Some Aggregation Operations on Octagonal Fuzzy Numbers and its Application to Decision Making

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***Abstract*—In this paper, we introduce a distance measure and some aggregation operations on octagonal fuzzy numbers and also investigate their fundamental properties. A fuzzy multi criteria decision making problem is handled using the proposed algorithm. Mathematical programming algorithm for computation and comparison of aggregation operators using mathematical software, MATHCAD 14 has been dealt with.**

***Index Terms*—Aggregation operators, distance measure, fuzzy multi criteria decision making, Mathcad program, Octagonal fuzzy numbers, TOPSIS.**

***MSC 2010 Codes* – 93C42, 90C70**

I. Introduction

AGGREGATION operations combining several fuzzy numbers, introduced by R R Yager [1] yields a single fuzzy number. The aggregation of information from different sources is vital and finds its applications in decision making, image processing, pattern recognition, etc.

Multiple criteria decision making (MCDM) involves making best selection among the alternatives in the presence of multiple criteria and constraints. The MCDM approach requires the courses of action being sorted or ranked under different criteria.

Hwang and Yoon [2] were first suggested classic Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. TOPSIS method is based on the idea that the most preferred alternative should be at the shortest distance from the ideal solution and the longest distance from the anti-ideal solution, using the concept of a metric. In real situations, quantifying the quality of the alternative under different criteria using numerals may not be precise. This difficulty is overcome by the use of fuzzy number. Some of the papers that dealt with fuzzy MCDM problems are [3] – [10].

In this paper, we use Octagonal Fuzzy Number(OFN) introduced in [11] to define the linguistic variables. Section II deals with the basic definitions and arithmetic operations on OFNs. Section III introduces a metric for OFNs and verifies the basic metric properties. Section IV introduces some aggregation operators on OFNs and investigates the fundamental properties that any aggregation operator should satisfy. Section V briefs the TOPSIS method for multi criteria decision making. In Section VI, a set mathematical programming algorithms using MathCAD 14 are presented that substantiates Sections III, IV and V through a numerical illustration. The outcome of the various aggregations are compared and presented in Section VII.

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II. Octagonal Fuzzy Numbers

Definition 2.1 [11]A fuzzy number $\tilde{A}$ is said to be a generalized octagonal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \le a_1 \\ m_1(x) & a_1 \le x \le a_2 \\ k & a_2 \le x \le a_3 \\ m_3(x) & a_3 \le x \le a_4 \\ w & a_4 \le x \le a_5 \\ m_5(x) & a_5 \le x \le a_6 \\ k & a_6 \le x \le a_7 \\ m_7(x) & a_7 \le x \le a_8 \\ 0 & x \ge a_8 \end{cases} \quad (1)$$

where $0 < k < 1$, $w = height(\tilde{A})$ $(w > k)$, $y = m_1(x)$ is the line joining the points $(a_1, 0)$ and (a_2, k), $y = m_3(x)$ is that of (a_3, k), $y = m_5(x)$ is that of (a_5, w) and (a_6, k), $y = m_7(x)$ is that of (a_7, k) and $(a_8, 0)$.

Definition 2.2 [12] The α-cut of an octagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$ given in Definition 2.1 for$\alpha \in (0, 1]$ is

$$[\tilde{A}] = [A^L_\alpha, A^R_\alpha] = \begin{cases} (A^L_\alpha)_1, (A^R_\alpha)_1] & \alpha \in [0, k] \\ (A^L_\alpha)_2, (A^R_\alpha)_2] & \alpha \in (k, 1] \end{cases} \quad (2)$$

where

$$(A^L_\alpha)_1 = a_1 + \tfrac{\alpha}{k}(a_2 - a_1), \quad (A^L_\alpha)_2 = a_3 + \tfrac{\alpha-k}{w-k}(a_4 - a_3),$$
$$(A^R_\alpha)_1 = a_8 - \tfrac{\alpha}{k}(a_8 - a_7), \quad (A^R_\alpha)_2 = a_5 + \tfrac{\alpha-w}{k-w}(a_6 - a_5) \quad (3)$$

Definition 2.3 A linguistic variable is a variable whose values are expressed in linguistic terms [13]

In this paper, let us express the linguistic variables in terms of OFN.

