Non-Uniform Slot Suction (Injection) into MHD Axisymmetric Boundary Layer Flow with Variable Viscosity

C. Poornima, G. Ashwini and A.T. Eswara

Abstract—**The influence of non-uniform slot suction (injection) into a steady, MHD boundary layer flow over a sphere with variable viscosity has been examined. The system of nonlinear, coupled partial differential equations governing the non-similar flow have been solved numerically and the solutions have been obtained from the starting point of the streamwise co-ordinate to the exact point of separation. The difficulties arising at the starting point of the streamwise co-ordinate, at the edges of the slot and at the point of separation have been overcome by applying an implicit finite –difference scheme along with quasilinearization technique. The results indicate that magnetic field has a significant effect on both skin friction and heat transfer coefficients. Further, both skin friction and heat transfer coefficients increase with the increase of variable viscosity parameter. It is found that boundary layer separation can be delayed by nonuniform slot suction and also by moving the slot downstream, but the non-uniform slot injection does the reverse.**

Index Terms— ***Skin friction and Heat transfer, Magnetic field, Variable viscosity, Slot suction (injection)***

***MSC 2010 Codes* —37N10, 76A02**

I. INTRODUCTION

MOST of the boundary layer flow and heat transfer problems contain thermo physical properties of the fluid among which variation of viscosity with temperature is an interesting aspect. Several investigators [1 - 3] have done their research work on the effect of variable viscosity on laminar boundary layer flow and heat transfer problems for various circumstances. In order to understand the phenomenon of the boundary layer separation, it is essential to study the effect of temperature dependent viscosity on momentum and heat transport phenomena.

It is well known that many boundary layer flow and heat transfer problems of current practical interest do not admit similarity transformations. A detailed analysis of the flow situation in the laminar boundary layer taking non-similarity into account is of prime importance, where the non-similarity may be due to the free stream velocity or due to the curvature of the body or due to the surface mass transfer or due to all these effects. A review of non-similarity solution methods along with citations of some relevant publications is given by Dewey and Gross [4]. Since then several researches [5 - 7] have attempted to study the behavior of non-similar boundary layer flows.

In many cases, mass transfer through a wall slot (i.e, mass transfer occurs in a small porous section of the body surface while there is no mass transfer in the remaining part of the body surface) into the boundary layer is of interest for the various potential applications including thermal protection, energizing of the inner portion of boundary layer in adverse pressure gradient, and skin friction reduction on control surfaces. Moreover, mass transfer through a slot strongly influences the development of a boundary layer along a surface and in particular can prevent or at least delay separation of the viscous region. Several investigators [8 – 10] have studied the effect of slot suction (injection) into a laminar compressible boundary layer by considering the interaction between the boundary layer and oncoming stream. Uniform mass transfer in a slot causes finite discontinuity at the leading and the trailing edges of the slot. The discontinuities can be avoided by choosing a non-uniform mass transfer distribution along a streamwise slot as has been discussed in Minkowycz et al. [11].

The study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. There has been a great interest in the study of magnetohydrodynamic flow and heat transfer in any medium due to the effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow has attracted the interest of many researchers [12 – 14] due to its applications in many

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engineering problems such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions.

In the present study, we investigate the effect of non-uniform slot suction (injection) (i.e., mass transfer in a small porous section of the body surface and remaining part of the body surface is solid) on the steady non-similar boundary layer flow over an axisymmetric body (sphere) with variable viscosity, and an applied magnetic field. The non-similar solutions have been obtained starting from the origin of the streamwise co-ordinate to the point of separation (zero skin friction in the streamwise direction) using quasilinearization technique with an implicit finite-difference scheme.

II. Mathematical Analysis

Consider the steady, laminar non-similar boundary layer flow of an incompressible electrically conducting fluid over a sphere when the free stream velocity and non-uniform mass transfer (slot suction /injection) vary with the axial distance along the surface. Let x and y be the curvilinear coordinates along and perpendicular to the boundary, respectively, u and v be the corresponding velocity components. The contour of the body of revolution is specified by the radii $r(x)$ of the section perpendicular to the axis (Fig.1). A transverse magnetic field B_0 is applied in y – direction normal to the body surface and it is assumed that magnetic Reynolds number is small.

