The Split and Nonsplit Eccentric Domination Number of a Graph

M. Bhanumathi and Sudha Senthil

***Abstract*—A subset D of the vertex set $V(G)$ of a graph G is said to be a dominating set if every vertex not in D is adjacent to at least one vertex in D. A dominating set D is said to be an eccentric dominating set if for every $v \in V - D$, there exists at least one eccentric point of v in D. An eccentric dominating set D of G is a nonsplit eccentric dominating set if the induced subgraph $< V - D >$ is connected. The minimum of the cardinalities of the nonsplit eccentric dominating sets of G is called the nonsplit eccentric domination number $\gamma_{nsed}(G)$ of G. An eccentric dominating set D of G is a split eccentric dominating set if the induced subgraph $< V - D >$ is disconnected. The minimum of the cardinalities of the split eccentric dominating sets of G is called the split eccentric domination number $\gamma_{sed}(G)$ of G.**

***Index Terms*—Domination, eccentric domination, split and nonsplit domination.**

***MSC 2010 Codes*– 05C69.**

I. Introduction

LET G be a finite, simple undirected graph on p vertices and q edges with vertex set $V(G)$ and edge set $E(G)$. For graph theoretic terminology refer Harary [1], Buckley and Harary [2].

In 2010, T.N. Janakiraman, M. Bhanumathi and S. Muthammai defined an eccentric domination in graphs [3]. V.R. Kulli and Janakiram introduced the concept of split and non split domination number of a graph in 1997 [4] and in 2000 [5]. Motivated by these, we have defined split and non split eccentric domination number of a graph.

Let G be a connected graph and v be a vertex of G. The eccentricity $e(v)$ of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v) : u \in V\}$. The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter $diam(G)$ is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$, v is a central vertex if $e(v) = r(G)$. The center $C(G)$ is the set of all central vertices. The central subgraph $< C(G) >$ of a graph G is the subgraph induced by the center. v is a peripheral vertex if $e(v) = d(G)$. The periphery $P(G)$ is the set of all peripheral vertices.

For a vertex v, each vertex at a distance $e(v)$ from v is an eccentric vertex of v. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G)/d(u, v) = e(v)\}$.

The open neighborhood $N(v)$ of a vertex v is the set of all vertices adjacent to v in V. $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood of v. For a vertex $v \in V(G)$, $N_i(v) = \{u \in V(G); d(u, v) = i\}$ is defined to be the i^{th} neighborhood of v in G.

A vertex cover of a graph G is a set of vertices that covers all the edges and an edge cover of G is a set of edges that covers all the vertices. The vertex (edge) covering number $\alpha_0(G)$. $(\alpha_1(G))$ of G is minimum cardinality of a vertex (edge) cover. A set S of vertices of G is independent if no two vertices in S are adjacent. The independence number $\beta_0(G)$ of G is the maximum cardinality of an independent set. A set F of edges of G is independent if no two edges in G are adjacent. The edge independence number $\beta_1(G)$ of G is the maximum cardinality among the independent sets of edges.

Clique of an undirected graph G is a subset of the vertex set $S \subseteq V$, such that for every two vertices in S, there exists an edge connecting the two. This is equivalent to saying that the subgraph induced by S is complete.

A maximal clique is a clique that cannot be extended by including one more adjacent vertex, that is, a clique which does not exist exclusively within the vertex set of a larger clique. The clique number $w(G)$ of a graph G is the number of vertices in the largest clique in G.

A set $S \subseteq V$ is said to be a dominating set in G, if every vertex in $V - S$ is adjacent to some vertex in S. A dominating set D is an independent dominating set, if no two vertices in D are adjacent that is D is an independent set.

The independent domination number $i(G)$ of a graph G is the minimum cardinality of an independent dominating set. A connected dominating set D is a dominating set D whose induced subgraph $< D >$ is connected. The connected domination number $\gamma_c(G)$ of a connected graph G is the minimum cardinality of a connected dominating set.

A dominating set D of a graph G is a split dominating set if the induced subgraph $< V - D >$ is disconnected. The split domination number $\gamma_s(G)$ of a graph G is the minimum cardinality of a split dominating set.

A dominating set D of a graph G is a nonsplit dominating set if the induced subgraph $< V - D >$ is connected. The nonsplit domination number $\gamma_{ns}(G)$ of a graph G is the minimum cardinality of a nonsplit dominating set.

