Unsteady MHD Flow through Porous Medium Past an Impulsively Started Inclined Oscillating Plate with Variable Temperature and Mass Diffusion in the Presence of Hall Current

U. S. Rajput and Gaurav Kumar

ABSTRACT- Unsteady MHD flow through porous medium past an impulsively started inclined oscillating plate with variable temperature and mass diffusion in the presence of Hall current is studied here. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The Governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity profile is discussed with the help of graphs drawn for different parameters like Grashof number, mass Grashof Number, Prandtl number, Hall current parameter, permeability parameter, phase angle, the magnetic field parameter and Schmidt number, and the numerical values of skin-friction have been tabulated.

Index Terms—**MHD flow, oscillating inclined plate, porous medium, Variable Temperature, Mass Diffusion and Hall current.**

***MSC 2010 Codes* —76D05, 76S05, 76W05.**

I. INTRODUCTION

The study of MHD flow through porous medium associated with heat and mass transfer plays important roles in different areas of science and technology, like chemical engineering, mechanical engineering, biological science, petroleum engineering, biomechanics, irrigation engineering and aerospace technology. Such problems frequently occur in petro-chemical industry, chemical vapour deposition on surfaces, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics. The influence of magnetic field on viscous, incompressible and electrically conducting fluid is of great importance in many applications such as magnetic material processing,
glass manufacturing control processes and purification of crude oil. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by
Lighthill[20]. The study of MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion was studied by Rajput and Kumar[5]. MHD flow between two parallel plates with heat transfer was investigated by Attia et al[10]. Raptis and Kafousias[12] have studied flow of a viscous fluid through a porous medium bounded by a vertical surface. Datta and Jana[15] have studied oscillatory magnetohydrodynamic flow past a flat plate will Hall effects. Soundalgekar[16] has investigated free convection effects on the oscillatory flow an infinite, vertical porous plate with constant suction. Heat transfer in flow through a porous medium bounded by infinite vertical plate under the action of magnetic field was studied by Raptis and Kafousias[13]. Longitudinal vortices in natural convection flow on inclined plates were studied by Sparrow and Husar[19].The researchers have studied the effect of Hall current in various flow models. Attia[9] has considered the effect of variable properties on the unsteady Hartmann flow with heat transfer considering the Hall effect. Attia and Ahmed[8] have studied the Hall effect on unsteady MHD couette flow and heat transfer of a Bingham fluid with suction and injection. Deka[6] has considered Hall effects on MHD flow past an accelerated plate. Muthucumaraswamy and Janakiraman[7] have analyzed mass transfer effect on isothermal vertical oscillating plate in the presence of chemical reaction. Hall effects on free and forced convective flow in a rotating channel was studied by Rao et al[14]. Pop [17] has investigated the effect of Hall current on hydromagnetic flow near an accelerated plate. Pop and Watanabe[11] have further studied Hall effects on magnetohydrodynamic boundary layer flow over a continuous moving flat plate. The effect of Hall current on the magneto hydrodynamic boundary layer flow past a semi-infinite plate was studied by Katagiri[18]. Combined effects of radiation and Hall current on MHD flow past an exponentially accelerated vertical plate in the presence of rotation were studied by Thamizhsudar and Pandurangan[2]. Maripala and Naikoti[1] have analyzed Hall effects on unsteady MHD free convection flow over a stretching sheet with variable viscosity and viscous dissipation. Sulochana[3] has investigated Hall effects on unsteady MHD three dimensional flow through a porous medium in a rotating parallel plates channel with effect of inclined magnetic field. Singh[4] has considered heat and mass transfer in MHD boundary layer flow past an inclined plate with viscous dissipation in porous medium. We are considering unsteady MHD flow through porous medium past an impulsively started inclined oscillating plate with variable

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temperature and mass diffusion in the presence of Hall current. The results are shown with the help of graphs and table.