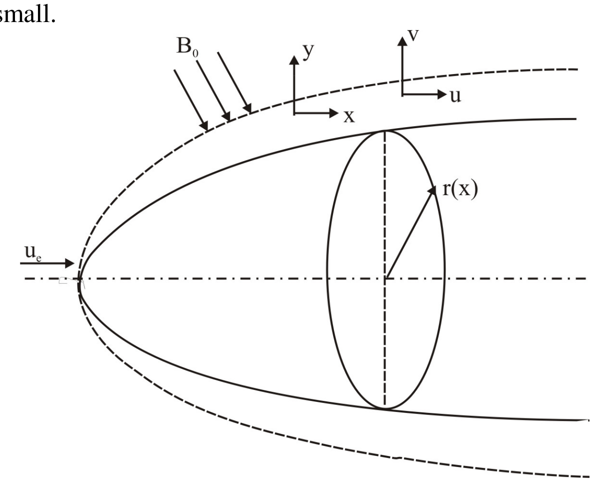

Fig.1 Flow Model and Co-ordinate System

The fluid is assumed to have constant physical properties except for the fluid viscosity (μ) which is assumed to be an inverse linear function of the temperature (T) [15].

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma(T - T_\infty)\right]$$

$$\frac{1}{\mu} = a(T - T_e)$$

where $a = \frac{\gamma}{\mu_\infty}$; $T_e = T_\infty - \frac{1}{\gamma}$;

Here, both a and T_e are constants and their values depend on the reference state and the thermal property of the fluid i.e., (γ). In general, $a > 0$ for liquids and $a < 0$ for gases. The blowing rate of the fluid is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. It is also assumed that the injected fluid possesses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature. Neglecting the effects of transverse curvature, the boundary layer equations governing the flow are given by:

$$\frac{\partial}{\partial x}(r^j u) + \frac{\partial}{\partial y}(r^j v) = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2}{\rho}\left(u - u_e\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions are given by

$$\begin{aligned} &u(x,0) = 0,\ v(x,0) = v_w(x),\ T(x,0) = T_w = \text{const} \\ &u(x,\infty) = u_e(x),\ T(x,\infty) = T_\infty = \text{const} \end{aligned} \tag{4}$$

Applying the following transformations:

$$\begin{aligned} &\xi = \int_0^x \frac{u_e}{u_\infty}\left(\frac{r}{L}\right)^{2j} d\left(\frac{x}{L}\right);\ \eta = \frac{u_e}{u_\infty}\left(\frac{\mathrm{Re}_L}{2\xi}\right)^{1/2}\frac{y}{L}\left(\frac{r}{L}\right)^j; \\ &\psi(x,y) = u_\infty L\left(\frac{2\xi}{\mathrm{Re}_L}\right)^{1/2} f(\xi,\eta);\ u = \left(\frac{L}{r}\right)^j \psi_y; \\ &v = -\left(\frac{L}{r}\right)^j \psi_x;\ T = T_\infty + \left(T_w - T_\infty\right) G(\xi,\eta) \end{aligned} \tag{5}$$

to (1) – (3), we see that the (1) (continuity equation) is identically satisfied and (2)&(3) reduce, respectively, to

$$F'' + \beta(\xi)\left(1 - \frac{G}{Ge}\right)\left(1 - F^2\right) + \left(1 - \frac{G}{Ge}\right) f F - PM\,(F - 1)\left(1 - \frac{G}{Ge}\right) + \frac{F'G'}{Ge - G} = \left(1 - \frac{G}{Ge}\right) 2\xi\left(F F_\xi - F' f_\xi\right) \tag{6}$$

$$G'' + f G' \mathrm{Pr} = 2\xi \mathrm{Pr}\left(F G_\xi - G' f_\xi\right) \tag{7}$$

where

$$\frac{u}{u_e} = f' = F$$

$$v = -\left(\frac{r}{L}\right)^j u_e\left(2\xi \mathrm{Re}_L\right)^{-1/2}\left(f + 2\xi f_\xi + F\eta(\beta + \alpha_1 - 1)\right);$$

$$\beta(\xi)=\left(\frac{2\xi}{u_e}\right)\left(\frac{du_e}{d\xi}\right);\quad \alpha_1=\left(\frac{2\xi j}{r}\right)\left(\frac{dr}{d\xi}\right);$$

$$M=\left(\frac{2}{3}\right)\frac{\sigma B_0^2 L}{\rho u_\infty};\ P=3\xi\left(\frac{L}{r}\right)^{2j}\left(\frac{u_\infty}{u_e}\right)^2;$$

$$\mathrm{Re}_L=\frac{\rho u_\infty L}{\mu};\quad \mathrm{Pr}=\frac{\nu}{\alpha}; \tag{8}$$

$$Ge=\frac{T_e-T_\infty}{T_w-T_\infty}=\frac{-1}{\gamma(T_w-T_\infty)};$$

$$f=\int_0^\eta F\,d\eta+f_w$$

$$f_w=-\xi^{-1/2}\left(\frac{\mathrm{Re}_L}{2}\right)^{1/2}\int_0^x\left(\frac{v_w}{u_\infty}\right)\left(\frac{r}{L}\right)^j d\left(\frac{x}{L}\right)$$

