Adaptive Sliding Mode Control for the Design of Chaos-Based Secure Communication Systems

Cheng-Fang Huang, Teh-Lu Liao and Jun-Juh Yan

***Abstract*— In this paper, we study the design of chaos-based secure communication using synchronized master-slave modified Chua's systems. We propose an adaptive sliding mode control (SMC) law to guarantee synchronization of the master and slave modified Chua's systems. Particularly, we introduce the concept of extended systems such that a continuous control input is obtained to avoid chattering phenomenon as frequently in the conventional systems with sliding mode control. Then, it becomes possible to ensure that the message signal embedded in the transmitter can be recovered in the receiver. Finally, simulation results demonstrate the SMC-based synchronization scheme's success in the secure communication application.**

***Index Terms*— Adaptive control; Secure communication; Chaos; Synchronization; Sliding mode control (SMC).**

***MSC 2010 Codes* — 93C40, 93C15, 93D20**

I. INTRODUCTION

IT is widely believed that that the studies for chaos synchronization are more and more popular in recent years. Synchronization of chaos is a key technology in designing a chaos-based secure communication system. As well known, the chaotic system is a very complex, dynamic nonlinear system and its response possesses many intrinsic characteristics such as broadband noise-like waveform, prediction difficulty, sensitivity to initial condition variations, etc. [1, 2]. Although it appears to be stochastic, it occurs in a deterministic nonlinear system under deterministic conditions.

Till now, many methods and techniques in synchronizing chaos have been proposed since the pioneering work of Pecora and Carroll in 1990 [1], such as sliding mode control [3, 4], observer-based control [5], feedback and non-feedback control [6, 7], adaptive control [8-9], backstepping control [10] etc. Moreover, the synchronization of chaotic circuits for the secure communication has received much attention in the literature [5, 11-18].

As well known, synchronization of chaos is a key technology in generating an identical chaotic waveform in both transmitter and receiver for signal decoding in communication systems.

The purpose of this study is to develop a SMC-based chaotic communication system. We first propose a new adaptive SMC-based control scheme to solve the synchronization problem of chaotic modified Chua's systems. The concept of extended systems developed in [19] is introduced such that a continuous adaptive SMC controller is obtained to avoid chattering phenomenon as frequently in the conventional sliding mode control systems.

A switching surface is first proposed, which makes it easy to guarantee the stability of the extended error dynamics in the sliding mode. And then, based on this switching surface, a continuous adaptive SMC is derived to not only guarantee the occurrence of the sliding motion but also avoid the chattering, even when the master system is undergoing unknown message signal. Moreover, the proposed continuous SMC synchronization scheme is then applied to establish a chaotic secure communication system as shown in Fig. 1. Since the proposed SMC controller is used to achieve synchronization between the transmitter and the receiver in communication systems, the message signal embedded by the master chaotic system in the transmitter can be recovered in the receiver by the continuous SMC controller and high performance communication can be obtained.

The remainder of this paper is organized as follows. Section II describes the structure of the proposed secure communication system and formulates the synchronization problem. In Section III, the switching surface which ensures the stability of the extended error system in the sliding mode is derived. Then a continuous adaptive SMC controller is designed to achieve the hitting. In Section IV, we show an illustrative example. Finally, conclusions are presented in Section V.

Throughout this paper, the notation $|w|$ is used to denote the absolute value of w. Moreover, for $x \in R^n$, let $\|x\| = (x^T x)^{1/2}$ denote the Euclidean vector norm. $sign(y)$ is the sign function of y, if $y > 0$, $sign(y) = 1$; if $y = 0$, $sign(y) = 0$; if $y < 0$, $sign(y) = -1$.

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II. System Description and Problem Formulation

For simplicity, we select the modified Chua's system for our design. However, the method developed in this paper can be easily extended for the other class of chaotic systems. Fig. 1 illustrates the proposed communication system designed in this paper, which consists of a transmitter and a receiver (master and slave modified Chua's circuits, respectively).