A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists at least one eccentric point of v in D. If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set. An eccentric dominating set D is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set.

The eccentric domination number $\gamma_{ed}(G)$ of a graph G is the minimum cardinality of a eccentric dominating set.

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In this paper, we have studied the split and nonsplit eccentric domination number in graphs.

II. Prior Results

Theorem 2.1: [3] An eccentric dominating set D is a minimal eccentric dominating set if and only if for each vertex $u \in D$, one of the following is true.

(i) u is an isolated vertex of D or u has no eccentric vertex in D.
(ii) There exists some $v \in V - D$ such that $N(v) \cap D = \{u\}$ or $E(v) \cap D = \{u\}$.

Theorem 2.2: [3] $\gamma_{ed}(C_n) = n/2$ if n is even.

$$\gamma_{ed}(C_n) = \begin{cases} \frac{n}{3} = \gamma(C_n) & \text{if } n = 3m \text{ and is odd} \\ \left\lceil \frac{n}{3} \right\rceil & \text{if } n = 3m+1 \text{ and is odd} \\ \left\lceil \frac{n}{3} \right\rceil + 1 & \text{if } n = 3m+2 \text{ and is odd.} \end{cases}$$

Theorem 2.3: [5] For any connected graph G, $(2p - q + 1)/2 \leq \gamma_{ns}(G) \leq p - w(G) + 1$, where $w(G)$ is the clique number of G.

Theorem 2.4: [5] Let D be a γ_{ns}-set of a connected graph G. If no two vertices in $V - D$ are adjacent to a common vertex in D, then $p - \xi(T) \leq \gamma_{ns}(G)$.

Theorem 2.5: [5] If T is a tree with $p \geq 3$ vertices, then $p - m \leq \gamma_{ns}(T)$. Here m denotes the number of vertices adjacent to end vertices.

Theorem 2.6: [5] For any graph G, $\gamma_{ns}(G) = p - 1$ if and only if G is a star.

Theorem 2.7: [6] For any graph G, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \gamma(G) \leq p - \Delta(G)$.

III. Main Results of Nonsplit Eccentric Domination

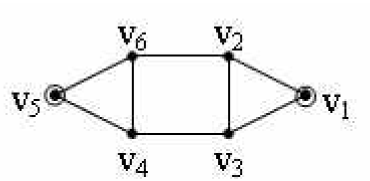
Definition 3.1: An eccentric dominating set D of G is a nonsplit eccentric dominating set if the induced subgraph $< V - D >$ is connected.

The nonsplit eccentric domination number $\gamma_{nsed}(G)$ of a graph G equals the minimum cardinality of a nonsplit eccentric dominating set. That is $\gamma_{nsed}(G) = \min|D|$, where the minimum is taken over D in $\mathcal{D}$, where $\mathcal{D}$ is the set of all minimal nonsplit eccentric dominating sets of G. $V(G)$ is a nonsplit eccentric dominating set for any graph G. Hence $\gamma_{nsed}(G)$ is a well defined parameter.

Observation 3.1:

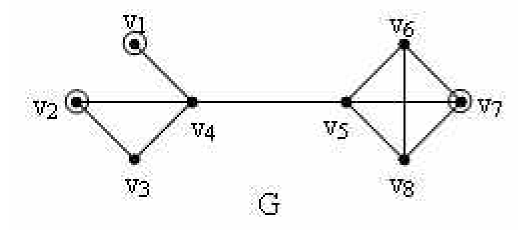
1) For any connected graph G, $\gamma(G) \leq \gamma_{ns}(G) \leq \gamma_{nsed}(G)$.
2) For any connected graph G, $\gamma(G) \leq \gamma_{ed}(G) \leq \gamma_{nsed}(G)$.
3) There are graphs with $\gamma_{ns}(G) = \gamma_{ed}(G)$ and $\gamma_{nsed}(G) = \gamma_{ns}(G)$.
4) There are graphs with $\gamma(G) = \gamma_{ed}(G) = \gamma_{nsed}(G)$.
5) There are graphs with $\gamma_{ed}(G) = \gamma_{ns}(G) < \gamma_{nsed}(G)$.