The transformed boundary conditions are:

$$F(\xi,0)=0,\ G(\xi,0)=1$$
$$F(\xi,\infty)=1,\ G(\xi,\infty)=0 \tag{9}$$

The skin friction coefficient and, heat transfer coefficient in the form of Nusselt number, can be expressed respectively as

$$C_f=\left(\frac{2Ge}{Ge-1}\right)\left(\frac{u_e}{u_\infty}\right)^2\left(\frac{r}{L}\right)^j\left(2\xi\,\mathrm{Re}_L\right)^{-1/2}F'_w$$
$$Nu=\left(\frac{\mathrm{Re}_L}{2\xi}\right)^{1/2}\left(\frac{r}{L}\right)^j\left(\frac{u_e}{u_\infty}\right)(-G'_w) \tag{10}$$

Here u and v are respectively, velocity components in x and y-directions of the flow; F is the dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ and η are transformed coordinates; ψ and f are the dimensional and dimensionless stream functions respectively; Pr is the Prandtl number; ρ,ν,α are respectively thermal diffusivity, density, kinetic viscosity; L is the characteristic length; R is the radius of the sphere; $r(x)$ is the radius of revolution of an axisymmetric body; Re_L is the Reynolds number; j = 1 for axisymmetric flows; f_w is the surface mass transfer; $u_\infty(x)$ is the free stream velocity; $u_e(x)$ is the inviscid flow velocity at the edge of the boundary layer and is a function of x; $v_w(x)$ denotes the surface mass transfer distribution; α_1 is a dimensionless parameter; β is the pressure gradient parameter; M is the nondimensional magnetic parameter. The subscripts e, w, ∞ denotes conditions at the edge of the boundary layer, on the wall and in the free stream, respectively; the subscripts x, y, ξ denote partial derivatives with respect to x, y, ξ respectively and prime $(')$ denotes derivatives with respect to η.

The dimensionless temperature G and viscosity ratio μ/μ_∞ are redefined as follows:

$$Ge=\frac{T-T_e}{T_w-T_\infty}+Ge$$

and hence $\dfrac{\mu}{\mu_\infty}=\dfrac{Ge}{Ge-G}$

where Ge is constant, called viscosity variation parameter, which is defined by

$$Ge=\frac{T_e-T_\infty}{T_w-T_\infty}=-\frac{1}{\gamma(T_w-T_\infty)}=\text{ constant}$$

And its value is determined by viscosity characteristics of the fluid under consideration and operating temperature difference $\Delta T=T_w-T_\infty$.

It may be remarked here that, if Ge is large (i.e., $Ge\to\infty$), the effect of variable viscosity can be neglected. On the other hand, for a smaller value of Ge, either the fluid viscosity changes markedly with temperature or operating temperature difference is high. In either case, the variable viscosity effect is expected to become very significant. Also, it may be noted here that, liquid viscosity varies differently with temperature than that of gas and therefore, it is important to note that $Ge<0$ for liquids and $Ge>0$ for gases when the temperature difference ΔT is positive.

Equations (6) and (7) under the conditions (9) represent steady state MHD boundary layer flow over any arbitrary bodies can be solved numerically if β and u_e, which depends on the shape of the body, are prescribed. In particular, we have studied the effect of non-uniform slot suction (injection) into laminar boundary layer flow over a sphere. The velocity distribution at the edge of the boundary layer, characterizing axisymmetric flow over a sphere, can be expressed as [16]:

$$\frac{u_e}{u_\infty}=\left(3/2\right)\sin(\bar{x});\ \bar{x}=\frac{x}{R};$$
$$j=1;\ L=R;\ \frac{r}{R}=\sin\bar{x} \tag{11}$$

where $\bar{x}$ is the dimensionless distance along the surface which give rise to non similarity in the flow. Consequently, the expressions for ξ, β, P, α_1, f_w are respectively given by

$$\xi=\frac{Q_1^2\,Q_3}{2};\ \beta=\left(3/2\right)\left[\cos\bar{x}\ Q_2^{-2}\,Q_3\right];$$
$$P=\left(2/3\right)Q_2^{1/2}\,Q_3;\qquad \alpha_1=\beta; \tag{12}$$

