On Total Edge Product Cordial Labeling

S. K. Vaidya and C. M. Barasara

***Abstract*—The total edge product cordial labeling is a variant of cordial labeling in general and total product cordial labeling in particular. Here we investigate total edge product cordial labeling of some cycle and path related graphs.**

***Index Terms*—Cordial Labeling, Product Cordial Labeling, Total Edge Product Cordial Labeling.**

***MSC 2010 Codes* - 05C38, 05C76, 05C78.**

I. Introduction

THE graph labeling is one of the concepts in graph theory which has attracted many researchers to work on it because of its diversified and rigorous application in fields such as design and analysis of communication networks, military surveillance, social sciences, optimization and linear algebra. Many types of graph labelings are available in the existing literature. Many types of graph labelings are available in the existing literature. A brief account of comprehensive bibliography of papers on the concept of graph labeling is available in *A dynamic survey of graph labeling* by Gallian [1].

We begin with finite, undirected and connected graph $G = (V(G), E(G))$ without loops and multiple edges. The members of $V(G)$ and $E(G)$ are commonly termed as graph elements. For the various grpah theoretic notations and terminology, we follow Balakrishnan and Ranganathan [2].

Definition 1.1 : A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices(edges) then the labeling is called *a vertex(an edge) labeling*.

In 1987, Cahit [3] has introduced the concept of cordial labeling. Some labeling schemes are also introduced with minor variations in cordial theme. Some of them are A - Cordial labeling, H - Cordial labeling, Prime cordial labeling, Product cordial labeling, Total product cordial labeling and Edge product cordial labeling.

Definition 1.2 : For a graph G, an edge labeling function $f : E(G) \to \{0, 1\}$ induces a vertex labeling function $f^* : V(G) \to \{0, 1\}$ defined as $f^*(v) = \prod\{f(uv)/uv \in E(G)\}$.

Let us denote $v_f(i)$ is the number of vertices of G having label i under f^* and $e_f(i)$ is the number of edges of G having label i under f for $i = 0, 1$.

The function f is called a *total edge product cordial labeling* of G if $|(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))| \leq 1$. A graph is called *total edge product cordial* if it admits total edge product cordial labeling.

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Vaidya and Barasara [4] have introduced the concept of total edge product cordial labeling and in the same paper they have investigated total edge product cordial labeling for some common graph families.

For the definitions of various graphs considered in the succeeding section the readers are advised to refer Vaidya and Barasara [5].

Here we investigate total edge product cordial labeling for some cycle and path related graphs.

II. Total Edge Product Cordial Labeling of Some Cycle Related Graphs

Theorem 2.1 : C_n^2 admits total edge product cordial labeling.

Proof : For the graph C_n^2 we have $|V(C_n^2)| = n$ and $|E(C_n^2)| = 2n$. To define $f : E(C_n^2) \to \{0, 1\}$, we will consider following four cases.

Case 1: When $n = 3$.

The graph C_3^2 is same as C_3 and it is total edge product cordial graph as proved by Vaidya and Barasara [4].

Case 2: For $n = 4k$ or $n = 4k + 2$, $k = 1, 2, \ldots$

$$f(v_{2i-1}v_{2i}) = 0; \quad 1 \leq i \leq \frac{n}{2}$$
$$f(v_iv_j) = 1; \quad \text{for remaining } \frac{3n}{2} \text{ edges}$$

Therefore the number of graph elements with label 0 is $\frac{3n}{2}$, the number of graph elements with label 1 is $\frac{3n}{2}$ and their difference is 0.

Case 3: For $n = 4k + 1$, $k = 1, 2, \ldots$

$$f(v_iv_{i+1}) = 0; \quad 1 \leq i \leq \frac{3n-3}{4}$$
$$f(v_iv_j) = 1; \quad \text{for remaining } \frac{5n+3}{4} \text{ edges}$$

Therefore the number of graph elements with label 0 is $\frac{3n-1}{2}$, the number of graph elements with label 1 is $\frac{3n+1}{2}$ and their difference is 1.

