Some Domination Parameters of the Intuitionistic Fuzzy Graph and its Properties

N. Vinoth Kumar and G. Geetha Ramani

Abstract— **Let G be an intuitionistic fuzzy graph IFG. Then S $\subseteq$ V is said to be a strong (weak) intuitionistic fuzzy dominating set of G if every vertex v$\in$ V-S is strongly (weakly) intuitionistic fuzzy dominated by some vertex u in S. We denote a strong (weak) intuitionistic fuzzy dominating set by sifd-set (wifd-set). Let G be a IFG. A set C $\subseteq$ V is a connected dominating set if C is a dominating set and the sub graph < C > induced by C is connected. Let G = (V, E) be a IFG. A set F $\subseteq$ V is an efficient dominating set if for every v $\in$ V-F then N[v] $\cap$ F=1. Let D $\subseteq$ V is said to be an independent dominating set of an IFG, if D is both an independent and a dominating set of G. The minimum cardinality among all minimal strong (weak), Connected, Efficient, Independent dominating set is called strong (weak), Connected, Efficient, Independent intuitionistic fuzzy domination number of G respectively, and is denoted by γ_{sif}(G) (γ_{wif}(G)), γ_{cif}(G), γ_{eif}(G), I_{if} (G) respectively.**

***Index Terms*— Connected intuitionistic fuzzy domination number, efficient intuitionistic fuzzy domination number, Independent Intuitionistic Fuzzy Domination Numbers, Strong (Weak) intuitionistic fuzzy domination number .**

***MSC 2010 Codes* — 05C72 , 05C69.**

I. Introduction

The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram [4] and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs [5] . C. Natarajan and S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. The concept of domination in intuitionistic fuzzy graphs was investigated by R. parvathi and G. Thamizhendhi. In this paper we develop the strong (weak) domination set, connected dominating set, efficient dominating set and independent dominating set of intuitionistic fuzzy graphs. Further introduce a dominating parameter of these sets and investigate the property of this domination parameter in intuitionistic fuzzy graph

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II. Basic Defenition

A. *Definition 2.1.*

An intuitionistic fuzzy graph (IFG) is of the form G=(V,E) , where V={v_1,v_2,…,v_n} such that μ_1:V→[0,1], γ_1: V→[0,1] denote the degree of membership and nonmember ship of the element $v_i \in$ V respectively and $0 \le \mu_1(v_i)+\gamma_1(v_i) \le 1$ for every vi$\in$ V,(i=1,2,…n) (ii) E $\subseteq$ V×V where μ_2:V×V→[0,1] and γ_2:V×V→[0,1] are such that $\mu_2(vi,vj) \le \mu_1(vi) \wedge \mu_1(vj)$, $\gamma_2(v_i,v_j) \le \gamma_1(v_i) \wedge \gamma_1(v_j)$ and $0\le \mu_2(v_i,v_j)+ \gamma_2(v_i,v_j)\le 1$.

B. *Definition 2.2.*

An arc (vi,vj) of an IFG G is called an strong arc if $\mu_2(vi ,vj) \le \mu_1(vi) \wedge \mu_1(vj)$ and $\gamma_2(v_i,v_j) \le \gamma_1(v_i) \wedge \gamma_1(v_j)$.

C. *Definition 2.3.*

Let G = (V,E) be a IFG. Then the cardinality of G is defined to be

$|G| = |\sum_{vi\in V} [(1+ \mu_1(v_i) - \gamma_1(v_i))/2] + \sum_{vi\in V} [(1 + \mu_2(v_i , v_j) - \gamma_2(v_i , v_j))/2]|$.

D. *Definition 2.4.*

The vertex cardinality of G is defined by $|V| = \sum_{vi\in v} [(1 + \mu_1(v_i) - \gamma_1(v_i))/2]$ for all $v_i \in V$. The edge cardinality of G is defined by

$|E| = \sum_{vi\in V}[(1 + \mu_2(v_i,v_j) -\gamma_2(v_i,v_j))/2]$ for all $(v_i,v_j) \in E$.