$$f_w = \begin{cases} 0, & \bar{x} \le \bar{x}_0 \\ A\,Q_1^{-1}\,Q_3^{-1/2}\,C(\bar{x},\bar{x}_0), & \bar{x}_0 \le \bar{x} \le \bar{x}_0^* \\ A\,Q_1^{-1}\,Q_3^{-1/2}\,C(\bar{x}_0^*,\bar{x}_0), & \bar{x} \ge \bar{x}_0^* \end{cases} \quad (13)$$

where the function

$$C(\bar{x},\bar{x}_0) = \frac{\sin\{(w^*-1)\bar{x} - w^*\bar{x}_0\} + \sin\bar{x}_0}{(w^*-1)} - \frac{\sin\{(w^*+1)\bar{x} - w^*\bar{x}_0\} - \sin\bar{x}_0}{(w^*+1)}$$

and

$$Q_1 = 1-\cos\bar{x}$$
$$Q_2 = 1+\cos\bar{x} \quad (14)$$
$$Q_3 = 2+\cos\bar{x}$$

Here $v_w(x)$ in expression (8) is taken as

$$v_w = \begin{cases} -u_\infty\left(\frac{\mathrm{Re}_L}{2}\right)^{-1/2} 2^{1/2} A\sin\{w^*(\bar{x}-\bar{x}_0)\}, & \bar{x}_0 \le \bar{x} \le \bar{x}_0^* \\ 0, & \bar{x} \le \bar{x}_0 \ \ and \ \ \bar{x} \ge \bar{x}_0^* \end{cases} \quad (15)$$

where w^* and $\bar{x}_0$ are the free parameters which determine the slot length and slot location. The function v_w is continuous for all values of $\bar{x}$ and it has a non-zero value only in the interval ($\bar{x}_0$, $\bar{x}_0^*$). The reason for taking such a type of function is that it allows the mass transfer to change slowly in the neighborhood of the leading and trailing edges of the slot. The parameter $A > 0$ for suction and $A < 0$ for injection.

It is convenient to express (6) and (7) in terms of $\bar{x}$ instead of ξ. Equation (12) gives the relation between ξ and $\bar{x}$ as

$$\xi\frac{\partial}{\partial\xi} = S(\bar{x})\frac{\partial}{\partial\bar{x}} \text{ where } S(\bar{x}) = \frac{1}{3}\tan\left(\bar{x}/2\right)Q_3\,Q_2^{-1} \quad (16)$$

Substituting (12) and (16) into (6) and (7), we obtain

$$F'' + \beta(\bar{x})\left(1-\frac{G}{Ge}\right)(1-F^2) + f\,F'\left(1-\frac{G}{Ge}\right) - PM\,(F-1)\left(1-\frac{G}{Ge}\right) + \frac{F'G'}{Ge-G} = 2S(\bar{x})\left(1-\frac{G}{Ge}\right)(F\,F_{\bar{x}} - F'\,f_{\bar{x}}) \quad (17)$$

$$G'' + f\,G'\,\mathrm{Pr} = 2S(\bar{x})\,\mathrm{Pr}\,(FG_{\bar{x}} - G'f_{\bar{x}}) \quad (18)$$

The boundary conditions (9) reduce to

$$F(\bar{x},0) = 0,\ \ G(\bar{x},0) = 1$$
$$F(\bar{x},\infty) = 1,\ G(\bar{x},\infty) = 0 \quad (19)$$

The skin friction and heat transfer coefficients can be written in terms of Nusselt number as

$$C_f\,(\mathrm{Re}_L)^{1/2} = \left(\frac{Ge}{Ge-1}\right)\left(9/2\right)\left[\frac{\sin\bar{x}\,Q_2}{Q_3^{1/2}}\right]F_w'$$
$$Nu\,(\mathrm{Re}_L)^{-1/2} = \left(3/2\right)\left[\frac{Q_2}{Q_3^{1/2}}\right](-G_w') \quad (20)$$

It is worth mentioning here that Kao and Elord [5] have studied the nonsimilar boundary layer flow, for axisymmetric bodies, considering (6) and (7) in the absence of slot suction (injection), without magnetic field and variable viscosity. Further, when M ≠ 0 (6) and (7) are exactly same as those of Meena and Nath [7], without slot suction (injection), for constant fluid properties (i.e., when $Ge \to \infty$).