Example 3.1:



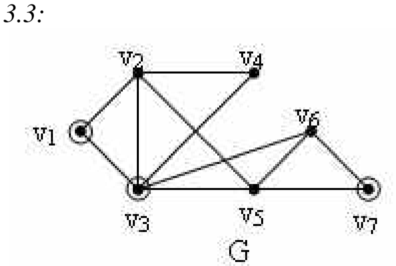
$D = \{v_1, v_5\}$ is a nonsplit eccentric dominating set
$V - D = \{v_2, v_3, v_4, v_6\}$
$\gamma_{nsed}(G) = 2$
$\gamma_{ed}(G) = 2$
$\gamma_{ns}(G) = 2$
Here $\gamma(G) = \gamma_{ed}(G) = \gamma_{ns}(G) = \gamma_{nsed}(G)$.

Example 3.2:



$D = \{v_1, v_2, v_7\}$ is a nonsplit eccentric dominating set.
$V - D = \{v_3, v_4, v_5, v_6, v_8\}$
$\gamma_{nsed}(G) = 3$
$\gamma(G) = 2$
$\gamma_{ed}(G) = 3$
$\gamma_{ns}(G) = 3$
Here $\gamma_{ed}(G) = \gamma_{ns}(G) = \gamma_{nsed}(G)$.

Example 3.3:



$D = \{v_1, v_3, v_7\}$ is a nonsplit eccentric dominating set
$V - D = \{v_2, v_4, v_5, v_6\}$
Here $\gamma(G) = \gamma_{ns}(G) = 2$
$\gamma_{ed}(G) = \gamma_{nsed}(G) = 3$
Therefore $\gamma_{nsed}(G) \neq \gamma_{ns}(G)$.

Theorem 3.1: Pendant vertices are members of every nonsplit eccentric dominating set (or) $e \leq \gamma_{nsed}(G)$, where e is the number of pendant vertices.

Proof: Let v be a vertex in G, such that, $deg_G(v) = 1$ and let D be a nonsplit eccentric dominating set.

If $v \in V - D$, then a vertex adjacent to v must be in D and hence $< V - D >$ is disconnected which is a contradiction. Therefore $v \in D$. Therefore every pendant vertices are members of every nonsplit eccentric dominating set (or) $e \leq \gamma_{nsed}(G)$. ■

Theorem 3.2: A nonsplit eccentric dominating set D is a minimal nonsplit eccentric dominating set if and only if for each vertex $u \in D$, one of the following is true.

(i) u is an isolated vertex of D or u has no eccentric vertex in D
(ii) There exists some $v \in VD$ such that $N(v) \cap D = \{u\}$ or $E(v) \cap D = \{u\}$
(iii) $N(u) \cap (V - D) = \phi$

Proof: Sufficiency is straight forward.

Conversely, suppose that D is a nonsplit eccentric dominating set such that for each $u \in D$ one of the conditions holds.

We show that D is a minimal nonsplit eccentric dominating set.

Suppose that D is not a minimal nonsplit eccentric dominating set. Then there exist a vertex $u \in D$ such that $D_1 = D - \{u\}$ is a nonsplit eccentric dominating set. Hence u is adjacent to at least one vertex v in D_1 and u has an eccentric point in D_1. Condition (i) does not hold.

Also if D_1 is a nonsplit eccentric dominating set, every element x in $V - D_1$ is adjacent to at least one vertex in D_1, and x has an eccentric point in D_1. That is every element in $V - D_1$ is adjacent to at least 1 vertex in D other than u or has an eccentric vertex in D other than u. Hence condition (ii) does not hold. Since $D - \{u\}$ is a nonsplit eccentric dominating set then $< V - D_1 >$ is connected and $u \in V - D_1$ implies $N(u) \cap (V - D_1) \neq \phi$ which contradicts the condition (iii). Thus we get a contradiction to our assumption that for each $u \in D$, one of the conditions holds. This proves the theorem. ■

Theorem 3.3:

(i) $\gamma_{nsed}(K_n) = 1$, for $n \geq 3$.

(ii) $\gamma_{nsed}(K_{m,n}) = 2$,

(iii) $\gamma_{nsed}(C_n) = n - 2$, for $n \geq 3$.

(iv) $\gamma_{nsed}(W_n) = 3$, for $n \geq 4$.

(v) $\gamma_{nsed}(T) = \gamma_{ns}(T)$ for any tree T.

Proof:

(i) when $G = K_n$, radius = diameter = 1.
Hence any vertex $u \in V(G)$ dominate other vertices and is also an eccentric point of other vertices and $< V - D >$ is connected. Hence $\gamma_{nsed}(K_n) = 1$.