Case 4: For $n = 4k + 3$, $k = 1, 2, \ldots$

$$f(v_iv_{i+1}) = 0; \quad 1 \leq i \leq \frac{3n-1}{4}$$
$$f(v_iv_j) = 1; \quad \text{for remaining } \frac{5n+1}{4} \text{ edges}$$

Therefore the number of graph elements with label 0 is $\frac{3n+1}{2}$, the number of graph elements with label 1 is $\frac{3n-1}{2}$ and their difference is 1.

Thus in all the cases described above the function f satisfies the condition for total edge product cordial labeling. Hence the result.

Illustration 2.2 : Total edge product cordial labeling of C_6^2 is shown in Figure 1.

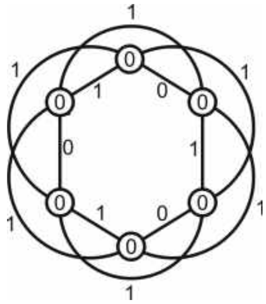

Figure 1

Theorem 2.3 : $D_2(C_n)$ admits total edge product cordial labeling.

Proof : For graph $D_2(C_n)$ let $v'_1, v'_2, ..., v'_n$ be the vertices of first copy of cycle C_n and $v''_1, v''_2, ..., v''_n$ be the vertices of second copy of cycle C_n. Then $|V(D_2(C_n))| = 2n$ and $|E(D_2(C_n))| = 4n$. To define $f : E(D_2(C_n)) \to \{0, 1\}$, we will consider following two cases.

Case 1: For $n = 2k,\ k = 1, 2, \ldots$

$$\begin{aligned} f(v'_i v'_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(v''_i v''_{i+1}) &= 0; \quad 1 \le i \le \frac{n}{2} \\ f(v'_1 v'_n) &= 1; \\ f(v'_i v''_j) &= 1; \quad \text{for all } 2n \text{ edges} \\ f(v''_i v''_j) &= 1; \quad \frac{n+2}{2} \le i \le n-1 \\ f(v''_1 v''_n) &= 1; \end{aligned}$$

Case 2: For $n = 2k+1,\ k = 1, 2, \ldots$

$$\begin{aligned} f(v'_i v'_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(v'_i v''_{i+1}) &= 0; \quad 1 \le i \le \frac{n+1}{2} \\ f(v'_1 v'_n) &= 1; \\ f(v'_i v''_j) &= 1; \quad \text{for remaining } \frac{3n-1}{2} \text{ edges} \\ f(v''_i v''_{i+1}) &= 1; \quad 1 \le i \le n-1 \\ f(v''_1 v''_n) &= 1; \end{aligned}$$

In both the cases the number of graph elements with label 0 is $3n$, the number of graph elements with label 1 is $3n$ and their difference is 0.

Hence the result.

Illustration 2.4 : Total edge product cordial labeling of $D_2(C_6)$ is shown in Figure 2.

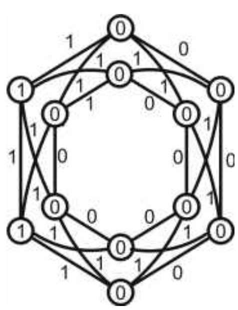

Figure 2

Theorem 2.5 : $M(C_n)$ admits total edge product cordial labeling.

Proof : For the graph $M(C_n)$ we have $|V(M(C_n))| = 2n$ and $|E(M(C_n))| = 3n$. To define $f : E(M(C_n)) \to \{0, 1\}$, we will consider following four cases.

Case 1: For $n = 4k,\ k = 1, 2, \ldots$

$$\begin{aligned} f(e_i e_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(e_i v_i) &= 0; \quad 1 \le i \le \frac{n}{4} \\ f(e_2 v_1) &= 0; \\ f(e_1 e_n) &= 1; \\ f(e_i v_j) &= 1; \quad \text{for remaining } \frac{7n-4}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n}{2}$, the number of graph elements with label 1 is $\frac{5n}{2}$ and their difference is 0.