III. Method of Solution

Applying quasilinearization technique [17], we replace the nonlinear partial differential equations (6) and (7) by an iterative sequence of linear equations as follows:

$$F''^{(k+1)} + X_1^{(k)}F'^{(k+1)} + X_2^{(k)}G'^{(k+1)} + X_4^{(k)}G^{(k+1)} + X_5^{(k)}F_\xi^{(k+1)} = U_1^{(k)} \quad (21)$$

$$G''^{(k+1)} + Y_1^{(k)}G'^{(k+1)} + Y_2^{(k)}G_\xi^{(k+1)} + Y_3^{(k)}F^{(k+1)} = U_2^{(k)} \quad (22)$$

where the coefficient functions with iterative index k are known and functions with iterative index $k+1$ are to be determined. The boundary conditions become

$$F^{(k+1)}(\xi,0) = 0$$
$$G^{(k+1)}(\xi,0) = 1$$
$$F^{(k+1)}(\xi,\infty) = 1 \quad (23)$$
$$G^{(k+1)}(\xi,\infty) = 0$$

The coefficients in (21) and (22) are given by

$$X_1^{(k)} = \left(1-\frac{G}{Ge}\right)f + \frac{G'}{Ge-G} + 2\xi f_\xi\left(1-\frac{G}{Ge}\right)$$

$$X_2^{(k)} = \beta\left(1-\frac{G}{Ge}\right)(-2F) - PM\left(1-\frac{G}{Ge}\right) - 2\xi F_\xi\left(1-\frac{G}{Ge}\right)$$

$$X_3^{(k)} = \frac{F'}{Ge-G}$$

$$X_4^{(k)} = \beta(1-F^2)\left(-\frac{1}{Ge}\right) + fF'\left(-\frac{1}{Ge}\right) - PM(F-1)\left(-\frac{1}{Ge}\right) + \frac{G'F'}{(Ge-G)^2} - 2\xi(FF_\xi - F'f_\xi)\left(-\frac{1}{Ge}\right)$$

$$X_5^{(k)} = -2\xi F\left(1-\frac{G}{Ge}\right)$$

$$U_1^{(k)} = -\beta(1+F^2)\left(1-\frac{G}{Ge}\right) - 2\xi F F_\xi\left(1-\frac{G}{Ge}\right) + \frac{F'G'}{Ge-G}$$
$$-\beta(1-F^2)\left(\frac{G}{Ge}\right) - f F'\left(\frac{G}{Ge}\right) + PM(F-1)\left(\frac{G}{Ge}\right)$$
$$+\frac{GG'F'}{(Ge-G)^2} + 2\xi(FF_\xi - F'f_\xi)\left(\frac{G}{Ge}\right) - PM\left(1-\frac{G}{Ge}\right)$$

$$Y_1^{(k)} = \Pr f + 2\xi f_\xi \Pr$$

$$Y_2^{(k)} = -2\xi F \Pr$$

$$Y_3^{(k)} = -2\xi G_\xi \Pr$$

$$U_2^{(k)} = -2\xi G_\xi \Pr F$$

The resulting linear partial differential equations (21) and (22) along with (23) were expressed in difference form, considering central difference in η-direction and backward difference in ξ-direction. The equations were then reduced to a system of linear algebraic equations with a block tridiagonal structure which is solved using Varga's algorithm [18]. In order to obtain grid independent numerical results, the grid sizes $\Delta\eta$ and $\Delta\bar{x}$ have been optimized. To achieve this, the computed values of physical parameters ($F'_w, -G'_w$) with a step size $\Delta\eta$ (keeping $\Delta\bar{x}$ fixed) are compared with those obtained using reduced step size viz., $(\Delta\eta/2), (\Delta\eta/4)$ and so on. Thus, the optimal values of the step sizes viz., $\Delta\eta = \Delta\bar{x} = 0.01$ has been used for computation, or $\bar{x} \le 1.5$. However, for $\bar{x} > 1.5$, finer step size for $\bar{x}$ has been used and in the neighborhood of the point of zero skin friction $\Delta\bar{x} = 0.0001$ is used. The value of η_∞ (i.e., the edge of the boundary layer) has been taken as 4.0 throughout the computation. A convergence criterion based on the relative difference between the current and the previous iteration has been used. The solution is assumed to have converged and the iterative process is terminated when

$$Max\left[\ \left|(F')_w^{(k+1)} - (F')_w^{(k)}\right|,\ \left|(G')_w^{(k+1)} - (G')_w^{(k)}\right|\ \right] < 10^{-4}$$

IV. RESULTS AND DISCUSSION

To assess the accuracy of our method of solution, we have compared our numerical results, in the absence of variable viscosity, with those of Kao and Elord [5] when magnetic field $M = 0$, as well as Meena and Nath [7] when $M \neq 0$, without slot suction (injection) [See Fig 2(a) & 2(b)]. The results are found to be in good agreement with those of [5,7].