(ii) $G = K_{m,n}$.
$V(G) = V_1 \cup V_2$, $|V_1| = m$ and $|V_2| = n$ such that each element of V_1 is adjacent to every vertex of V_2 and vice versa. Let $D = \{u, v\}$, $u \in V_1$, $v \in V_2$. u dominates all the vertices of V_2 and it is eccentric to elements of $V_1 - \{u\}$. Similarly v dominates all the vertices of V_1 and it is eccentric to elements of $V_2 - \{v\}$. Hence D is a minimum eccentric dominating set and $< V - D >$ is connected. Therefore D is a minimum nonsplit eccentric dominating set. Hence $\gamma_{nsed}(K_{m,n}) \leq 2$. Also $\gamma_{nsed}(K_{m,n}) \geq 2$ since $\gamma_{ed}(G) \geq 2$ when $G \neq K_n$. Thus $\gamma_{nsed}(K_{m,n}) = 2$.

(iii) Let $G = C_n$.
Let $u, v \in V$ be any two adjacent vertices in G. Then $D = V - \{u, v\}$ is a nonsplit dominating set since $V - D$ is connected and every point of $V - D$ has an eccentric point in D. Therefore D is a nonsplit eccentric dominating set. Therefore $\gamma_{nsed}(G) \leq n - 2$. Also any set D containing less than $n - 2$ vertices is either not a dominating set or not connected. Therefore $\gamma_{nsed}(G) \geq n - 2$. Thus $\gamma_{nsed}(G) = n - 2$.

(iv) $\gamma_{nsed}(W_n) = 3$, for $n \geq 4$.
When $G = W_n$, let $D = \{u, v, w\}$ where u, v are non central adjacent vertices and w is the central vertex. Then D is a minimum eccentric dominating set and $V - D$ is connected. Therefore D is a minimum nonsplit eccentric dominating set of G. Therefore $\gamma_{nsed}(W_n) \leq 3$, for $n \geq 4$ and $\gamma_{nsed}(W_n) \geq \gamma_{ed}(W_n) = 3$. Thus $\gamma_{nsed}(W_n) = 3$.

(v) If G is a tree, every pendent vertex is in a nonsplit dominating set. Hence any γ_{ns}-set is an eccentric dominating set. Therefore $\gamma_{ns}(T) = \gamma_{nsed}(T)$ for any tree.

■

Theorem 3.4: For any connected graph G, $\gamma_{nsed}(G) + \Delta(G) = 2p - 2$, if and only if $G \cong K_{1,p-1}$.

Proof: When $G \cong K_{1,p-1}$, $\gamma_{nsed}(T) = p-1$ and $\Delta(G) = p - 1$.

Therefore $\gamma_{nsed}(G) + \Delta(G) = 2p - 2$.

Conversely, $\gamma_{nsed}(G) + \Delta(G) = 2p - 2$ is possible if $\gamma_{nsed}(G) = p - 1$ and $\Delta(G) = p - 1$. But, $\gamma_{nsed}(G) = p - 1$ is possible if and only if G is a star. ■

Theorem 3.5: Let G be a connected graph which is not complete then $(2q - p - 1)/2 \leq \gamma_{nsed}(G) \leq p - w(G) + 1$, where $w(G)$ is the clique number of G.

Proof: By Theorem 2.3 in [5], $\gamma_{ns}(G) \leq \gamma_{nsed}(G)$, $(2q - p - 1)/2 \leq \gamma_{ns}(G)$ which implies that $(2q - p - 1)/2 \leq \gamma_{nsed}(G)$.

Let S be a set of vertices of G such that $< S >$ is complete with $|S| = w(G)$. Let $D = (V - S) \cup \{u\}$, $u \in S$ such that u is further from vertices of $V - D$. $< V - D >$ is a complete graph on $w(G) - 1$ vertices. $< V - D >$ is connected. Therefore D is a nonsplit dominating set. Also if $v \in V - D$, its eccentric vertex is in D only, since $< V - D >$ is complete. Therefore D is a nonsplit eccentric dominating set of G. Thus, $\gamma_{nsed}(G) \leq p - w(G) + 1$. ■

Theorem 3.6: Let D be a γ_{nsed}-set of a connected graph. If no two vertices in $V - D$ are adjacent to a common vertex in D, then $\gamma_{nsed}(G) + \xi(T) \geq p$ where $\xi(T)$ is the maximum number of end vertices in any spanning tree T of G.

