([0,a];X)\to C([0,a];X)$ by

$$(Qu)(t)=U_u(t,0)g(u)+\int_0^t U_u(t,s)\int_0^s f(s,u(\tau),(Gu)(\tau),(Hu)(\tau))d\tau ds, t\in[0,a] \tag{3.3}$$

for all $u\in C([0,a];X)$. We can show that Q is continuous.

We denote by

$W_0 = \{u \in C([0,a];X), \| u(t) \| \le m(t),$
$\| Gu(t) \| \le n(t)\}$

and $\| Hu(t) \| \le q(t)$ for all $t \in [0,a]$. Then $W_0 \subseteq C([0,a];X)$ is bounded and convex.

Define $W_1 = \overline{conv}K(W_0)$, where $\overline{conv}$ means the closure of the convex hull in C([0,a]);X. As $U_u(t,s)$ is equicontinuous, g is compact and $W_0 \subseteq C([0,a];X)$ is bounded due to Lemma (2.7) and hypothesis (H4), $W_1 \subseteq C([0,a];X)$ is bounded closed convex nonempty and equicontinuous on [0,a]. For any $u \in Q(W_0)$, we know

$$\| u(t) \| \le \| U_u(t,0) \| \| g(u) \| + \int_0^t \| U_u(t,s) \| \int_0^s \| f(s,u(t),(Gu)(\tau),(Hu)(\tau)) \| d\tau ds$$

$$\le MN + M\int_0^t \int_0^s h(s, \| u(\tau) \|, \| Gu(\tau) \|, \| Hu(\tau) \|)\, d\tau ds$$

$$\le MN + M\int_0^t \int_0^s h(s, m(\tau), n(\tau), q(\tau)) d\tau ds$$

$$\le MN + M\int_0^t h(s, m(s), n(s), q(s)) ds$$

= m(t)

for $t \in [0,a]$. It implies that $Qu \in W_0$

Let $W_{n+1} = \overline{conv}Q(W_n)$, for n = 1,2……. Form above we know that $\{W_n\}_{n=1}^{\infty}$ is a decreasing sequence of bounded, closed, convex, equicontinuous on [0,a] and nonempty subsets in C([0,a]),X).

Now for $n \ge 1$ and $t \in [0,a], W_n(t)$ and $Q(W_n(t))$ are bounded subsets of X, hence, for any $\varepsilon > 0$, there is a sequence $\{u_k\}_{k=1}^{\infty} \subset W_n$ such that

$$\beta(W_{n+1}(t)) = \beta(QW_n(t))$$

$$\le 2\beta\Big(\int_0^t U_u(t,s) \int_0^s f\big(s, \{u_k(\tau)\}_{k=`}^{\infty}\big), \big(G\{u_k(\tau)\}_{k=1}^{\infty}\big), \big(H\{u_k(\tau)\}_{k=1}^{\infty}\big) d\tau ds\Big) + \varepsilon$$

$$\le 4M\int_0^t \int_0^s \beta\Big(f\big(s, \{u_k(\tau)\}_{k=1}^{\infty}, \big(G\{u_k(\tau)\}_{k=1}^{\infty}\big), \big(H\{u_k(\tau)\}_{k=1}^{\infty}\big) d\tau ds\Big)\Big) + \varepsilon$$

$$\le 4M\int_0^t \int_0^s \Big(\eta_1(s)\beta\big(\{u_k(\tau)\}_{k=1}^{\infty} + \eta_2(s)\, \beta\big(G\{u_k(\tau)\}_{k=1}^{\infty}\big) + \eta_3(s)\beta\big(H\{u_k(\tau)\}_{k=1}^{\infty}\big)\big) d\tau ds\Big) + \varepsilon$$

$$\le 4M\int_0^t \int_0^s \Big(\eta_1(s)\beta\big(\{u_k(\tau)\}_{k=1}^{\infty}\big) + k_1\eta_2(s)\, \beta\big(\{u_k(\tau)\}_{k=1}^{\infty}\big) k_2\eta_3(s)\beta\big(\{u_k(\tau)\}_{k=1}^{\infty}\big) d\tau ds + \varepsilon$$

$$\le 4M\int_0^s \beta(W_n(\tau)) d\tau \int_0^t (\eta_1(s) + k_1\eta_2(s) + k_2\eta_3(s)) ds + \varepsilon$$

$$\le 4MK\int_0^t \beta(W_n(s)) ds + \varepsilon$$

Since $\varepsilon > 0$ is arbitrary, if follows from the above inequality that

$$\beta(QW_{n+1}(t)) \le 4MK\int_0^t \beta(W_n(s)) ds \quad (3.4)$$

for all $t \in [0,a]$. Since W_n is decreasing for n, we define

$$f_n(t) = \lim_{n\to\infty} \beta(W_n(t))$$

for all $t \in [0,a]$. From (3.4), we have

$$f_{n+1}(t) \le 4MK\int_0^t f_n(s) ds$$

for $t \in [0,a]$, which implies that $f_n(t) = 0$ for all $t \in [0,a]$.

We know that $\lim_{n\to\infty} \beta_c(W_n) = 0$. Using Lemma (2.1), we know that $W = \bigcap_{n=1}^{\infty} W_n$ is convex compact and noempty in $C([0,a];X)$ and $Q(W) \subset W$. By Schauder's fixed point theorem, there exists at least one mild solution u of the initial value problem (1.1)-(1.2), where $u \in W$ is a fixed point of the continuous map Q.