The message signal $m(t)$ is embedded into the chaotic system in transmitter and the state of master modified Chua's circuit system is simultaneously transmitted to the receiver. A continuous adaptive SMC controller is given in the receiver to achieve synchronization. And then, the input message signal $m(t)$ can be recovered on the side of receiver by the continuous adaptive SMC controller. Before constructing the secure communication system, the first problem undertaken here is how to design a continuous SMC controller to solve the synchronization problem of systems. The master-slave chaotic systems in Fig. 1 are defined below, respectively [20].

Master System X_m :

$$\begin{aligned} \dot{x}_m &= p(y_m - \tfrac{1}{7}(2x_m^3 - x_m)) + m(t) \\ \dot{y}_m &= x_m - y_m + z_m \\ \dot{z}_m &= -qy_m \end{aligned} \tag{1}$$

Slave System X_s :

$$\begin{aligned} \dot{x}_s &= p(y_s - \tfrac{1}{7}(2x_s^3 - x_s)) + d_n(t) + u(t) \\ \dot{y}_s &= x_s - y_s + z_s \\ \dot{z}_s &= -qy_s \end{aligned} \tag{2}$$

where $p > 0$ and $q > 0$ are system parameters, $u(t)$ is the control input proposed later to synchronize master and slave systems (1) and (2), $m(t)$ and $d_n(t)$ are the bounded embedded message and external noise, respectively, which satisfy $|m(t)| \le \delta_m \in R^+$ and $|d_n(t)| \le \delta_n \in R^+$.

It is assumed that the magnitude of δ_n is much smaller than that of δ_m..

Let us define the state errors between the master system (1) and slave system (2) as follows:

$$\begin{aligned} e_x &= x_s - x_m \\ e_y &= y_s - y_m \\ e_z &= z_s - z_m \end{aligned} \tag{3}$$

Then the dynamics of the error system is determined directly from (1) and (2) as follows:

$$\begin{aligned} \dot{e}_x &= p[e_y - e_x(\tfrac{2}{7}(x_s^2 + x_s x_m + x_m^2) - \tfrac{1}{7})] \\ &\quad - m(t) + d_n(t) + u \\ \dot{e}_y &= e_x - e_y + e_z \\ \dot{e}_z &= -qe_y \end{aligned} \tag{4}$$

For deriving the main results, the following assumption is made.

Assumption 1: There exists an unknown and sufficiently large constant κ satisfying

$$\begin{aligned} &\left| \frac{d}{dt} \{ p[e_y - e_x(\tfrac{2}{7}(x_s^2 + x_s x_m + x_m^2) - \tfrac{1}{7})] \right. \\ &\qquad \left. - m(t) + d_n(t) \} \right| \le \kappa < \infty \end{aligned} \tag{5}$$

Remark 1: It is noteworthy that the unknown but existing constant κ is only introduced to prove Theorem 1 later and it is not necessary to know the precise value of κ for our adaptive control design. Thus we can suppose that κ is large enough, even with $\kappa \to \infty$, such that the assumption of (5) will always hold.

Now we newly introduce the concept of extended systems and extend the error dynamics (4) as

$$\begin{aligned} \dot{e}_x &= p[e_y - e_x(\tfrac{2}{7}(x_s^2 + x_s x_m + x_m^2) - \tfrac{1}{7})] \\ &\quad - m(t) + d_n(t) + u = e_E \\ \dot{e}_y &= e_x - e_y + e_z \\ \dot{e}_z &= -qe_y \\ \dot{e}_E &= \frac{d}{dt}[p(e_y - e_x(\tfrac{2}{7}(x_s^2 + x_s x_m + x_m^2) - \tfrac{1}{7})) \\ &\quad - m(t) + d_n(t)] + \dot{u} \end{aligned} \tag{6}$$

The goal of this paper is that for any given modified Chua's circuit systems as (1) and (2), an adaptive SMC controller is designed such that the asymptotical stability of the resulting extended error system (6) can be achieved in the sense that $\lim_{t\to\infty} \|e(t)\| \to 0$, where $e(t) = [e_x, e_y, e_z, e_E]$. Then, the message signal embedded in the transmitter can be recovered in the receiver.