Definition 2.4 The multiplication of two OFNs $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8; k, w)$ when $a_1 \ge 0$ and $b_1 \ge 0$ is defined as

$$\tilde{A}.\tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7, a_8b_8; k, w) \quad (4)$$

III. Metric measure for OFN

In this paper, we define the distance between two OFNs and verify the metric properties.

Definition 3.1 [14] Distance for Intervals: Let $f(x) = (b-a)x+a$ and $g(x) = (d-c)x+c$ where $x \in 0,1]$. The distance between two intervals $[a,b]$ and $[c,d], a \leq b,\ c \leq d$, is defined as

$$d_I^{(p)}([a,b],[c,d]) = (D_I^{(p)}([a,b],[c,d]))^{\frac{1}{p}} \tag{5}$$

where

$$D_I^{(p)}([a,b],[c,d]) = ||f(x)-g(x)||_{L_p}^p \tag{6}$$

where $||.||$ is the usual norm in the L_p space on $[0,1]$ $(p>1)$.

Remark 3.1 The above distance for intervals is considered in our study, because it takes every point in the interval into account to define the distance between the intervals.

Definition 3.2 Let $f_1(\alpha,x,\tilde{A}) = ((A_\alpha^R)_1 - (A_\alpha^L)_1)x + (A_\alpha^L)_1$ and $f_2(\alpha,x,\tilde{A}) = ((A_\alpha^R)_2 - (A_\alpha^L)_2)x + (A_\alpha^L)_2$, where $(A_\alpha^L)_i,\ (A_\alpha^R)_i, i = 1,2$ are as defined before, then define

$$d(\alpha,\tilde{A},\tilde{B}) = \begin{cases} f(\alpha,x,\tilde{A},\tilde{B}) & if\ 0 \leq \alpha \leq k \\ g(\alpha,x,\tilde{A},\tilde{B}) & if\ k < \alpha \leq w \end{cases}$$

where $f(\alpha,x,\tilde{A},\tilde{B}) = \sqrt{\int_0^1 |f_1(\alpha,x,\tilde{A}) - f_1(\alpha,x,\tilde{B})|^2 dx}$

and $g(\alpha,x,\tilde{A},\tilde{B}) = \sqrt{\int_0^1 |f_2(\alpha,x,\tilde{A}) - f_2(\alpha,x,\tilde{B})|^2 dx}$

and the distance between two octagonal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is

$$D(\tilde{A},\tilde{B}) = \sup_{\alpha \in [0,1]} d(\alpha,\tilde{A},\tilde{B}) \tag{7}$$

Proposition 3.1 The proposed distance $D(\tilde{A},\tilde{B})$ satisfies the metric properties for OFN.

Proof.

i. $d(\alpha,\tilde{A},\tilde{B})$ is a positive square root of square of a modulus. Thus $d(\alpha,\tilde{A},\tilde{B}) \geq 0$. Since $D(\tilde{A},\tilde{B})$ is the supremum of $d(\alpha,\tilde{A},\tilde{B})$, $D(\tilde{A},\tilde{B})$ is also ≥ 0.

ii. We let $\tilde{A} = \tilde{B}$, when they have the same $\alpha-$cut. Thus $f_1(\alpha,x,\tilde{A}) = f_1(\alpha,x,\tilde{B})andf_2(\alpha,x,\tilde{A}) = f_2(\alpha,x,\tilde{B}) \Rightarrow d(\alpha,\tilde{A},\tilde{B}) = 0$ for all $\alpha \in 0,1]$. Thus $D(\tilde{A},\tilde{B}) = 0$.

iii. Since $|f_1(\alpha,x,\tilde{A}) - f_1(\alpha,x,\tilde{B})| = |f_1(\alpha,x,\tilde{B}) - f_1(\alpha,x,\tilde{A})|$, $|f_2(\alpha,x,\tilde{A}) - f_2(\alpha,x,\tilde{B})| = |f_2(\alpha,x,\tilde{B}) - f_2(\alpha,x,\tilde{A})|$, then $d(\alpha,\tilde{A},\tilde{B}) = d(\alpha,\tilde{B},\tilde{A}) \Rightarrow D(\tilde{A},\tilde{B}) = D(\tilde{B},\tilde{A})$

iv.