The vertex cardinality of an IFG is called the order of G and it is denoted by O (G). The cardinality of the edges in G is called the size of G, it is denoted by S (G).

E. *Definition 2.5.*

The complement of an IFG,G = (V,E) is an IFG $\overline{G} = (\overline{V}, \overline{E})$, where (i) $\overline{V} = V$, (ii) $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\gamma_{1i}} = \gamma_{1i}$ for all i = 1,2,..n (iii) $\overline{\mu_{2ij}} = \min(\mu_{1i},\mu_{1j}) - \mu_{2ij}$ and $\overline{\gamma_{2ij}} = \max(\gamma_{1i},\gamma_{1j}) - \gamma_{2ij}$ for all i,j =1,2,…n..

F. Definition 2.6.

The effective degree of a vertex v in a IFG, G = (V, G) is defined to be sum of the strong edges incident at v. It is denoted by d_E (v). The minimum degree of G is $\delta_E(G)=\min\{d_E(v)/v\in V\}$. The maximum degree of G is $\Delta_E(G)= \max\{d_E(v)/v\in V\}$. Two vertices Vi and Vj are said to be neighborhood in IFG there is a strong arc between v_i and v_j

G. Definition 2.7.

An IFG H'= (V',E,) is said to be an intuitionistic fuzzy sub graph of G =(V,E) if V'⊆V and E'⊆E. That is $\mu'_{1i} \leq \mu_{1i;}$ $\gamma'_{1i} \geq \gamma_{1i}$ and $\mu'_{2ij} \leq \mu_{2ij}$; $\gamma'_{2ij} \geq \gamma_{2ij}$ for every i,j=1,2,…n.

H. Definition 2.8.

A path in an IFG is a sequence of distinct vertices $v_1,v_2,\ldots,v_n$ such that either one of the following is satisfied (i) $\mu_2(v_i,v_j) > 0$, $\gamma_2(v_i,v_j) > 0$ for some i and j (ii) $\mu_2(v_i,v_j) = 0,\gamma_2(v_i,v_j) > 0$ for some i and j (iii) $\mu_2(v_i,v_j) > 0,\gamma_2(v_i,v_j)= 0$ for some i and j.

I. Definition 2.9.

Let G = (V,E) be an IFG . Let u, v ∈ V, we say that u dominates v in G if there exist a strong arc between them. A subset D ⊆ V is said to be dominating set in G if for every v∈ V-D, there exist u in S such that u dominates v.

J. Definition 2.10.

A dominating set D of IFG is said to be minimal dominating set if no proper subset of S is a dominating set. Minimum cardinality among all minimal dominating set is called the intuitionistic fuzzy domination number, and is denoted by γ_{if} (G).

III. Some Domination Parameters in Intuitionistic Fuzzy Graphs

A. Strong (Weak) Intuitionistic Fuzzy Domination Number

Let u and v be any two vertices of an IFG G. Then u strongly dominates v (v weakly dominates u) if (i) strong arc between u and v (ii) $d_N(u) \geq d_N(v)$.

A subset S of V is called a strong (weak) dominating set in IFG G for every v ∈V-S there exist u∈S such that u strongly dominates v (v weakly dominates u) it is denoted by sifd-set (wifd-set)

A strong (weak) dominating set S of an IFG is said to be minimal strong (weak) dominating set if no proper subset of S is a strong (weak) dominating set of G. The minimum cardinality among all minimal strong (weak) dominating set is called strong (weak) intuitionistic fuzzy domination number of G ,and is denoted by $\gamma_{sif}(G)(\gamma_{wif}(G))$.

B. Connected Intuitionistic Fuzzy Domination Number

Let G be a IFG. A set C ⊆V is a connected dominating set if C is a dominating set and the sub graph <C> induced by C is connected.