III. Switching Surface and Continuous Adaptive SMC Design

To complete the design of secure communication as shown in Fig. 1, we need to propose a continuous SMC scheme to stabilize the extended error dynamics (6) and achieve synchronization. In the following, we separate the design of continuous SMC scheme into two major phases.

First, we select an appropriate switching surface such that the sliding motion on the sliding manifold is stable. Second, we establish a continuous SMC law which guarantees the attraction of the sliding manifold. To assure the error dynamics (6) in the sliding manifold can be stable asymptotically, the designed switching surface $s(t)$ corresponding to $e(t)$ is given as follows:

$$s(t) = k_x e_x + k_y e_y + k_z e_z + e_E \tag{7}$$

where $s \in R$ and $k_x, k_y, k_z \in R$ are designed constants. According to the works in [21, 22], when the system can operate in the sliding mode, i.e. $s(t) = 0$, the following equation must be satisfied

$$s(t) = k_x e_x + k_y e_y + k_z e_z + e_E = 0 \tag{8}$$

and

$$\dot{s}(t) = k_x \dot{e}_x + k_y \dot{e}_y + k_z \dot{e}_z + \dot{e}_E = 0 \tag{9}$$

From (8), it is obtained

$$e_E = -k_x e_x - k_y e_y - k_z e_z \tag{10}$$

By (6) and (10), we have

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \end{bmatrix} = \begin{bmatrix} -k_x & -k_y & -k_z \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = A \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} \tag{11}$$

Obviously, the error dynamics (11) is exponentially stable if the constants k_x, k_y, k_z are suitable chosen such that the eigenvalues of matrix *A* in (11) are with negative real parts. Also the convergence rate of (11) can be determined by the eigenvalues of matrix *A*. Furthermore, by (10), e_E converges to zero when e_x, e_y and e_z converge to zero.

After establishing an appropriate sliding surface, the next step is to establish a robust control law not only to guarantee the occurrence of the sliding mode but also ensure that the state trajectory can stay on the sliding mode $s = 0$ thereafter even undergoing the unknown message signal. Before stating the adaptive SMC controller design, the Barbalat Lemma is given below.

Lemma 1 (Barbalat Lemma)[23]. If $F : R \to R$ is a uniformly continuous function for $t \geq 0$ and if the limit of the integral

$$\lim_{t\to\infty} \int_0^t |F(\lambda)| d\lambda \tag{12}$$

exists and is finite, then

$$\lim_{t\to\infty} F(t) = 0 \tag{13}$$

To ensure the occurrence of the sliding motion, an adaptive SMC scheme is proposed as

$$\begin{aligned} \dot{u}(t) = &-[k_y e_x - (k_y + qk_z)e_y + k_y e_z + k_x e_E] \\ &- r\hat{\kappa}(t) sign(s(t)), u(0) = u_0 \end{aligned} \tag{14}$$

where $r > 1$ and u_0 is the bounded initial value of $u(t)$.

The adaptive law is

$$\dot{\hat{\kappa}}(t) = \theta |s(t)|, \ \hat{\kappa}(0) = \kappa_0 \tag{15}$$

where $\theta > 0$ and κ_0 is the bounded initial value of $\hat{\kappa}(t)$.

The adaptive SMC controller (14) can be also written in the integral form as

$$\begin{aligned} u(t) = -\int_0^t &\{[k_y e_x - (k_y + qk_z)e_y + k_y e_z + k_x e_E] \\ &+ r\hat{\kappa}(t) sign(s(t))\} dt + u_0 \end{aligned} \tag{16}$$

and

$$\hat{\kappa}(t) = \theta \int_0^t |s(t)| dt + \kappa_0 \tag{17}$$

Remark 2: In the conventional SMC controller, the control scheme is often discontinuous which causes ‘chattering’ in the sliding mode. However, the proposed adaptive controller as shown in (15) or (17) is continuous. Therefore, the ‘chattering’ in the sliding mode will be removed. Furthermore, the property of ‘chattering free’ makes it possible and easy to recover the embedded message on the receiver side.