Numerical results of the local skin friction and heat transfer coefficient as well as the velocity and temperature profiles are presented in the following paragraphs for various physical parameters viz.,variable viscosity parameter (Ge), slot suction (injection) parameter (A), magnetic parameter (M), for Pr = 0.7(air).

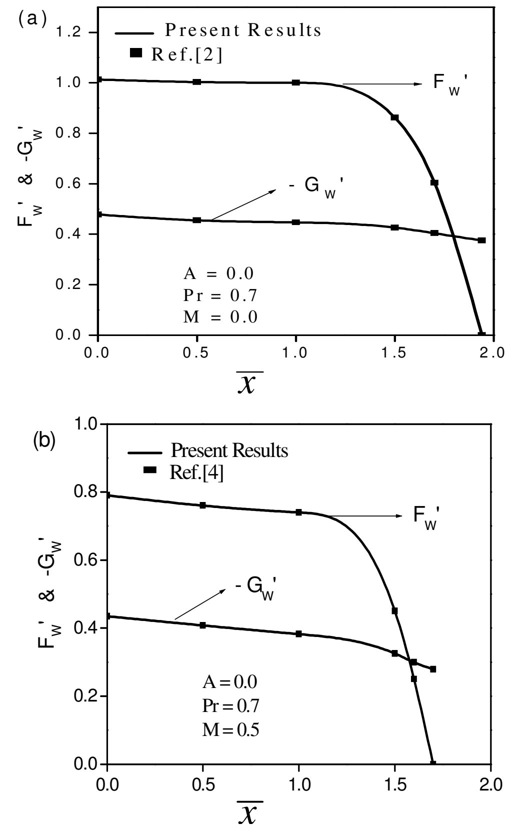

Fig 2. Comparison of skin friction and heat transfer results with those of (a) Kao and Elord [2] (b) Meena and Nath [4]

Fig. 3 shows the effect of magnetic field on skin friction [$C_f(\mathrm{Re}_L)^{1/2}$] and heat transfer [$Nu(\mathrm{Re}_L)^{-1/2}$] coefficients, in the presence of variable viscosity parameter (Ge), without slot suction (injection). It is clear that skin friction and heat transfer coefficients increase with the increase in magnetic field M. In fact, the percentage of increase in $C_f(\mathrm{Re}_L)^{1/2}$ from $M = 0.0$ to $M = 1.0$ is about 74.07% whereas; the percentage of increase in $Nu(\mathrm{Re}_L)^{-1/2}$ is about 2.39% at $\bar{x} = 1.0$ when $0 \le M \le 1.0$.

In Fig. 4, the influence of variable viscosity on skin friction [$C_f(\mathrm{Re}_L)^{1/2}$] and heat transfer [$Nu(\mathrm{Re}_L)^{-1/2}$] coefficients is shown. It is observed that as variable viscosity increases both skin friction and heat transfer coefficients increase without the effect of slot suction (injection) parameter. To be more specific, the percentage of increase in $C_f(\mathrm{Re}_L)^{1/2}$ from $Ge = 1.5$ to $Ge = 3.0$ is about 187.23% whereas, the percentage of increase in $Nu(\mathrm{Re}_L)^{-1/2}$ is about 9.7% at $\bar{x} = 1.0$ when $1.5 \le Ge \le 3.0$. Also, it is evident from Fig.3 and 4 that both magnetic field as well as variable viscosity help in delaying boundary layer separation.

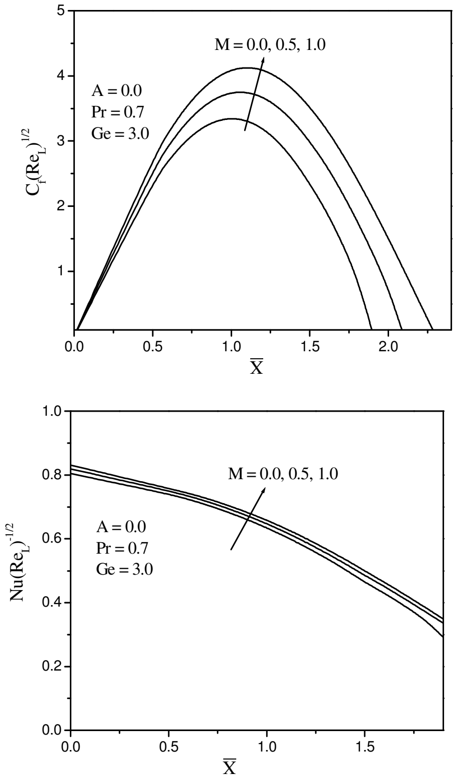

Fig. 3: Effects of magnetic parameter with variable viscosity on (a) skin friction coefficient and (b) heat transfer coefficient