$$D(\tilde{A},\tilde{C}) = \sup_{\alpha \in [0,1]} d(\alpha,\tilde{A},\tilde{C}) \tag{8}$$

$$= \sup_{\alpha \in [0,1]} \begin{cases} \sqrt{\int_0^1 |f_1(\alpha,x,\tilde{A}) - f_1(\alpha,x,\tilde{C})|^2 dx} \\ \text{if } 0 \leq \alpha \leq k \\ \sqrt{\int_0^1 |f_2(\alpha,x,\tilde{A}) - f_2(\alpha,x,\tilde{C})|^2 dx} \\ \text{if } k < \alpha \leq w \end{cases} \tag{9}$$

$$\leq \sup_{\alpha \in 0,1]} \begin{cases} \sqrt{\int_0^1 |f_1(\alpha,x,\tilde{A}) - f_1(\alpha,x,\tilde{B})|^2 dx} \\ \text{if } 0 \leq \alpha \leq k \\ \sqrt{\int_0^1 |f_2(\alpha,x,\tilde{A}) - f_2(\alpha,x,\tilde{B})|^2 dx} \\ \text{if } k < \alpha \leq w \end{cases} \tag{10}$$

$$+ \sup_{\alpha \in 0,1]} \begin{cases} \sqrt{\int_0^1 |f_1(\alpha,x,\tilde{B}) - f_1(\alpha,x,\tilde{C})|^2 dx} \\ \text{if } 0 \leq \alpha \leq k \\ \sqrt{\int_0^1 |f_2(\alpha,x,\tilde{B}) - f_2(\alpha,x,\tilde{C})|^2 dx} \\ \text{if } k < \alpha \leq w \end{cases} \tag{11}$$

$$= D(\tilde{A},\tilde{B}) + D(\tilde{B},\tilde{C}). \tag{12}$$

IV. Aggregation Operations on OFNs

Aggregation operations[1] on fuzzy sets are operations which yield a single fuzzy number by combining several fuzzy numbers. In this section let us define some possible aggregation operations for OFNs.

Definition 4.1 Let

$$\tilde{A}_i = (a_1^i, a_2^i, \ldots, a_8^i; k_i, w_i), i \in \{1,2,...,n\}$$

be a set of n OFNs, then we define

(i) Minimum aggregation:

$$\begin{aligned} &min\_aggreg(\tilde{A}_1,...,\tilde{A}_n) \\ &= \left(\min(a_1^1,...,a_1^n),...,\min(a_8^1,...,a_8^n)\right) \end{aligned} \tag{13}$$

(ii) Maximum aggregation:

$$\begin{aligned} &max\_aggreg(\tilde{A}_1,...,\tilde{A}_n) \\ &= \left(\max(a_1^1,...,a_1^n),...,\max(a_8^1,...,a_8^n)\right) \end{aligned} \tag{14}$$

(iii) Arithmetic Mean:

$$\begin{aligned} &Ari\_aggreg(\tilde{A}_1,...,\tilde{A}_n) \\ &= \left(\frac{a_1^1+...+a_1^n}{n},...,\frac{a_8^1+...+a_8^n}{n}\right) \end{aligned} \tag{15}$$

(iv) Geometric Mean:

$$\begin{aligned} &Geo\_aggreg(\tilde{A}_1,...,\tilde{A}_n) \\ &= \left((a_1^1...a_1^n)^{\frac{1}{n}},...,(a_8^1...a_8^n)^{\frac{1}{n}}\right), \end{aligned} \tag{16}$$

(17)

(v) Harmonic Mean:

$$\begin{aligned} &Har\_aggreg(\tilde{A}_1,...,\tilde{A}_n) \\ &= \left(\frac{n}{\frac{1}{a_1^1}+...+\frac{1}{a_1^n}},...,\frac{n}{\frac{1}{a_8^1}+...+\frac{1}{a_8^n}}\right), \end{aligned} \tag{18}$$

$$\text{provided } a_1^i > 0,\ i = \{1,2,...,n\} \tag{19}$$

(vi) Mixed Mean:

$$\begin{aligned} &Mix\_aggreg(\tilde{A}_1, ..., \tilde{A}_n) \\ &= (\min(a_1^1, ..., a_1^n), \tfrac{n}{\frac{1}{a_2^1}+...+\frac{1}{a_2^n}}, \\ &\tfrac{n}{\frac{1}{a_3^1}+...+\frac{1}{a_3^n}}, (a_4^1...a_4^n)^{\frac{1}{n}}, (a_5^1...a_5^n)^{\frac{1}{n}}, \\ &\tfrac{a_6^1+...+a_6^n}{n}, \tfrac{a_7^1+...+a_7^n}{n}, \\ &\max(a_8^1, ..., a_8^n)) \end{aligned} \tag{20}$$