Let C ⊆V is said to be minimal connected dominating set if C-{u} is not connected dominating set. The minimum cardinality among all the minimal connected dominating set is called connected dominating number of IFG G, and is denoted by γ_{cif} (G).

C .Efficient Intuitionistic Fuzzy Domination Number

Let G = (V, E) be a IFG. A set F ⊆ V is an efficient dominating set if for every v ∈ V-F then N[v]∩F = 1.

The efficient intuitionistic fuzzy domination number is the minimum cardinality among all the efficient dominating set in G, and is denoted by γ_{eif} (G).

D .Independent Intuitionistic Fuzzy Domination Numbers

Two vertices in an IFG, G = (V,E) are said to be independent if there is no strong arc between them.

A subset I of V is said to be independent set of G if no strong arc between the pair of vertices in I.

Let D ⊆ V is said to be an independent dominating set of an IFG, if D is both an independent and a dominating set of G. The independent dominating number of an IFG G is the minimum cardinality among all the independent dominating set of G, and it is denoted by I_{if} (G).

A SIFD-set (WIFD-set) S of an IFG G is said to be an independent strong (weak) dominating set of G, if it is independent. The minimum cardinality of an independent strong (weak) dominating set is called the independent strong(weak) intuitionistic fuzzy dominating number and it is denoted by $I_{sif}(G)$ $(I_{wif}(G))$.

Example: Let G: (V, E) be a fuzzy graph with V = {a, b, c, d, e, f, g, h}, the membership functions of vertices and edges are given bellow,

$((\mu_1(a),\gamma_1(a)) = (.3,.6)$, $((\mu_1(b),\gamma_1(b)) = (.2,.3)$,
$((\mu_1(c),\gamma_1(c)) = (.4,.5)$, $((\mu_1(d),\gamma_1(d)) = (.6,.2)$,
$((\mu_1(e),\gamma_1(e)) = (.7,.3)$, $((\mu_1(f),\gamma_1(f)) = (.5,.4)$,
$((\mu_1(g),\gamma_1(g)) = (.4,.5)$, $((\mu_1(h),\gamma_1(h)) = (.2,.1)$

And

$((\mu_2(ab),\gamma_2(ab)) = (.2,.6)$, $((\mu_2(bc),\gamma_2(bc)) = (.2,.5)$,
$((\mu_2(bd),\gamma_2(bd)) = (.2,.3)$, $((\mu_2(dc),\gamma_2(dc)) = (.1,.5)$,
$((\mu_2(ce),\gamma_2(ce)) = (.4,.5)$, $((\mu_2(ef),\gamma_2(ef)) = (.5,.4)$,
$((\mu_2(fg),\gamma_2(fg)) = (.4,.5)$, $((\mu_2(eh),\gamma_2(eh)) = (.2,.3)$.

For the given intuitionistic fuggy graph in example, γ_{sif} (G) = 1.6 ,γ_{wif} (G) = 2.5, since {b, e, g} and {a ,c ,d ,g ,h} are the minimal sifd-set and wifd-set respectively. γ_{cif} (G) = 2.15, γ_{eif} (G) = 1.45, since {b,c,e,f} and {b,g,h} are the connected dominating set and efficient domination of the IFG in fig.1 respectively.

For an IFG in example, I_f (G) = 1.45, since {b, g , h} a minimum dominating set of G, and O(G) = 4.2, S(G) = 3.95 , $\Delta_E(G) = 1.55$, $\Delta_N(G) = 2.2$, $\delta_E(G) = 0.3$, $\delta_N(G) = 0.45$.

Remark: If S is a minimal sifd-set, then V-S need not be a wifd-set for example, consider the IFG in example, S = {b, e , g}but the vertices of V-S can't be a weakly dominating set of G.