Next, the proposed adaptive SMC (14) will be proved to be able to drive the extended error dynamics (6) onto the sliding mode $s(t) = 0$.

Theorem 1. Consider the extended error dynamics (6), if the control input $u(t)$ is suitably designed as (14) with adaptation law (15), then the trajectory of the error dynamics (6) converges to the switching surface $s(t) = 0$.

Proof. Consider the following Lyapunov function candidate

$$V(t) = \tfrac{1}{2}(s^2 + \theta^{-1}\delta^2); \theta > 0 \tag{18}$$

where $\delta(t) \in R$ denotes the adaptation error (to be defined).

Taking the derivative of $V(t)$ with respect to time, one has

$$\begin{aligned} \dot{V}(t) &= s\dot{s} + \theta^{-1}\delta\dot{\delta} \\ &= s(k_x \dot{e}_x + k_y \dot{e}_y + k_z \dot{e}_z + \dot{e}_E) + \theta^{-1}\delta\dot{\delta} \\ &= s\{\frac{d}{dt}[p(e_y - e_x(\tfrac{2}{7}(x_s^2 + x_s x_m + x_m^2) - \tfrac{1}{7})) \\ &\quad - m(t) + d_n(t))] + k_y e_x - (k_y + qk_z)e_y \\ &\quad + k_y e_z + k_x e_E + \dot{u}\} + \theta^{-1}\delta\dot{\delta} \\ &\leq |s| \ \kappa - r \cdot \hat{\kappa}(t) \cdot \ s \cdot sign(s) + \theta^{-1} \delta\dot{\delta} \\ &\leq [\kappa - r\hat{\kappa}(t)]|s| + \theta^{-1}\delta\dot{\delta} \end{aligned} \tag{19}$$

Now let $\delta(t) = \kappa - \hat{\kappa}(t)$ denote the adaptation error. Since κ is constant, $\dot{\kappa} = 0$ and the following expression holds.

$$\dot{\delta}(t) = -\dot{\hat{\kappa}}(t) \tag{20}$$

Inserting (20) into the R.H.S. of inequality (19), this yields

$$\dot{V}(t) \le \{[\overbrace{\kappa - \hat{\kappa}(t)}^{\delta}] + (1-r)\hat{\kappa}(t)\}|s| - \theta^{-1}\delta\dot{\hat{\kappa}}(t) \tag{21}$$

By placing (15) into (21), we get

$$\dot{V}(t) \le (1-r)\hat{\kappa}(t)|s| \le -F(t) \le 0 \tag{22}$$

where $F(t) = (r-1)\hat{k}(t)|s| \ge 0.$ Integrating the above equation from zero to *t*, it yields

$$V(0) \ge V(t) + \int_0^t F(\lambda)d\lambda \ge \int_0^t F(\lambda)d\lambda. \tag{23}$$

As $t \to \infty$, the above integral is always less than or equal to $V(0)$. Since $V(0)$ is positive and finite, $\lim_{t\to\infty}\int_0^t F(\lambda)d\lambda$ exists and is finite.

Thus according to Barbalat Lemma (see **Lemma 1**), we obtain

$$\lim_{t\to\infty} F(t) = \lim_{t\to\infty}(r-1)\hat{\kappa}(t)|s| = 0. \tag{24}$$

Since both $(r-1)$ and $\hat{\kappa}(t)$ are greater than zero, Equation (24) implies $s = 0$. The theorem is therefore proved. ■

The following theorem is introduced to guarantee the asymptotical stability of the closed-loop error system.

Theorem 2. The error system (4) driven by the controller $u(t)$ expressed in (14) with adaptation law (15) is globally stable.