Proof: By Theorem 2.4, $\gamma_{ns}(G) + \xi(T) \geq p$. Therefore $\gamma_{nsed}(G) + \xi(T) \geq \gamma_{ns}(G) + \xi(T) \geq p$. ■

Theorem 3.7: $\gamma_{nsed}(T) = e$ if and only if each vertex of degree at least 2 is a support, where e is the number of pendant vertices in T. In this case $T = G^+$.

Proof: Assume each vertex of degree at least 2 is a support. If S is the set of all pendant vertices in T. Then S is a dominating set in T such that $< V - S >$ is connected and every point of $< V - S >$ has an eccentric point in S. Therefore S is a nonsplit eccentric dominating set of T.

Therefore

$$\gamma_{nsed}(T) \leq e \tag{1}$$

By Theorem 3.1,

$$e \leq \gamma_{nsed}(T) \tag{2}$$

From (1) and (2), $e = \gamma_{nsed}(T)$.

Conversely, let u be a vertex in T such that $deg(u) \geq 2$. Let D be a γ_{nsed}-set of T, which contains only the end vertices. If u is not a support of T, then u is not adjacent to any of the vertices in D which is a contradiction. Hence the result. ■

Theorem 3.8: For any tree with p vertices, $\gamma_c(T) + \gamma_{nsed}(T) \geq p$.

Proof: If D_1 is the set of all cut vertices of T with $|D_1| = p_1$, then $\gamma_c(T) = p_1 = p - e$. If D_2 is the set of all end (or) pendant vertices of T with $|D_2| = p_2$ then $\gamma_{nsed}(T) \leq p_2 = e$. But $|V(T)| = p_1 + p_2 = p$. This implies $\gamma_c(T) + \gamma_{nsed}(T) \geq p - e + e = p$. By the Theorem 3.7, equality holds if and only if each vertex of degree at least 2 is a support. ■

Theorem 3.9: If T is a tree with $p \geq 3$ vertices, then $p - m \leq \gamma_{nsed}(T)$ here m denote the number of vertices adjacent to end vertices.

Proof: The proof follows from Theorem 2.5. ■

Theorem 3.10: For any connected graph G with $p \geq 2$, $\left\lceil \frac{p}{\Delta+1} \right\rceil \leq \gamma_{nsed}(G) \leq 2q - p + 1$. Also, if $\gamma_{nsed}(G) = 2q - p + 1$ if and only if G is a star.

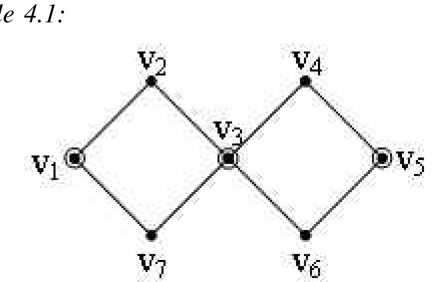
Proof: $\left\lceil \frac{p}{\Delta+1} \right\rceil \leq \gamma(G) \leq \gamma_{nsed}(G)$ from Theorem 2.7 and Observation 3.1. For any graph G, $\gamma_{nsed}(G) \leq p - 1 = 2(p-1)-(p-1) \leq 2q-p+1$. Therefore $\gamma_{nsed}(G) \leq 2q-p+1$. Also if $\gamma_{nsed}(G) = 2q - p + 1$. Then $2q - p + 1 \leq p - 1$ and so $q \leq p - 1$. Hence $q = p - 1$. Therefore G is a tree, by the Theorem 2.6 and 3.3, $\gamma_{nsed}(G) = p - 1$ if and only if G is a star. ■

IV. Split Eccentric Domination

Definition 4.1: An eccentric dominating set D of G is a split eccentric dominating set, if the induced subgraph $< V - D >$ is disconnected.

The split eccentric domination number $\gamma_{sed}(G)$ of a graph G equals the minimum cardinality of a split eccentric dominating set. That is $\gamma_{sed}(G) = \min |D|$, where the minimum is taken over D in $\mathcal{D}$ where $\mathcal{D}$ is the set of all minimal split eccentric dominating sets of G. $V(G)$ is a split eccentric dominating set for any graph G. Hence $\gamma_{sed}(G)$ is an well defined parameter. Obviously $\gamma(G) \leq \gamma_{ed}(G) \leq \gamma_{sed}(G)$.

Example 4.1:



$D = \{v_1, v_3, v_5\}$ is a split eccentric dominating set.
$V - D = \{v_2, v_4, v_6, v_7\}$
$\gamma_{sed}(G) = 3$.