IV. MAIN RESULTS

Theorem 3.1:

Let D be a minimal sifd-set of an intuitionistic fuzzy graph G. Then for each $v \in D$ if and only if one of the following holds

(i) No vertex in D strongly dominates v

(ii) There exists $u \in V\text{-}D$ such that v is the only vertex in D which strongly dominates u.

Proof: Assume that D is a minimal sifd-set of G .Then for every vertex $v \in D$, D-u is not a strong dominating set and hence there exist $u \in V\text{-}(D\text{-}\{v\})$ which is not strongly dominated by any vertex in D-{v}. If u = v, we get, u is not strongly dominated by vertex in D. If $v \neq u$, u is not strongly dominated by D-{u} but u is strongly dominated by D, then the vertex u is strongly dominated by a vertex v in D.

Conversely, assume that D is a strong dominating set and for each vertex $v \in D$, one of the two conditions holds. Suppose D is not minimal strong dominating set, then there exists a vertex $v \in D$, D-{v} is a strong dominating set. Hence v is strongly dominated by at least one vertex in D-{v}, the condition one does not hold. If D-{v} is a strong dominating set then every vertex in V-D is a strongly dominated by at least one vertex in D-{v}, the second condition does not holds, which contradicts to our assumption that at least one of these conditions holds. So d is minimal strong dominating set.

■

Theorem 3.2:

Let D be a minimal wifd-set of an intuitionistic fuzzy graph G. Then for each $v \in D$ if and only if one of the following holds

(i) No vertex in D weekly dominates v

(ii) There exists $u \in V\text{-}D$ such that v is the only vertex in D which weakly dominates u. ■

Proposition 3.3:

For a intuitionistic fuzzy graph G of order O (G),

(i) $\gamma_{if}(G) \leq \gamma_{sif}(G) \leq O(G)\text{-}\Delta_N(G) \leq O(G)\text{-}\Delta_E(G)$

(ii) $\gamma if(G) \leq \gamma_{wif}(G) \leq O(G)\text{-}\delta_N(G) \leq O(G)\text{-}\delta_E(G)$

Proof: Since every sifd-set (wifd-set) is intuitionistic fuzzy dominating set of IFG G, $\gamma_{if}(G) \leq \gamma_{sif}(G)$ and $\gamma_{if}(G) \leq \gamma_{wif}(G)$. Let $u,v \in V$, if $d_N(u) = \Delta_N(G)$ and $d_N(v) = \delta_N(G)$. Then V-N(u) is a sifd-set but not minimal and V-N(v) is a wifd-set but not minimal. Therefore $\gamma_{sif}(G) \leq |V - N(u)|_{if}$ i.e. $|V\text{-}N(u)|_{if} = |V|\text{-}|N(u)|$

$\Rightarrow O(G) - d_N(u)$

$\Rightarrow O(G) - \Delta_N(G)$

$\Rightarrow \gamma_{sif}(G) \leq O(G)\text{-}\Delta_N(G)$

and , $\gamma_{wif}(G) \leq |V\text{-}N(v)|_{if}$ i.e. $|V\text{-}N(v)|_{if} = |V|\text{-}|N(v)|$

$\Rightarrow O(G) - d_N(v)$

$\Rightarrow O(G) - \delta_N(G)$

$\Rightarrow \gamma_{wif}(G) \leq O(G)\text{-}\delta_N(G)$

Further since $\Delta_E(G) \leq \Delta_N(G)$ and $\delta_E(G) \leq \delta_N(G)$

$\Rightarrow \gamma_{if}(G) \leq \gamma_{sif}(G) \leq O(G)\text{-}\Delta_N(G) \leq O(G)\text{-}\Delta_E(G)$

$\Rightarrow \gamma_{if}(G) \leq \gamma_{wif}(G) \leq O(G)\text{-}\delta_N(G) \leq O(G)\text{-}\delta_E(G)$. ■

Theorem 3.4:

Let G be an IFG, then

$\gamma_{if}(G) \leq \gamma_{cif}(G)$ and $\gamma_{if}(G) \leq \gamma_{eif}(G)$

Proof: Let G be an IFGH. Let C de a connected dominating set. C is also a dominating set.