Proof. Using the concept of extended systems, the error dynamics (4) can be extended as the extended error dynamics (6). When the extended error dynamics (6) is driven by the control input $u(t)$ given in (14) with adaptation law (15), as previously discussed in Theorem 1, the trajectory of the error dynamics system (6) surely converges to the sliding mode $s = 0$. Thus the equivalent error dynamics in the sliding mode is obtained as

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \end{bmatrix} = \begin{bmatrix} -k_x & -k_y & -k_z \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = A \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} \tag{25}$$

Furthermore, since the design parameters k_x, k_y and k_z are specified to ensure $\lambda_{\max}(A) < 0$, the stability of (25) is surely guaranteed, i.e. $\lim_{t\to\infty}\left\|\begin{bmatrix} e_x & e_y & e_z \end{bmatrix}\right\| = 0.$

Furthermore, by the relation

$$s(t) = k_x e_x + k_y e_y + k_z e_z + e_E = 0,$$

$e_E(t)$ is also stable, i.e. $\lim_{t\to\infty} e_E(t) = 0.$

Consequently, the asymptotical stability of the closed-loop error system is also ensured. The theorem is therefore proved. ■

When the error dynamics converges to zero as discussed in Theorem 2, we can obtain the following result from (4):

$$\begin{aligned} \dot{e}_x &= p(e_y - e_x(\tfrac{2}{7}(x_2^2 + x_2x_1 + x_1^2) + \tfrac{1}{7}) \\ &\quad - m(t) + d_n(t) + u = 0 \\ \dot{e}_y &= e_x - e_y + e_z = 0 \\ \dot{e}_z &= -qe_y = 0 \end{aligned} \tag{26}$$

Then, from (26), we can deduce that

$$\lim_{t\to\infty}(u(t) - m(t) + d_n(t)) = 0 \tag{27a}$$

If the magnitude of δ_n is much smaller than that of δ_m, then we have

$$u(t) = m(t) - d_n(t) \cong m(t). \tag{27b}$$

In other words, the message signal $m(t)$ can be recovered in the receiver from the continuous control input $u(t)$.

IV. Numerical Simulation

In this section, simulation results are presented to demonstrate and verify the performance of the present design. The parameters p and q are chosen as $p = 10$ and $q = \frac{100}{7}$ [20] in the simulation to ensure the existence of chaos for the master system (1).

Assume $d_n(t) = 0.01\sin(10t)$ and the initial states of the master system (1) are

$$x_m(0) = 0.65,\ y_m(0) = 0,\ z_m(0) = 0$$

and initial states of the slave system (2) are

$$x_s(0) = -1,\ y_s(0) = 1,\ z_s(0) = -2.$$

For simulation, we embed a sine wave $m(t) = 0.2\sin(2t)$ into the dynamics of master system. And then, as described above, the proposed design procedure can be summarized as follows:

Step 1: According to (11), we select

$$k_x = 8,\ k_y = 3.7143 \text{ and } k_z = 6.32$$

such that $\lambda(A) = (-2, -3, -4)$ which results in a stable sliding mode. Therefore the switching surface equation is obtained as

$$s(t) = 8e_x + 3.7143e_y + 6.32e_z + e_E \tag{28}$$

Step 2: From (16) and (17), the continuous control input is determined as

$$u(t) = -\int_0^t \{ [k_y e_x - (k_y + qk_z)e_y + k_y e_z + k_x e_E] + r\hat{\kappa}(t) sign(s(t)) \} dt + u_0 \quad (29)$$

where $r = 1.1 > 1$ to guarantee the existence of the sliding motion and $u_0 = 0$.

The adaptive law is

$$\hat{\kappa}(t) = \theta \int_0^t |s(t)| dt + \kappa_0 \quad (30)$$

where $\theta = 1$ and $\hat{\kappa}_0 = 1$.

The simulation results are shown in Fig. 2-5 under the proposed continuous adaptive SMC (29). Fig. 2 shows the time responses of corresponding $s(t)$ and adaptation parameter $\hat{\kappa}(t)$. Fig. 3 and 4 shows, respectively, the state and error state responses of the controlled master-slave modified Chua's system. From the simulation result, it shows that the trajectory of error dynamics do converge to $s(t) = 0$ and the synchronization error also converges to zero. Thus the proposed adaptive SMC works well and master and slave modified Chua's systems from different initial values are indeed achieving chaos synchronization. Also the chattering does not appear due to the continuous control input. Finally, Fig. 5 depicts the simulations of chaotic secure communication for message signal given above. The solid line indicates the transmitted message signal $m(t)$ and the dash line denotes the recovered message under the effect of external noise $d_n(t)$. Clearly, these results prove that the master and slave systems synchronization can be achieved as well as the hidden message for secure communication can be recovered in the slave system.