IV. EXISTENCE FOR LIPSCHITZ g

In the previous section, we obtained the existence results when g is compact but without the compactness of $\{U_u(t,s)\}_{0\le s\le t\le b}$ or f.

In this section, we discuss the equation (1.1)-(1.2) when g is Lipschitz and f is not Lipschitz. Now, we replace (H_2) by (H_2').

(H_2') There exist a constant $L\in\left(0,\dfrac{1}{M}\right)$ such that

$\| g(u)-g(v)\| \le L\| u-v\|$ for every

$u,v\in C([0,a];X)$.

Theorem 4.1. Let (H1)-(H5) and (H_2') be satisfied. Then the equation (1.1)-(1.2) has at least one mild solution provided that

$$ML+4MKa<1 \qquad (4.1)$$

Proof. Define the operator $Q:C([0,B];\to C(0,B];X)$ by

$(Qu)(t)=(Q_1u)(t)+(Q_2u)(t)$

With

$$(Q_1u)(t)=U_u(t,0)\,g(u),$$

$$(Q_2u)(t)=\int_0^t U_u(t,s)\int_0^t f\left(s,u(\tau),(Gu)(\tau)(Hu)(\tau)\right)d\tau ds$$

For all $u\in C([0,B];X)$. Define

$$W_0=\{u\in C([0,B];X):\| u(t)\|\le m(t)\,\forall t\in[0,a]\},$$

and let $W=\overline{conv}QW_0$. Then from the proof of Theorem 3.1 we know that W is a bounded closed convex and equicontinuous subset of C([0,B];X) and $QW\subset W$. We shall prove that Q is β_c-contraction on W. Then by using Darbo-Sadovskii's fixed point theorem to get a fixed point of Q in W, which is a mild solution of (1.1)-(1.2).

First, for every bounded subset $B\subset W$, from the (H_2') and Lemma 2.1 we have

$$\beta_c(Q_1B)=\beta_c\left(U_B(t,0)\,g(B)\right)\le ML\beta_c(B). \qquad (4.2)$$

Next, for every bounded subset $B\subset W$, for $t\in[0,a]$ and every $\varepsilon>0$, there is a sequence $\{u_k\}_{k=1}^{\infty}\subset B$, such that

$$\beta(Q_2B(t))\le 2\beta\left(\{Q_2u_k(t)\}_{n=1}^{\infty}\right)+\varepsilon$$

Since B and Q_2B are equicontinuous, from Lemma 2.1,2.4,2.5 and hypothesis (H5) that

$$\beta(Q_2B(t))\le 2M\int_0^t\beta\Big(\int_0^s f(s,\{u_k(\tau)\}_{k=1}^{\infty},$$
$$\left(G\{u_k(\tau)\}_{k=1}^{\infty}\right)\left(H\{u_k(\tau)\}_{k=1}^{\infty}\right)d\tau)ds\Big)+\varepsilon$$
$$\le 4M\int_0^t\int_0^s\Big(\eta_1(s)\beta\left(\{u_k(\tau)\}_{k=1}^{\infty}\right)+$$
$$\eta_2(s)\beta\left(G\{u_k(\tau)\}_{k=1}^{\infty}\right)+$$
$$\eta_3(s)\beta\left(H\{u_k(\tau)\}_{k=1}^{\infty}\right)\Big)d\tau ds+\varepsilon$$
$$\le 4M\int_0^t\int_0^s\Big(\eta_1(s)\beta\left(\{u_k(\tau)\}_{k=1}^{\infty}\right)+$$
$$k_1\eta_2(s)\beta\left(G\{u_k(\tau)\}_{k=1}^{\infty}\right)+$$
$$k_2\eta_3(s)\beta\left(H\{u_k(\tau)\}_{k=1}^{\infty}\right)\Big)d\tau ds+\varepsilon$$
$$\le 4M\int_0^t\int_0^s\Big(\eta_1(s)\beta(B(\tau))+$$
$$k_1\eta_2(s)\beta(B(\tau))+$$
$$k_2\eta_3(s)\beta(B(\tau))\Big)d\tau ds+\varepsilon$$
$$\le 4MK\int_0^s\beta(B(\tau))d\tau+\varepsilon$$
$$\le 4MK\int_0^a\beta(B(s))ds+\varepsilon$$
$$\le 4MKa\beta_c(B)+\varepsilon$$

Taking supremum in $t\in[0,a]$,

We have $\beta_c(Q_2B)\le 4MKa\beta_c(B)+\varepsilon$

Since $\varepsilon>0$ is arbitrary, we have

$$\beta_c(Q_2B)\le 4MKa\beta_c(B)$$

for any bounded $B\subset W$. Now, for any subset $B\subset W$, due to Lemma 2.1, (4.2) and (4.3) we have

$$\beta_c(QB)\le\beta_c(Q_1B)+\beta_c(Q_2B)$$
$$\le ML\beta_c(B)+4MKa\beta c(B)$$
$$\le(ML+4MKa)\beta_c(B) \qquad (4.3)$$

By (4.1) we know that Q is a β_c-contraction on W. By Lemma 2.2, there is a fixed point u of Q in W, which is a solution of (1.1)-(1.2).

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