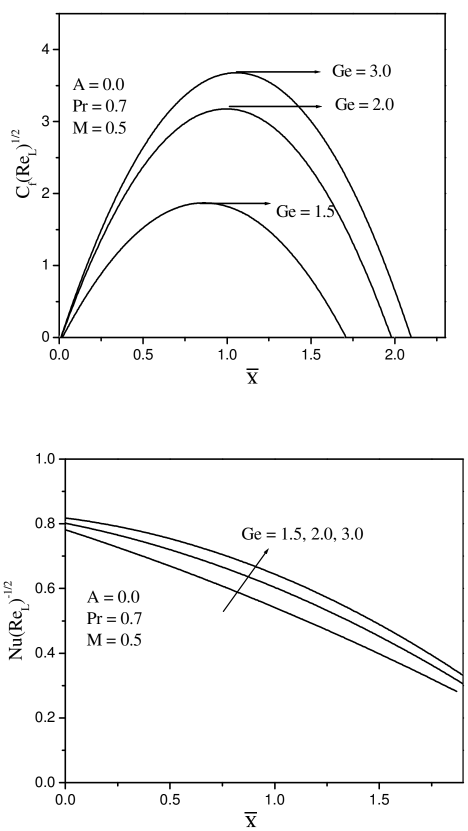

Fig.4: Effects of variable viscosity on (a) skin friction coefficient and (b) heat transfer coefficient without slot suction (injection)

Fig. 5 and 6 reveal the effect of slot suction parameter (A > 0) and slot injection parameter (A < 0) on skin friction [$C_f(\mathrm{Re}_L)^{1/2}$] and heat transfer [$Nu(\mathrm{Re}_L)^{-1/2}$] coefficients. It is observed that, due to slot suction (A > 0) the skin friction and heat transfer coefficients increase and attain their maximum value before trailing edge of the slot. Finally, both skin friction and heat transfer coefficients decrease from their maximum value and $C_f(\mathrm{Re}_L)^{1/2}$ reaches zero (i.e, separation of flow occurs), but $Nu(\mathrm{Re}_L)^{-1/2}$ remains finite. The effect of slot suction is to move the point of separation downstream, i.e., it delays the boundary layer separation. Fig. 5 also shows that when the slot location moves downstream i..e., when the slot is moved from $[0.5, 1.0]$ to $[1.5, 2.0]$, the point of separation also move downstream which in turn delay in the boundary layer separation. In Fig. 6, the effect of slot injection (A< 0) on $C_f(\mathrm{Re}_L)^{1/2}$ and $Nu(\mathrm{Re}_L)^{-1/2}$ is shown. It is clear that slot injection advances boundary layer separation and reduces heat transfer inside the slots.