Case 2: For $n = 4k+1,\ k = 1, 2, \ldots$

$$\begin{aligned} f(e_i e_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(e_i v_i) &= 0; \quad 1 \le i \le \frac{n-1}{4} \\ f(e_2 v_1) &= 0; \\ f(e_1 e_n) &= 1; \\ f(e_i v_j) &= 1; \quad \text{for remaining } \frac{7n-3}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n-1}{2}$, the number of graph elements with label 1 is $\frac{5n+1}{2}$ and their difference is 1.

Case 3: For $n = 4k+2,\ k = 1, 2, \ldots$

$$\begin{aligned} f(e_i e_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(e_i v_i) &= 0; \quad 1 \le i \le \frac{n+2}{4} \\ f(e_1 e_n) &= 1; \\ f(e_i v_j) &= 1; \quad \text{for remaining } \frac{7n-2}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n}{2}$, the number of graph elements with label 1 is $\frac{5n}{2}$ and their difference is 0.

Case 4: For $n = 4k+3,\ k = 0, 1, \ldots$

$$\begin{aligned} f(e_i e_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(e_i v_i) &= 0; \quad 1 \le i \le \frac{n+1}{4} \\ f(e_1 e_n) &= 1; \\ f(e_i v_j) &= 1; \quad \text{for remaining } \frac{7n-1}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n-1}{2}$, the number of graph elements with label 1 is $\frac{5n+1}{2}$ and their difference is 1.

Thus in all the cases described above the function f satisfies the condition for total edge product cordial labeling. Hence the result.

Illustration 2.6 : Total edge product cordial labeling of $M(C_6)$ is shown in Figure 3.

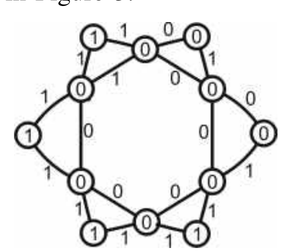

Figure 3

Theorem 2.7 : $T(C_n)$ admits total edge product cordial labeling.

Proof : For the graph $T(C_n)$ we have $|V(T(C_n))| = 2n$ and $|E(T(C_n))| = 4n$. To define $f : E(T(C_n)) \to \{0, 1\}$, we will consider following two cases.

<u>**Case 1:** For $n = 2k,\ k = 1, 2, \ldots$</u>

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(e_i e_{i+1}) &= 0; \quad 1 \le i \le \frac{n}{2} \\ f(e_i e_j) &= 1; \quad \frac{n+2}{2} \le i \le n-1 \\ f(e_1 e_n) &= 1; \\ f(v_1 v_n) &= 1; \\ f(e_i v_j) &= 1; \quad \text{for all } 2n \text{ edges} \end{aligned}$$

<u>**Case 2:** For $n = 2k+1,\ k = 1, 2 \ldots$</u>

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(e_i v_i) &= 0; \quad 1 \le i \le \frac{n+1}{2} \\ f(e_i e_{i+1}) &= 1; \quad 1 \le i \le n-1 \\ f(e_1 e_n) &= 1; \\ f(v_1 v_n) &= 1; \\ f(e_i v_j) &= 1; \quad \text{for remaining } \frac{n-1}{2} \text{ edges} \end{aligned}$$

In both the cases we have the number of graph elements with label 0 is $3n$, the number of graph elements with label 1 is $3n$ and their difference is 0.

Hence the result.

Illustration 2.8 : Total edge product cordial labeling of $T(C_6)$ is shown in Figure 4.

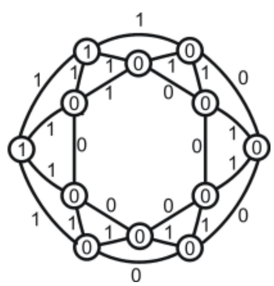

Figure 4

Theorem 2.9 : $S'(C_n)$ admits total edge product cordial labeling.

Proof : Let C_n be the graph with vertices $v_1, v_2, \ldots, v_n$. For graph $S'(C_n)$ added vertices corresponding to $v_1, v_2, \ldots, v_n$ are $v'_1, v'_2, \ldots, v'_n$. Then $|V(S'(C_n))| = 2n$ and $|E(S'(C_n))| = 3n$. To define $f : E(S'(C_n)) \to \{0, 1\}$, we will consider following four cases.