The above mentioned aggregation operations satisfies the fundamental properties of aggregation namely:

Proposition 4.1 *(Idempotency)* $*\_aggreg(\tilde{A}, ..., \tilde{A}) = \tilde{A}$.

Proof. The proof is straight forward from the definition.

Proposition 4.2 (*Boundedness*) Let $\tilde{A}_1, ..., \tilde{A}_n$ be n octagonal fuzzy numbers, then $\min(\tilde{A}_1, ..., \tilde{A}_n) \leq *\_aggreg(\tilde{A}_1, ..., \tilde{A}_n) \leq \max(\tilde{A}_1, ..., \tilde{A}_n)$

Proof. The proof follows directly from the properties of minimum, maximum, arithmetic, geometric and harmonic means of real numbers.

Proposition 4.3 (*Monotonicity*) Let $\tilde{A}_1, ..., \tilde{A}_n$ and $\tilde{B}_1, ..., \tilde{B}_n$ be $2n$ octagonal fuzzy numbers such that $a_k^i < b_k^i,\ k = 1, 2, ..., 8$ for each $i = 1, 2, ..., n$ then $(*\_aggreg(\tilde{A}_1, ..., \tilde{A}_n))_k < (*\_aggreg(\tilde{B}_1, ..., \tilde{B}_n))_k, k = 1, 2, ..., 8$ *that is* $*\_aggreg$ *is monotonic increasing in all its arguments.*

Proof. Given that $a_k^i < b_k^i,\ k = 1, 2, ..., 8$
for each $i = 1, 2, ..., n$
The above inequality implies

$$\min(a_k^1, ..., a_k^n) < \min(b_k^1, ..., b_k^n), \tag{21}$$

$$\max(a_k^1, ..., a_k^n) < \max(b_k^1, ..., b_k^n), \tag{22}$$

$$\frac{a_k^1 + ... + a_k^n}{n} < \frac{b_k^1 + ... + b_k^n}{n}, \tag{23}$$

$$(a_k^1...a_k^n)^{\frac{1}{n}} < (b_k^1...b_k^n)^{\frac{1}{n}}, \tag{24}$$

$$\frac{1}{a_k^1} + ... + \frac{1}{a_k^n} > \frac{1}{b_k^1} + ... + \frac{1}{b_k^n} \tag{25}$$

$$\Rightarrow \frac{n}{\frac{1}{a_k^1} + ... + \frac{1}{a_k^n}} < \frac{n}{\frac{1}{b_k^1} + ... + \frac{1}{b_k^n}} \tag{26}$$

Thus $*\_aggreg$ is monotonic increasing in all its arguments.

Proposition 4.4 (*Symmetry*) $*\_aggreg$ is a symmetric function in all its arguments.

Proof. From the definition of the aggregation operations , it is clear that $*\_aggreg(\tilde{A}_1, ..., \tilde{A}_n) = *\_aggreg(\tilde{A}_{p(1)}, ..., \tilde{A}_{p(n)})$ for any permutation *p* on n numbers.

V. TOPSIS

TOPSIS is a popular approach to MCDM problems proposed by Hwang and Yoon[2]. Chen[15] extended the TOPSIS method to fuzzy group decision making situations. In this section, we propose the TOPSIS method for OFNs.

Let a MCDM problem having n alternatives, $A_1, A_2, ..., A_n$, m criteria, $C_1, C_2, ..., C_m$ and k decision makers, $D_1, D_2, ..., D_k$. Each decision maker evaluates the the n alternatives based on m criteria , also the weightage of each criteria. All evaluation are considered as OFNs.