Observation 4.1: For any tree T with $diam(T) > 3$, $\gamma_{sed}(T) = \gamma_{ed}(T)$

Observation 4.2: For a tree T which is not a path with $r(T) = 2$ and $diam(T) = 3$, that is $T = \bar{K}_m + K_1 + K_1 + \bar{K}_n$, $\gamma_{ed} = \gamma_{sed} = 3$ or 4.

$\gamma_{sed}(G) = 4$ if both $m, n > 1$ otherwise $\gamma_{sed}(G) = 3$.

Observation 4.3: For a tree T with $\gamma(T) = 2$ and $diam(T) = 4$ and has more than two peripheral vertices, $\gamma_{ed}(G) = \gamma_{sed}(G) \leq deg\ u + 2$, where u is the central vertex, $\gamma_{ed} = \gamma_{sed} \leq \gamma + 2$.

In the following theorem, we first characterize minimal split eccentric dominating set of a graph.

Theorem 4.1: A split eccentric dominating set D is a minimal split eccentric dominating set if and only if for each vertex $u \in D$, one of the following is true.

(i) u is an isolated vertex D or u has no eccentric vertex in D.
(ii) There exists some $v \in V - D$ such that $N(v) \cap D = \{u\}$ or $E(v) \cap D = \{u\}$.
(iii) $(V - D) \cup \{u\}$ is connected.

Proof: Sufficiency is straight forward.

Conversely, suppose that D is a split eccentric dominating set such that for each $u \in D$, one of the conditions holds. We show that D is a minimal split eccentric dominating set. Suppose that D is not a minimal split eccentric dominating set (i.e.,) $\exists$ a vertex $u \in D$ such that $D_1 = \{D - (u)\}$ is a split eccentric dominating set. Hence u is adjacent to at least one vertex v in D_1 and u has an eccentric point in D_1. This implies condition (i) does not hold.

Also if D_1 is a split eccentric dominating set, every element x in $V - D$ is adjacent to at least one vertex in D_1, and x has an eccentric point in D_1. Hence condition (ii) does not hold. Suppose D is not minimal. $D - \{u\}$ is a split eccentric dominating set then $< V - D_1 >$ is disconnected which contradicts the condition (iii).

This is a contradiction to our assumption that for each $u \in D$, one of the conditions holds. This proves the theorem. ■

Theorem 4.2:

(i) $\gamma_{sed}(K_n) = n - 1$, for $n \geq 3$.
(ii) $\gamma_{sed}(K_{m,n}) = \min(m + 1, n + 1)$.
(iii) $\gamma_{sed}(P_4) = 3$ and $\gamma_{ed}(P_4) = 2$
for $n \geq 5$, $\gamma_{ed}(P_n) = \gamma_{sed}(P_n)$.
$\gamma_{sed}(P_n) = \lceil n/3 \rceil$ if $n = 3k + 1$
$\gamma_{sed}(P_n) = \lceil n/3 \rceil$ if $n = 3k$ or $3k + 2$
(iv) $\gamma_{sed}(C_n) = n/2$ if $n$is even

$$\gamma_{ed}(C_n) = \begin{cases} \frac{n}{3} = \gamma(C_n) & \text{if } n = 3m \text{ and is odd} \\ \left\lceil \frac{n}{3} \right\rceil & \text{if } n = 3m + 1 \text{ and is odd} \\ \left\lceil \frac{n}{3} \right\rceil + 1 & \text{if } n = 3m + 2 \text{ and is odd.} \end{cases}$$

Proof: The proof follows from Theorem 2.2. ■

Theorem 4.3: $\gamma_{sed}(W_4) = 4$, $\gamma_{sed}(W_5) = 4$, $\gamma_{sed}(W_n) = 3$ for $n \geq 6$.

Proof: When $G = W_n$, let $D = \{u, v, w\}$ where u and v are any two non central vertices such that in C_n $d(u, v) \geq 3$ and w is the central vertex. Then D is a minimum eccentric dominating set of G and $V - D$ is disconnected. Therefore D is a split eccentric dominating set of G. Therefore $\gamma_{sed}(W_n) = 3$, for $n \geq 6$. ■

Theorem 4.4: $\gamma_{sed}(K_{1,n}) = 2$.