<u>**Case 1:** For $n = 4k,\ k = 1, 2, \ldots$</u>

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \le i \le n-3 \\ f(v_i v'_{i+1}) &= 0; \quad 1 \le i \le \frac{n+4}{4} \\ f(v_{n-1} v_n) &= 0; \\ f(v_{n-2} v_{n-1}) &= 1; \\ f(v_n v_1) &= 1; \\ f(v_i v'_j) &= 1; \quad \text{for remaining } \frac{7n-4}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n}{2}$, the number of graph elements with label 1 is $\frac{5n}{2}$ and their difference is 0.

<u>**Case 2:** For $n = 4k+1,\ k = 1, 2, \ldots$</u>

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(v_i v'_{i+1}) &= 0; \quad 1 \le i \le \frac{n+3}{4} \\ f(v_n v_1) &= 1; \\ f(v_i v'_j) &= 1; \quad \text{for remaining } \frac{7n-3}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n+1}{2}$, the number of graph elements with label 1 is $\frac{5n-1}{2}$ and their difference is 1.

<u>**Case 3:** For $n = 4k+2,\ k = 1, 2, \ldots$</u>

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(v_i v'_{i+1}) &= 0; \quad 1 \le i \le \frac{n+2}{4} \\ f(v_n v_1) &= 1; \\ f(v_i v'_j) &= 1; \quad \text{for remaining } \frac{7n-2}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n}{2}$, the number of graph elements with label 1 is $\frac{5n}{2}$ and their difference is 0.

<u>**Case 4:** For $n = 4k+3,\ k = 0, 1, \ldots$</u>

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \le i \le n-1 \\ f(v_i v'_{i+1}) &= 0; \quad 1 \le i \le \frac{n+1}{4} \\ f(v_n v_1) &= 1; \\ f(v_i v'_j) &= 1; \quad \text{for remaining } \frac{7n-1}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n-1}{2}$, the number of graph elements with label 1 is $\frac{5n+1}{2}$ and their difference is 1.

Thus in all the cases described above the function f satisfies the condition for total edge product cordial labeling. Hence the result.

Illustration 2.10 : Total edge product cordial labeling of $S'(C_6)$ is shown in Figure 5.

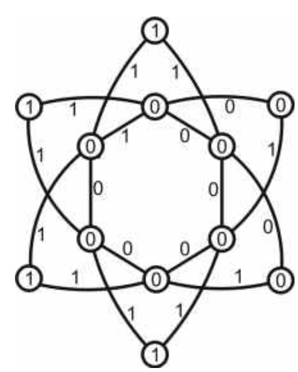

Figure 5

III. Total Edge Product Cordial Labeling of Some Path Related Graphs

Theorem 3.1 : P_n^2 admits total edge product cordial labeling except for $n = 2$.

Proof : For the graph P_n^2 we have $|V(P_n^2)| = n$ and $|E(P_n^2)| = 2n-3$. To define $f : E(P_n^2) \rightarrow \{0,1\}$, we will consider following four cases.

Case 1: For $n = 2$

The graph P_2^2 is same as P_2 and it is not total edge product cordial graph as proved by Vaidya and Barasara [4].

Case 2: For $n = 4k+1,\ k = 1, 2, \ldots$

$$
\begin{aligned}
f(v_i v_{i+1}) &= 0; \quad 1 \leq i \leq \frac{3n-11}{4} \\
f(v_{n-1} v_n) &= 0; \\
f(v_i v_j) &= 1; \quad \text{for remaining } \frac{5n-5}{4} \text{ edges}
\end{aligned}
$$

Therefore the number of graph elements with label 0 is $\frac{3n-3}{2}$, the number of graph elements with label 1 is $\frac{3n-3}{2}$ and their difference is 0.

Case 3: For $n = 4k+3,\ k = 0, 1, \ldots$

$$
\begin{aligned}
f(v_i v_{i+1}) &= 0; \quad 1 \leq i \leq \frac{3n-5}{4} \\
f(v_i v_j) &= 1; \quad \text{for remaining } \frac{5n-7}{4} \text{ edges}
\end{aligned}
$$

Therefore the number of graph elements with label 0 is $\frac{3n-3}{2}$, the number of graph elements with label 1 is $\frac{3n-3}{2}$ and their difference is 0.