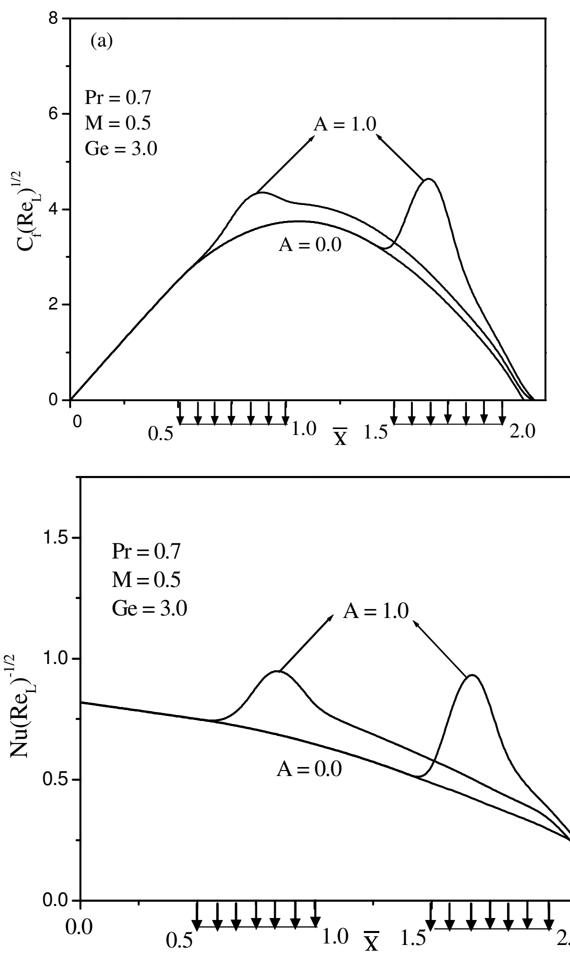

Fig. 5: Effect of slot suction (A > 0) on the (a) skin friction and (b) heat transfer coefficients at slot location $[\bar{x}_0, \bar{x}_0^*] = [0.5, 1.0]$ & $[1.5, 2.0]$

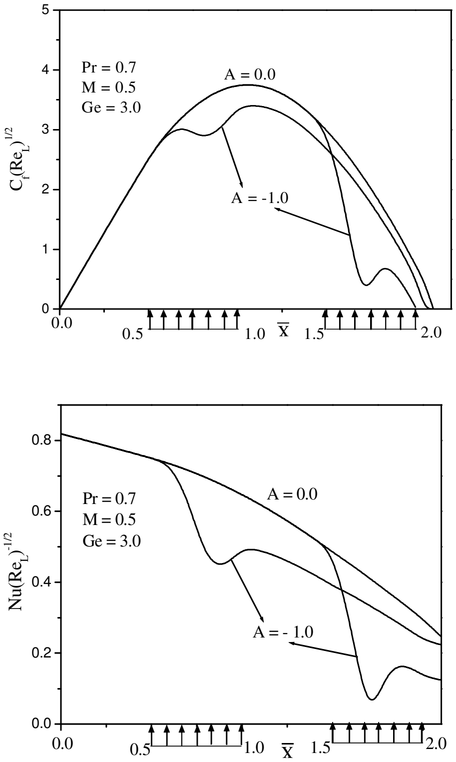

Fig. 6: Effect of slot injection (A < 0) on the (a) skin friction and (b) heat transfer coefficients at slot location

$[\overline{x}_0, \overline{x}_0^*] = [0.5,\ 1.0]\ \&\ [1.5,\ 2.0]$

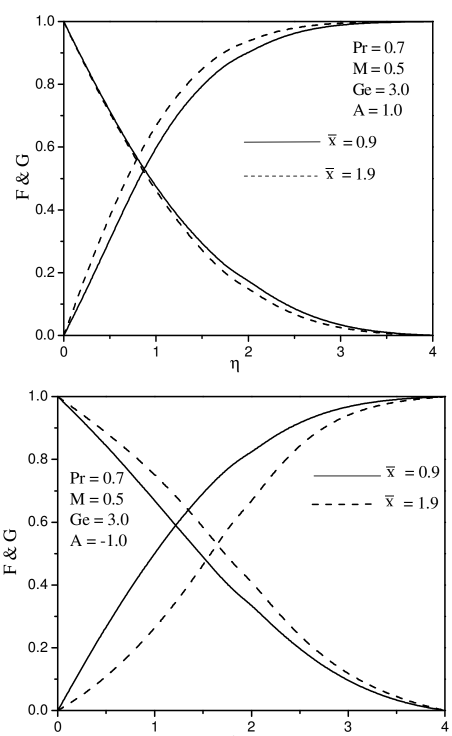

Fig.7: The velocity and temperature profile for (a) suction and (b) injection at streamwise locations

The velocity and temperature profiles, for different streamwise locations, in the case of suction (A> 0) as well as injection (A< 0) are shown in Fig.7. From Fig .7(a), it is observed that the momentum and thermal boundary layer thickness decrease due to slot suction, while from Fig.7(b) it is obvious that the slot injection does the reverse.

V. Conclusions

The effect of slot suction (injection) into steady, MHD nonsimilar boundary layer flow over a sphere has been studied in the presence of variable viscosity. In order to obtain the numerical results, a stable implicit finite difference scheme has been employed along with quasilinearization technique. The results indicate that both skin friction and heat transfer coefficients increases with the increase of variable viscosity parameter as well as magnetic parameter. Further, there is delay in the boundary layer separation during slot suction (A > 0) and also by moving the slot downstream, while slot injection (A < 0) has reverse effect on boundary layer separation.

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