II. MATHEMATICAL ANALYSIS

In this paper we have considered MHD flow between two parallel electrically non conducting plates inclined at an angle α from vertical. x axis is taken along the plate and z normal to it. A transverse magnetic field B_0 of uniform strength is applied on the flow. The viscous dissipation and induced magnetic field has been neglected due to its small effect. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞ and the concentration level C_∞ everywhere in the fluid is same in stationary condition. At time t > 0, the plate starts oscillating in its own plane with frequency ω and temperature of the plate is raised to T_w and the concentration level near the plate is raised linearly with respect to time. Due to the Hall effect there will be two components of the momentum equation, the flow modal is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial z^2} + g\beta Cos\alpha (T - T_\infty) + g\beta^* Cos\alpha (C - C_\infty) - \frac{\sigma B_0^2 (u + mv)}{\rho(1+m^2)} - \frac{\upsilon u}{K}, \tag{1}$$

$$\frac{\partial v}{\partial t} = \upsilon \frac{\partial^2 v}{\partial z^2} + \frac{\sigma \mu^2 B_0^2 (mu - v)}{\rho(1+m^2)} - \frac{\upsilon v}{K}, \tag{2}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2}, \tag{3}$$

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2}, \tag{4}$$

with the corresponding initial and boundary conditions:

$$\left.\begin{aligned} & t \le 0: u = 0,\ v = 0,\ T = T_\infty,\ C = C_\infty, \ \text{for all } z \\ & t > 0: u = u_0 Cos\omega t,\ v = 0, \\ & T = T_\infty + (T_w - T_\infty)\frac{u_0^2 t}{\upsilon}, \\ & C = C_\infty + (C_w - C_\infty)\frac{u_0^2 t}{\upsilon}, \ at\ z = 0, \\ & u \to 0,\ v = 0,\ T \to T_\infty,\ C \to C_\infty \ \text{ as } z \to \infty \end{aligned}\right\} \tag{5}$$

Here u is the Primary velocity, v - the secondary velocity, g- the acceleration due to gravity, β - volumetric coefficient of thermal expansion, t- time, $m(=\omega_e \tau_e)$ is the Hall parameter with ω_e - cyclotron frequency of electrons and τ_e - electron collision time, K- the permeability parameter, T- temperature of the fluid, β^* - volumetric coefficient of concentration expansion, C- species concentration in the fluid, ν - the kinematic viscosity, ρ - the density, C_p - the specific heat at constant pressure, k- thermal conductivity of the fluid, D- the mass diffusion coefficient, T_w - temperature of the plate at z= 0, C_w - species concentration at the plate z= 0, B_0 - the uniform magnetic field, σ - electrical conductivity.

The following non-dimensional quantities are introduced to transform equations (1), (2), (3) and (4) into dimensionless form:

$$\left.\begin{aligned} & \bar{z} = \frac{z u_0}{\upsilon}, \bar{u} = \frac{u}{u_0}, \bar{v} = \frac{v}{u_0}, \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \\ & S_c = \frac{\upsilon}{D}, P_r = \frac{\mu C_p}{k}, G_r = \frac{g\beta\upsilon (T_w - T_\infty)}{u_0^3}, \\ & \bar{\omega} = \frac{\omega\upsilon}{u_0^2}, M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, G_m = \frac{g\beta^* (C_w - C_\infty)}{u_0^3}, \\ & \mu = \rho\upsilon, \bar{C} = \frac{(C - C_\infty)}{(C_w - C_\infty)}, \bar{t} = \frac{t u_0^2}{\upsilon}, \bar{K} = \frac{u_0}{\upsilon^2} K. \end{aligned}\right\} \tag{6}$$

where $\bar{u}$ is the dimensionless Primary velocity, $\bar{v}$ - the secondary velocity, $\bar{t}$ - dimensionless time, θ - the dimensionless temperature, $\bar{C}$ - the dimensionless concentration, $\bar{K}$ - the dimensionless permeability parameter, G_r - thermal Grashof number, G_m - mass Grashof number, μ - the coefficient of viscosity, P_r - the Prandtl number, S_c - the Schmidt number, M- the magnetic parameter.