Case 4: For $n = 4k$ or $n = 4k+2,\ k = 1, 2, \ldots$

In Vaidya and Barasara [5] it has been proved that P_n^2 is edge product cordial graph for even n and P_n^2 is of even order. Every edge product cordial graph of either even size or even order is total edge product cordial as proved by Vaidya and Barasara [4].

Hence from case 1 to case 4 the result follows.

Illustration 3.2 : Total edge product cordial labeling of P_6^2 is shown in Figure 6.

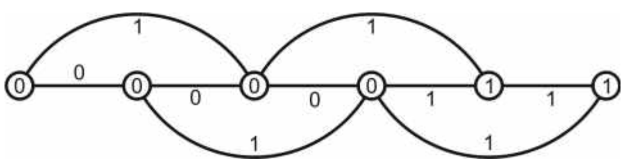

Figure 6

Theorem 3.3 : $D_2(P_n)$ admits total edge product cordial labeling except for $n = 2$.

Proof : For graph $D_2(P_n)$ let $v'_1, v'_2, ..., v'_n$ be the vertices of first copy of path P_n and $v''_1, v''_2, ..., v''_n$ be the vertices of second copy of path P_n. Then $|V(D_2(P_n))| = 2n$ and $|E(D_2(P_n))| = 4n-4$. To define $f : E(D_2(P_n)) \rightarrow \{0,1\}$, we will consider following three cases.

Case 1: When $n = 2$

The graph $D_2(P_n)$ is same as C_4 and it is not total edge product cordial graph as proved by Vaidya and Barasara [4].

Case 2: For $n = 2k,\ k = 2, 3, \ldots$

$$
\begin{aligned}
f(v'_i v'_{i+1}) &= 0; \quad 1 \leq i \leq n-1 \\
f(v''_i v''_{i+1}) &= 0; \quad 1 \leq i \leq \frac{n-2}{2} \\
f(v'_i v''_j) &= 1; \quad \text{for all } 2n-2 \text{ edges} \\
f(v''_i v''_j) &= 1; \quad \frac{n}{2} \leq i \leq n-1
\end{aligned}
$$

Case 3: For $n = 2k+1,\ k = 1, 2, \ldots$

$$
\begin{aligned}
f(v'_i v'_{i+1}) &= 0; \quad 1 \leq i \leq n-1 \\
f(v'_i v''_{i+1}) &= 0; \quad 1 \leq i \leq \frac{n-1}{2} \\
f(v'_i v''_j) &= 1; \quad \text{for remaining } \frac{3n-3}{2} \text{ edges} \\
f(v''_i v''_j) &= 1; \quad 1 \leq i \leq n-1
\end{aligned}
$$

In case 2 and 3, we have the number of graph elements with label 0 is $3n-2$, the number of graph elements with label 1 is $3n-2$ and their difference is 0.

Hence the result.

Illustration 3.4 : Total edge product cordial labeling of $D_2(P_6)$ is shown in Figure 7.

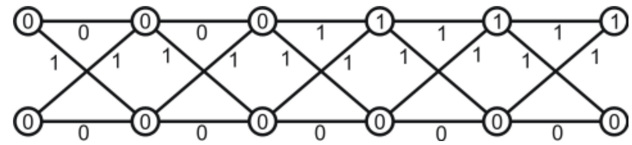

Figure 7

Theorem 3.5 : $M(P_n)$ admits total edge product cordial graph.

Proof : For the graph $M(P_n)$ we have $|V(M(P_n))| = 2n-1$ and $|E(M(P_n))| = 3n-4$. To define $f : E(M(P_n)) \rightarrow \{0,1\}$, we will consider following three cases.