Proof: $G = K_{1,n}$. Let $D = \{u, v\}$, v-central vertex. The central vertex dominate all vertices in $V - D$ and u is an eccentric point of vertices of $V - D$ and also $< V - D >$ is disconnected. Hence $\gamma_{ed}(K_{1,n}) = 2$, $n \geq 2$. ■

Theorem 4.5: If G is a tree which is not a path then, $\gamma_{sed}(G) \leq \alpha_0(G) + 2$.

Proof: Let S be a maximal independent set. Therefore $V - S$ is a split dominating set. Let u and v be end vertices at distance = diameter. Therefore $(V - S) \cup \{u, v\}$ is an eccentric split dominating set.

$\Rightarrow \gamma_{sed}(G) \leq p - \beta_0(G) + 2$
$= \alpha_0(G) + 2$
$\Rightarrow \gamma_{sed}(G) \leq \alpha_0(G) + 2$. ■

Theorem 4.6: If G is a tree which is not a path, then $\gamma_i(G) + \gamma_{sed}(G) \leq p + 2$.

Proof: We know that $\gamma_i(G) \leq \beta_0(G)$ by the Theorem 4.5, $\gamma_{sed}(G) \leq \alpha_0(G) + 2$. $\gamma_i(G) + \gamma_{sed}(G) \leq \alpha_0(G) + \beta_0(G) + 2$. $\gamma_i(G) + \gamma_{sed}(G) \leq p + 2$. ■

Theorem 4.7: If G is any graph, then $\gamma_{sed}(G) \geq 2$.

Proof: $\gamma_{ed}(G) \leq \gamma_{sed}(G)$. $\gamma_{ed}(G) = 1$ if and only if and only if $G = K_n$ and $\gamma_{sed}(K_n) = n - 1$. But we know that $\gamma_{sed}(K_n) = n - 1$. This implies $\gamma_{sed}(G) \geq 2$. ■

Theorem 4.8: Let G be a triangle free graph. If G is of radius 2 with a unique central vertex u, then $\gamma_{sed}(G) \leq n - deg(u)$.

Proof: If G is of radius 2 with a unique central vertex u, then u dominates $N(u)$. $D = \{V - N(u)\}$ is a dominating set and each vertex in $N(u)$ has an eccentric vertex in D only and also the vertices of $N(u)$ are disconnected, since G has no triangles. Therefore D is a split eccentric dominating set of cardinality $n - deg(u)$. Therefore $\gamma_{sed}(G) \leq n - deg(u)$. ■

Theorem 4.9:

(i) If G is a unicentral tree of radius 2, then $\gamma_{sed}(G) \leq n - deg(u)$, where u is the central vertex.

(ii) If G is a bicentral tree of radius 2, then $\gamma_{sed}(G) \leq n - deg(u) + 1$, where u is a central vertex.

Proof:

(i) If G is an unicentral tree of radius 2 then, $V - Nn(u)$ is a split eccentric dominating set. Therefore $\gamma_{sed}(G) \leq n - deg\ u$.

(ii) Let u and v be the central vertices of G and w be the end vertex. Therefore $[V - N(u)] \cup w$ is a split eccentric dominating set. Therefore $\gamma_{sed}(G) \leq n - deg\ u + 1$. ■

Theorem 4.10: For any graph G with an end vertex which is not an eccentric vertex $\gamma_{ed}(G) = \gamma_{sed}(G)$.

Proof: Let v be an end vertex of G and let D be a γ_{ed} set of G. Suppose $v \in D$. Since v is not an eccentric vertex we can form D' such that D' is an eccentric dominating set and $|D| = |D'|$ (by deleting u from D and adding its support to D). Also $< V - D' >$ is disconnected. Hence $\gamma_{ed}(G) = \gamma_{sed}(G)$. ■

Theorem 4.11: For any graph G with end vertices, if there exists a γ_{ed}-set which does not contain an end vertex then $\gamma_{ed}(G) = \gamma_{sed}(G)$.

Proof: Let D be a γ_{ed} set which does not contain the end vertex v. D contains $u \in V(G)$, $uv \in E(G)$. $< V - D >$ is disconnected. D is a split eccentric dominating set. Therefore $\gamma_{ed}(G) = \gamma_{sed}(G)$. ■

V. Conclusion

Here we have studied the split and non split eccentric domination number of some families of graph, and also studied some bounds for the split and non split eccentric domination number of a graph. We have already studied split and non split eccentric domination in trees and split and non split eccentric domatic number of a graph.

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