$\gamma_{if}(G) \leq \gamma_{cif}(G)$

Similarly we get $\gamma_{if}(G) \leq \gamma_{eif}(G)$. ■

Theorem 3.5:

Let G be a IFG, then $\gamma_{sif}(G) \leq \gamma_{wif}(G)$

Proof: Let S, W be the minimal strong and weak dominating set respectively. Let $d_N(u) = \Delta_N(G)$ and $d_N(v) = \delta_N(G)$ note that V-N (u) is a strong dominating set and V-N(v) is a weak dominating set of G

$\gamma_{sif}(G) \leq |V\text{-}N(u)|_{if}$

$\gamma_{sif}(G) \leq O(G) - \Delta_N(G) \rightarrow(1)$

and $\gamma_{wif}(G) \leq |V\text{-}N(v)|_{if} = O(G) - \delta_N(G) \rightarrow(2)$

We know that $O(G) - \Delta_N(G) \leq O(G) - \delta_N(G)$

Using (1) and (2) we get $\gamma_{sif}(G) \leq \gamma_{wif}(G)$. ■

Theorem 3.6:

Let G be an IFG, then $\gamma_{cif}(G) \leq \gamma_{eif}(G)$.

Proof: Let C, E' be the minimum complete, efficient dominating set of G respectively. Let $v \in V\text{-}C$ then $|N[v] \cap C| \geq 1$ for connected dominating set of G and $|N[v] \cap C| = 1$ for efficient dominating set of G.

$\Rightarrow |C| \geq |E'|$

$\Rightarrow \gamma_{cif}(G) \leq \gamma_{eif}(G)$. ■

Lemma 3.7:

Let G be a IFG. If D is an independent strong intuitionistic fuzzy dominating set of G then $D \cap V_{\Delta N} \neq \phi$. Here $V_{\Delta N} = \{v \in V / d_N(v) = \Delta_N(G)\}$

Proof: Let $v \in V_{\Delta N}$, since D is independent strong intuitionistic fuzzy set, $v \in D$ or there exist a vertex $u \in D$ such that $(u, v) \in E$ is a strong arc in G for which $d_N(u) \geq d_N(v)$. $v \in D$, $v \in V_{\Delta N}$ and $v \in D$ clearly $D \cap V_{\Delta N} \neq \phi$.

If $v \notin D$, $d_N(u) \geq d_N(v)$ sine D is strong dominating set ,if $d_N(u) > d_N(v)$ is impossible, since $d_N(v) = \Delta_N(G)$ therefore $d_N(u) = d_N(v)$ this implies $u \in V_{\Delta N}$, therefore $D \cap V_{\Delta N} \neq \phi$. ■

Lemma 3.8:

Let G be an IFG. If D is an independent weak intuitionistic fuzzy dominating set of G then $D \cap V_{\delta N} \neq \phi$. Here $V_{\delta N} = \{v \in V / d_N(v) = \delta_N(G)\}$. ■

Proposition 3.9:

For an intuitionistic fuzzy graph G. $I_{sif} \leq O(G) - \Delta_N(G)$

Proof: Let S be an independent strong intuitionistic fuzzy dominating set. Then by lemma.3.7. $D \cap V_{\Delta N} \neq \phi$. Let $v \in D \cap V_{\Delta N}$. Since D is independent this implies $D \cap N(v) = \phi$.

$\Rightarrow D \subseteq |V - N(v)|_{if} \Rightarrow I_{sif}(G) \leq |D|_{if} \leq O(G) - d_N(v)$

$\Rightarrow I_{sif}(G) \leq O(G) - \Delta_N(G)$ ■

Proposition. 3.10:

For an intuitionistic fuzzy graph G. $I_{wif} \leq O(G) - \delta_N(G)$.