Thus the model becomes

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + G_r Cos\alpha\, \theta + G_m Cos\alpha\, \bar{C} - \frac{M(\bar{u} + m\bar{v})}{(1+m^2)} - \frac{1}{\bar{K}}\bar{u}, \tag{7}$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{M(m\bar{u} - \bar{v})}{(1+m^2)} - \frac{1}{\bar{K}}\bar{v}, \tag{8}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c}\frac{\partial^2 \bar{C}}{\partial \bar{z}^2}, \tag{9}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \bar{z}^2}, \tag{10}$$

with the following boundary conditions:

$$\bar{t} \le 0: \bar{u} = 0,\ \bar{v} = 0,\ \theta = 0,\ \bar{C} = 0, \ \text{for all } \bar{z}$$

$$\bar{t} > 0 : \bar{u} = Cos\bar{\omega}\bar{t},\ \bar{v} = 0,\ \theta = \bar{t},\ \bar{C} = \bar{t},\ \text{at } \bar{z} = 0, \quad (11)$$
$$\bar{u} \to 0,\ \bar{v} \to 0,\ \theta \to 0,\ \bar{C} \to 0,\ \text{as } \bar{z} \to \infty.$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2} + G_r Cos\,\alpha\,\theta + G_m Cos\,\alpha\,C - \frac{M(u+mv)}{(1+m^2)} - \frac{1}{K}u, \quad (12)$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} + \frac{M(mu-v)}{(1+m^2)} - \frac{1}{K}v, \quad (13)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2}, \quad (14)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial z^2}, \quad (15)$$

with the following boundary conditions:

$$\left.\begin{array}{l} t \le 0 : u = 0,\ v = 0,\ \theta = 0,\ C = 0,\ \text{for all } z \\ t > 0 : u = Cos\omega t,\ v = 0,\ \theta = t,\ C = t \ \text{at } z=0 \\ u \to 0,\ v \to 0,\ \theta \to 0,\ C \to 0 \quad \text{as } z \to \infty. \end{array}\right\} \quad (16)$$

Writing the equations (12) and (13) in Combined form:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + G_r Cos\,\alpha\,\theta + G_m Cos\,\alpha\,C - qa, \quad (17)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2}, \quad (18)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial z^2}, \quad (19)$$

with the following boundary conditions:

$$\left.\begin{array}{l} t \le 0 : q = 0,\ \theta = 0,\ C = 0, \quad \text{for all } z, \\ t > 0 : q = Cos\omega t,\ \theta = t,\ C = t, \quad \text{at } z=0, \\ q \to 0,\ \theta \to 0,\ C \to 0 \quad \text{as } z \to \infty. \end{array}\right\} \quad (20)$$

Here q= u + i v, $a = \frac{M(1-im)}{1+m^2} + \frac{1}{K}$.

The dimensionless governing equations (17) to (19), subject to the boundary conditions (20), are solved by the usual Laplace - transform technique.

The solution obtained is as under:

$$C = t\left\{(1 + \frac{z^2 S_c}{2t}) erfc[\frac{\sqrt{S_c}}{2\sqrt{t}}] - \frac{z\sqrt{S_c}}{\sqrt{\pi}\sqrt{t}} e^{-\frac{z^2}{4t}S_c}\right\},$$

$$\theta = t\left\{(1 + \frac{z^2 P_r}{2t}) erfc[\frac{\sqrt{P_r}}{2\sqrt{t}}] - \frac{z\sqrt{P_r}}{\sqrt{\pi}\sqrt{t}} e^{-\frac{z^2}{4t}P_r}\right\},$$