Case 1: For $n = 4k+1,\ k = 1, 2, \ldots$

$$
\begin{aligned}
f(e_i e_{i+1}) &= 0; \quad 1 \leq i \leq n-2 \\
f(e_i v_{i+1}) &= 0; \quad 1 \leq i \leq \frac{n-1}{4} \\
f(e_2 v_2) &= 0; \\
f(e_i v_j) &= 1; \quad \text{for remaining } \frac{7n-11}{4} \text{ edges}
\end{aligned}
$$

Therefore the number of graph elements with label 0 is $\frac{5n-5}{2}$, the number of graph elements with label 1 is $\frac{5n-5}{2}$ and their difference is 0.

Case 2: For $n = 4k+3,\ k = 0, 1, \ldots$

$$
\begin{aligned}
f(e_i e_{i+1}) &= 0; \quad 1 \leq i \leq n-2 \\
f(e_i v_{i+1}) &= 0; \quad 1 \leq i \leq \frac{n+1}{4} \\
f(e_i v_j) &= 1; \quad \text{for remaining } \frac{7n-9}{4} \text{ edges}
\end{aligned}
$$

Therefore the number of graph elements with label 0 is $\frac{5n-5}{2}$, the number of graph elements with label 1 is $\frac{5n-5}{2}$ and their difference is 0.

Case 3: For $n = 4k,\ k = 1, 2, \ldots$ or $n = 4k+2,\ k = 0, 1, \ldots$

In Vaidya and Barasara [5] it has been proved that $M(P_n)$ is edge product cordial graph for even n and $M(P_n)$ is of even size. Every edge product cordial graph of either even size or even order is total edge product cordial as proved by Vaidya and Barasara [4].

Hence from case 1 to case 3 the result follows.

Illustration 3.6 : Total edge product cordial labeling of $M(P_6)$ is shown in Figure 8.

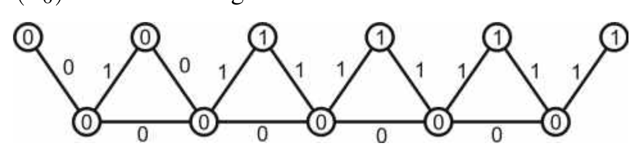

Figure 8

Theorem 3.7 : $T(P_n)$ admits total edge product cordial graph.

Proof : For the graph $T(P_n)$ we have $|V(T(P_n))| = 2n - 1$ and $|E(T(P_n))| = 4n - 5$. To define $f : E(T(P_n)) \rightarrow \{0, 1\}$, we will consider following two cases.

Case 1: For $n = 2k + 1,\ k = 1, 2, \ldots$

In Vaidya and Barasara [5] it has been proved that $T(P_n)$ is edge product cordial graph for even n and $T(P_n)$ is of even order. Every edge product cordial graph of either even size or even order is total edge product cordial as proved by Vaidya and Barasara [4].

Case 2: For $n = 2k,\ k = 1, 2, \ldots$

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \leq i \leq n - 1 \\ f(v_i e_i) &= 0; \quad 1 \leq i \leq \frac{n-2}{2} \\ f(e_i e_j) &= 1; \quad 1 \leq i \leq n - 2 \\ f(e_i v_j) &= 1; \quad \text{for remaining } \frac{3n-2}{2} \text{ edges} \end{aligned}$$

Thus we have the number of graph elements with label 0 is $3n - 3$, the number of graph elements with label 1 is $3n - 3$ and their difference is 0.

Hence the result.

Illustration 3.8 : Total edge product cordial labeling of $T(P_6)$ is shown in Figure 9.

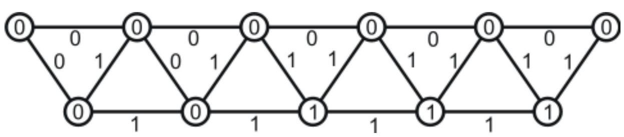

Figure 9

Theorem 3.9 : $S'(P_n)$ admits total edge product cordial graph.

Proof : Let P_n be the graph with vertices $v_1, v_2, \ldots, v_n$. For graph $S'(P_n)$ added vertices corresponding to $v_1, v_2, \ldots, v_n$ are $v'_1, v'_2, \ldots, v'_n$. Then $|V(S'(P_n))| = 2n$ and $|E(S'(P_n))| = 3n - 3$. To define $f : E(S'(P_n)) \rightarrow \{0, 1\}$, we will consider following four cases.