(i) Aggregate the evaluations of each decision maker based on any one of the methods presented in Definition 4.1 . Thus, we obtain a decision matrix $DM = \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & ... & \tilde{x}_{1m} \\ \tilde{x}_{21} & \tilde{x}_{22} & ... & \tilde{x}_{2m} \\ \vdots & \vdots & & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & ... & \tilde{x}_{nm} \end{pmatrix}$ and a weight vector $W = \begin{pmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \vdots \\ \tilde{w}_m \end{pmatrix}$, where $\tilde{x}_{ij}$ is the aggregated evaluations of all k decision makers of the n alternatives based on the m criteria and $\tilde{w}_j$ is the aggregated weight of the j^{th} criteria.

(ii) Associate the weights of the criteria to evaluation of the alternatives, i.e. obtain the weighted decision matrix as $\tilde{v}_{ij} = \tilde{w}_j\tilde{x}_{ij}, i \in \{1, 2, ..., n\},\ j \in \{1, 2, ..., m\}$.

(iii) Determine the positive ideal alternative A^+ and negative ideal alternative A^- as follows:

$$A^+ = \begin{pmatrix} \tilde{v}_1^+ & \tilde{v}_2^+ & ... & \tilde{v}_m^+ \end{pmatrix}, \ where\ \ \tilde{v}_j^+ = \min\_aggreg(\tilde{v}_{1j}, \tilde{v}_{2j}, ..., \tilde{v}_n \tag{27}$$

(iv) Obtain the seperation of each alternative from the positive ideal solution and the negative ideal solution, using the distance measure introduced in 3.2 and calculate

$$D_i^+ = \sum_{j=1}^{m} D(\tilde{v}_{ij}, \tilde{v}_j^+), \quad i \in \{1, 2, ..., n\} \tag{28}$$

$$D_i^- = \sum_{j=1}^{m} D(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i \in \{1, 2, ..., n\} \tag{29}$$

(v) Calculate the relative closeness to the ideal alternatives as:

$$RC_i = \frac{D_i^-}{D_i^+ + D_i^-}, \quad i \in \{1, 2, ..., n\} \tag{30}$$

The bigger the RC_i value, the better is the alternative.

VI. Numerical Illustrations

In this section, let us consider a numerical example taken from [15], to compare the different aggregation methods that are proposed for OFNs. The considered problem consists of three alternatives evaluated under five criteria by three decision makers.

The problem and the mathematical programs for aggregations are presented using MATHCAD 14, a mathematical software. The linguistic terms ranging from VERY LOW(VL), LOW(L), MEDIUM LOW(ML), MEDIUM(M), MEDIUM HIGH(MH), HIGH(H) to VERY HIGH(VH) are considered and are described by OFNs whose entries are rated in zero to ten scale which incorporates fuzziness in the system.

Linguistic Term Set For Attributes and Weights:

$$VP := \begin{pmatrix}0\\0\\0\\0\\1\\3\\5\\7\end{pmatrix} \quad P := \begin{pmatrix}0\\0\\0\\1\\3\\5\\7\\9\end{pmatrix} \quad MP := \begin{pmatrix}0\\0\\1\\3\\5\\7\\9\\10\end{pmatrix} \quad F := \begin{pmatrix}0\\1\\3\\5\\7\\9\\10\\10\end{pmatrix} \quad MG := \begin{pmatrix}1\\3\\5\\7\\9\\10\\10\\10\end{pmatrix} \quad G := \begin{pmatrix}3\\5\\7\\9\\10\\10\\10\\10\end{pmatrix} \quad VG := \begin{pmatrix}5\\7\\9\\10\\10\\10\\10\\10\end{pmatrix}$$

$VL := VP \quad L := P \quad ML := MP \quad M := F \quad MH := MG \quad H := G \quad VH := VG$

Number of Decision Makers: $q := 3$

Number of Alternatives: $m := 3 \quad i := 1..m$

Number of Attributes: $n := 5 \quad j := 1..n$

Importance of Attributes Matrix:

$$wkj := \begin{pmatrix} H & VH & VH & VH & M \\ VH & VH & H & VH & MH \\ MH & VH & H & VH & MH \end{pmatrix}$$

Evaluation matrix of Decision Maker 1:

$$xij1 := \begin{pmatrix} MG & G & F & VG & F \\ G & VG & VG & VG & VG \\ VG & MG & G & G & G \end{pmatrix}$$

Evaluation matrix of Decision Maker 2:

$$xij2 := \begin{pmatrix} G & MG & G & G & F \\ G & VG & VG & VG & MG \\ G & G & MG & VG & G \end{pmatrix}$$