$$q = \frac{1}{4}e^{-itw}A_{15} + \frac{zCos\alpha}{4a^2\sqrt{\pi}}[\sqrt{\pi}G_r\{-A_9 + \sqrt{a}e^{-\sqrt{a}z}A_2 + \frac{1}{2zA_{13}A_3} + \frac{2e^{-\sqrt{a}z}A_1P_r}{z} + \frac{2A_{13}A_3P_r}{z}\} - G_rP_r\{-aA_{10} + \frac{1}{z\sqrt{P_r}}A_{13}\sqrt{\pi}A_4 + \frac{2\sqrt{\pi}A_{11}}{z\sqrt{P_r}} - \frac{2a\sqrt{\pi}tA_{11}}{z\sqrt{P_r}} + \frac{1}{zA_{13}\sqrt{\pi}A_8\sqrt{P_r}} - \frac{2\sqrt{\pi P_r}A_{11}}{z}\} - \sqrt{\pi}G_m\{A_9 - \sqrt{a}e^{-\sqrt{a}z}A_2 - \frac{2e^{-\sqrt{a}z}A_1S_c}{z} + \frac{2A_{14}A_5S_c}{z}\} + \sqrt{S_c}G_m\{-aA_{16} + \frac{1}{z\sqrt{\pi S_c}A_{14}A_7} + \frac{2\sqrt{\pi}A_{12}}{z\sqrt{S_c}} + \frac{1}{zA_{14}\sqrt{\pi S_c}A_6} - \frac{2A_{12}\sqrt{\pi S_c}}{z}\}].$$

The expressions for the constants involved in the above equations are given in the appendix.

III. SKIN FRICTION

The dimensionless skin friction at the plate z=0

$$\left(\frac{dq}{dz}\right)_{z=0} = \tau_x + i\tau_y,$$

Separating real and imaginary part in $\left(\frac{dq}{dz}\right)_{z=0}$, the dimensionless skin – friction component $\tau_x = \left(\frac{du}{dz}\right)_{z=0}$ and $\tau_y = \left(\frac{dv}{dz}\right)_{z=0}$ can be computed.

IV. RRESULT AND DISCUSSIONS

The velocity profile for different parameters like, thermal Grashof number Gr, magnetic field parameter M, Hall parameter m, Prandtl number Pr and time t is shown in figures 1.0 to 2.9. It is observed from figures 1.0 and 2.0 that the primary and secondary velocities of fluid decrease when the angle of inclination (α) is increased. It is observed from figure 1.1 and 2.1, when the mass Grashof number is increased then the velocities are increased. From figures 1.2 and 2.2 it is deduced that when thermal Grashof number Gr is increased then the velocities are increased. It is observed that velocities increase when permeability parameter is increased (figures 1.3 and 2.3). If Hall current parameter m is increases then the primary velocity increases and secondary velocity decreases (figures 1.4 and 2.4). It is observed from figures 1.5 and 2.5 that the effect of increasing values of the parameter M results in decreasing u and increasing v. it is deduced that when phase angle ωt is increased then the velocities get decreased (figures 1.6 and 2.6). Further, it is observed that velocities decrease when Prandtl number is increased (figures 1.7 and 2.7). When the Schmidt number is increased then the velocities get decreased (figures 1.8 and 2.8). Further, from figures 1.9 and 2.9 it is observed that velocities increase with time.

Skin friction is given in table1. The value of τ_x increases with the increase in thermal Grashof number, mass Grashof Number, Permeability parameter, Hall currents parameter and time, and it decreases with the angle of inclination of plate, the magnetic field, phase angle, Prandtl number and Schmidt number. The value of τ_y increases with the increase in thermal Grashof number, the magnetic field, Permeability parameter, mass Grashof Number and time, and it decreases with the angle of inclination of plate, Hall current parameter, phase angle, Prandtl number and Schmidt number.