Case 1: For $n = 3$

Total edge product cordial labeling of $S'(P_3)$ is shown in Figure 10.

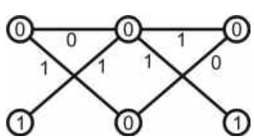

Figure 10

Case 2: For $n = 4k,\ k = 1, 2, \ldots$ or $n = 4k + 2,\ k = 0, 1, \ldots$

In Vaidya and Barasara [5] it has been proved that $S'(P_n)$ is edge product cordial graph for even n and $S'(P_n)$ is of even order. Every edge product cordial graph of either even size or even order is total edge product cordial as proved by Vaidya and Barasara [4].

Case 3: For $n = 4k + 1,\ k = 1, 2, \ldots$

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \leq i \leq n - 1 \\ f(v_i v'_{i+1}) &= 0; \quad 1 \leq i \leq \frac{n-1}{4} \\ f(v_i v'_j) &= 1; \quad \text{for remaining } \frac{7n-7}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n-3}{2}$, the number of graph elements with label 1 is $\frac{5n-3}{2}$ and their difference is 0.

Case 4: For $n = 4k + 3,\ k = 1, 2, \ldots$

$$\begin{aligned} f(v_i v_{i+1}) &= 0; \quad 1 \leq i \leq n - 1 \\ f(v_i v'_{i+1}) &= 0; \quad 1 \leq i \leq \frac{n-3}{4} \\ f(v_3 v'_2) &= 0; \\ f(v_i v'_j) &= 1; \quad \text{for remaining } \frac{7n-9}{4} \text{ edges} \end{aligned}$$

Therefore the number of graph elements with label 0 is $\frac{5n-3}{2}$, the number of graph elements with label 1 is $\frac{5n-3}{2}$ and their difference is 0.

Hence from case 1 to case 4 the result follows.

Illustration 3.10 : Total edge product cordial labeling of $S'(P_6)$ is shown in Figure 11.

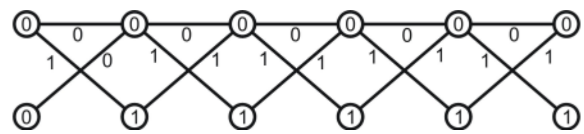

Figure 11

IV. CONCLUDING REMARKS

The vertex analogue of the present work is termed as total product cordial labeling which was introduced by Sundaram *et al.* [6] while we discuss here the concept of total edge product cordial labeling. These two concepts are totally independent as mentioned below.

1) $T(C_n)$ is total edge product cordial as well as total product cordial.
2) P_2 is not total edge product cordial but it is total product cordial.
3) $K_{2,3}$ is total edge product cordial but not total product cordial.
4) C_4 is neither total edge product cordial nor total product cordial.

REFERENCES

[1] J.A. Gallian, "A dynamic survey of graph labeling", *The Electronic Journal of Combinatorics*, vol. 19, #DS6, 2012.

[2] R. Balakrishnan and K. Ranganathan, "A Textbook of Graph Theory", 2/e, *Springer*, New York, 2012.

[3] I. Cahit, "Cordial Graphs: A weaker version of graceful and harmonious Graphs", *Ars Combinatoria*, vol. 23, pp. 201–207, 1987.

[4] S.K. Vaidya and C.M. Barasara, "Total Edge Product Cordial Labeling of Graphs", *Malaya Journal of Matematik*, vol. 3 no.1, pp. 55–63, 2013.

[5] S.K. Vaidya and C.M. Barasara, "Edge Product Cordial Labeling in the Context of Some Graph Operations", *International Journal of Mathematics and Scientific Computing*, vol. 3, no. 1, pp. 4–7, 2013.

[6] M. Sundaram, R. Ponraj and S. Somasundaram, "Total product cordial labeling of graphs", *Bulletin of Pure and Applied Sciences (Mathematics & Statistics)*, vol. 25E, pp. 199–203, 2006.