Theorem. 3.11:

If G is an IFG of order O(G), then

$O(G) \backslash [1+\Delta_N(G)] \leq \gamma_{if}(G) \leq O(G) - \Delta_N(G)$.

Proof: Let S be a minimum dominating set of G. Then $V(G) \subseteq \cup_{v \in S} N(v)$

$\Rightarrow |V(G) - S| \leq |S|\,|N(v)|$

$\Rightarrow |V(G)| - |S| \leq |S|\,|N(v)|$

$\Rightarrow O(G) - \gamma_{if}(G) \leq \gamma_{if}(G).d_N(V)$

$\gamma_{if}(G) \geq O(G) \backslash [1 + \Delta_N(G)]$

Let v be a vertex of G with $d_N(V) = \Delta_N(G)$, then V-N(v) is a dominating set of G.

$\Rightarrow |S| \leq |V - N(v)|$

$\gamma_{if}(G) \leq O(G) - \Delta_N(G)$ ■

Theorem 3.12:

If G is an intuitionistic fuzzy graph of order O(G). Let σ1,σ2 is the last two minimum cardinality of the vertex in G, then

$\sigma_2 + 2\sigma_1 \leq \gamma_f(G) + \gamma_f(\overline{G}) \leq O(G) + \sigma_1 \quad \rightarrow (A)$

$\sigma_1^2 + \sigma_1\sigma_2 \leq \gamma_f(G).\gamma_f(\overline{G}) \quad \rightarrow (B)$

Proof: Let σ_1 is the minimum cardinality among all the vertices in G. If $\gamma_f(G) = \sigma_1(v)$ or $\gamma_f(\overline{G}) = \sigma_1(v)$. This implies there is a strong arc between v and the other vertices in G. Then $\gamma_f(\overline{G}) \geq \sigma_1 + \sigma_2$ and $\gamma_f(G) \geq \sigma_1 + \sigma_2$. Therefore $\gamma_f(G) + \gamma_f(\overline{G}) \geq \sigma_1 + \sigma_1 + \sigma_2$ and $\gamma_f(G).\gamma_f(\overline{G}) \geq \sigma_1.(\sigma_1+\sigma_2)$.

$\Rightarrow \gamma_f(G) + \gamma_f(\overline{G}) \geq 2\sigma_1+\sigma_2 \quad \rightarrow (1)$

$\Rightarrow \gamma_f(G).\gamma_f(\overline{G}) \geq \sigma_1^2 + \sigma_1\sigma_2 \quad \rightarrow (2)$

from (1) and (2) the lower bound of (A), (B) are follows immediately.

If G has an isolated vertex, then $\gamma_f(G) \leq O(G)$ and $\gamma_f(\overline{G}) = \sigma_1$ while if $\overline{G}$ has an isolated vertex, then $\gamma_f(\overline{G}) \leq O(G)$ and $\gamma_f(G) = \sigma_1$. So in this case

$\gamma_f(G) + \gamma_f(\overline{G}) \leq O(G) + \sigma_1 \quad \rightarrow (3)$

If neither G nor $\overline{G}$ has isolated vertices then $\gamma_f(G) \leq O(G)/2$ and $\gamma_f(\overline{G}) \leq O(G)/2$

$\Rightarrow \gamma_f(G) + \gamma_f(\overline{G}) \leq O(G) \quad (\therefore O(G) \leq O(G) + \sigma_1)$.

$\Rightarrow \gamma_f(G) + \gamma_f(\overline{G}) \leq O(G) + \sigma_1$ ■

CONCULISION

In this paper we have introduce the concept of strong (weak) domination set, independent dominating set, effective dominating set, connected dominating set for an intuitionistic fuzzy graph . Some interesting results related with the above are proved. Further, the authors proposed to introduce new dominating parameters in intuitionistic fuzzy graph and apply these concepts to the intuitionistic fuzzy graph models.

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