V. Conclusions

By introducing the concept of extended systems, a continuous adaptive sliding mode controller has been derived to guarantee the synchronization between the master and slave systems. Then the proposed scheme has been also successfully applied to establish the secure communication system. Numerical simulations have verified the effectiveness of the proposed method.

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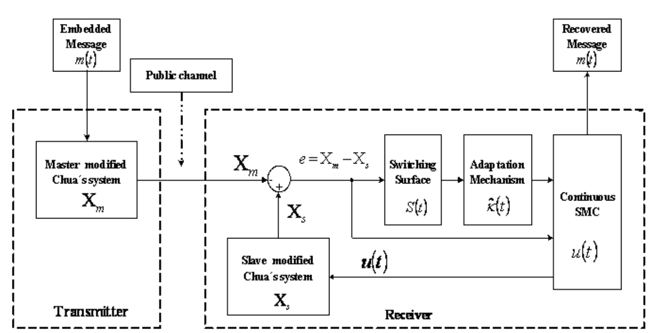

Fig. 1. Block diagram of Chaotic Communication System

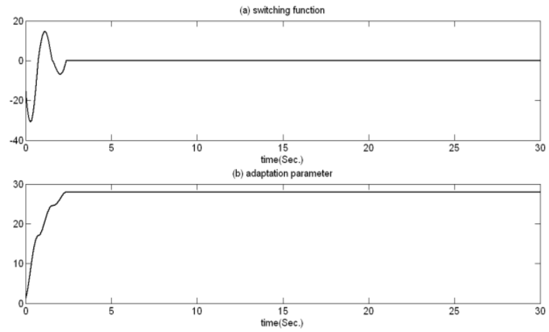

Fig. 2. (a) Switching function $s(t)$; (b) Adaptation Parameter $\hat{\kappa}(t)$.

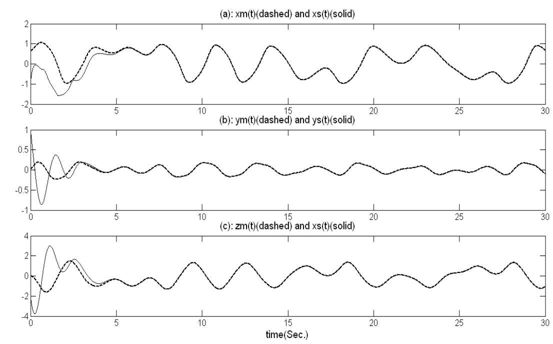

Fig. 3. The State Responses of Controlled Master-Slave systems.

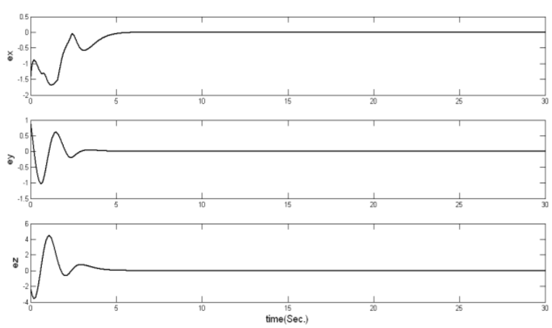

Fig. 4. The Time Responses of Synchronization Error

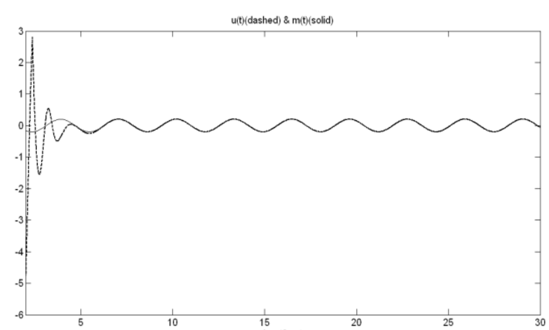

Fig. 5. The Original and Recovered Message Signals.