Evaluation matrix of Decision Maker 3:

$$xij3 := \begin{pmatrix} MG & F & G & VG & F \\ MG & VG & G & VG & G \\ F & VG & VG & MG & MG \end{pmatrix}$$

Minimum & Maximum of n Octagonal Fuzzy Numbers:

```
MIN(V) := | i ← 1
          | for j ∈ 1..8
          |   | m_j ← (V_i)_j
          |   | n ← rows(V)
          |   | for i ∈ 1..n
          |   |   m_j ← (V_i)_j  if  m_j > (V_i)_j
          | m^T
```

```
MAX(V) := | i ← 1
          | for j ∈ 1..8
          |   | m_j ← (V_i)_j
          |   | n ← rows(V)
          |   | for i ∈ 1..n
          |   |   m_j ← (V_i)_j  if  m_j < (V_i)_j
          | m^T
```

Arithmetic & Geometric Aggregation of n Octagonal Fuzzy Numbers:

```
ARI(V) := | i ← 1
          | for j ∈ 1..8
          |   | n ← rows(V)
          |   | m_j ← ( Σ_{i=1}^{n} (V_i)_j ) / n
          | m^T
```

```
GEO(V) := | i ← 1
          | for j ∈ 1..8
          |   | n ← rows(V)
          |   | m_j ← [ Π_{i=1}^{n} (V_i)_j ]^(1/n)
          | m^T
```

Harmonic Aggregation of n Octagonal Fuzzy Numbers:

```
HAR(V) := | i ← 1
          | for j ∈ 1..8
          |   | n ← rows(V)
          |   | for i ∈ 1..n
          |   |   (V_i)_j ← 1  if  (V_i)_j = 0
          |   | m_j ← n / ( Σ_{i=1}^{n} 1/(V_i)_j )
          | m^T
```

Mixed Aggregation of n Octagonal Fuzzy Numbers:

```
MIX(V) := | i ← 1
          | n ← rows(V)
          | m_1 ← (V_i)_1
          | for i ∈ 1..n
          |   m_1 ← (V_i)_1  if  m_1 > (V_i)_1
          | for j ∈ 2..3
          |   | n ← rows(V)
          |   | for i ∈ 1..n
          |   |   (V_i)_j ← 1  if  (V_i)_j = 0
          |   | m_j ← n / ( Σ_{i=1}^{n} 1/(V_i)_j )
          | for j ∈ 4..5
          |   m_j ← [ Π_{i=1}^{n} (V_i)_j ]^(1/n)
          | for j ∈ 6..7
          |   m_j ← ( Σ_{i=1}^{n} (V_i)_j ) / n
          | m_8 ← (V_i)_8
          | for i ∈ 1..n
          |   m_8 ← (V_i)_8  if  m_8 < (V_i)_8
          | m^T
```

The relative closeness to the ideal alternative using different aggregations are given below:

$$RC\_min = \begin{pmatrix} 0.179 \\ 1 \\ 0.433 \end{pmatrix} \quad RC\_ari = \begin{pmatrix} 0.044 \\ 0.987 \\ 0.527 \end{pmatrix} \quad RC\_geo = \begin{pmatrix} 0.052 \\ 1 \\ 0.495 \end{pmatrix}$$

$$RC\_har = \begin{pmatrix} 0.076 \\ 0.997 \\ 0.49 \end{pmatrix} \quad RC\_mix = \begin{pmatrix} 0.065 \\ 1 \\ 0.482 \end{pmatrix} \quad RC\_max = \begin{pmatrix} 0 \\ 0.667 \\ 1 \end{pmatrix}$$

VII. Conclusion

Some aggregation operations for OFNs are introduced and are verified for their basic properties like idempotency, boundedness, monotonicity and symmetry. min_aggreg and max_aggreg are dealing with extreme entries. If one of the decision makers is biased and evaluates an alternative under low grade, then the aggregated values under geo_aggreg, har_aggreg and mix_aggreg tends to be in the lower side. Compared to the other aggregations, ari_aggreg provides a central value of the aggregated numbers. The comparison of the aggregations are computed using MCDM under TOPSIS method and the usage of interactive mathematical design software, MATHCAD 14 reveals effective comparisons.

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