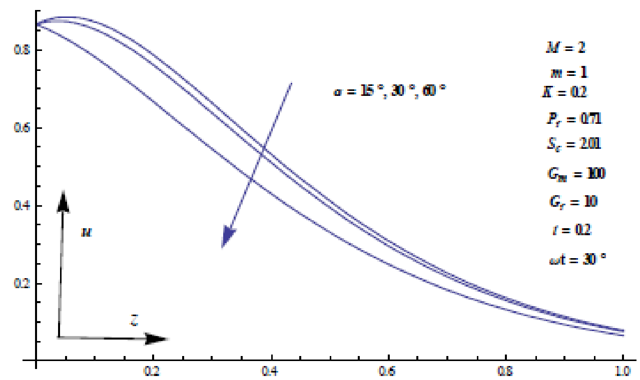

Figure1.0: u v/s z

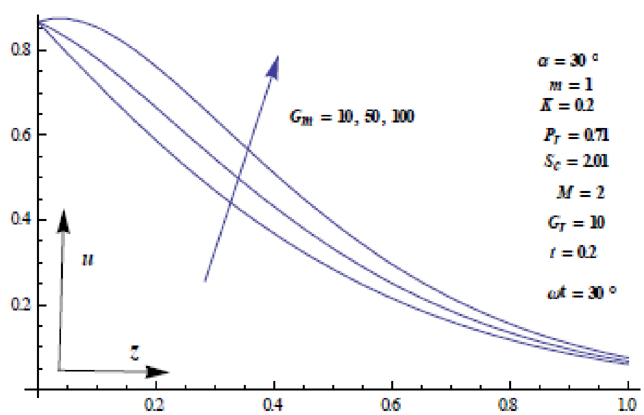

Figure1.1: u v/s z

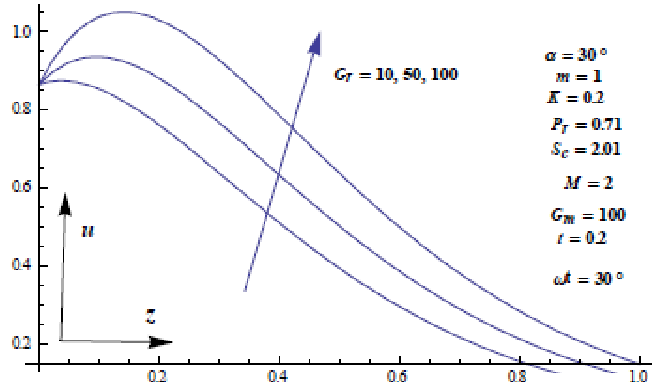

Figure1.2: u v/s z

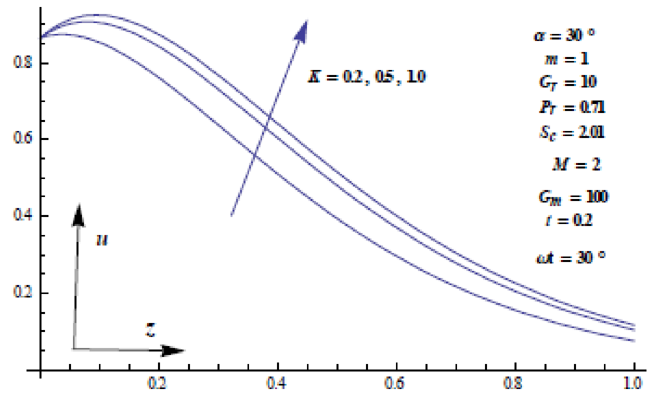

Figure1.3: u v/s z

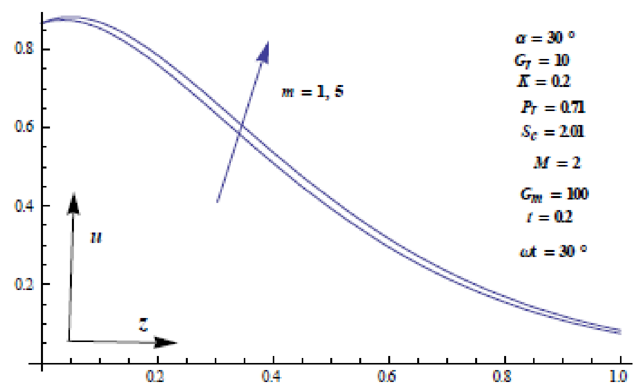

Figure1.4: u v/s z

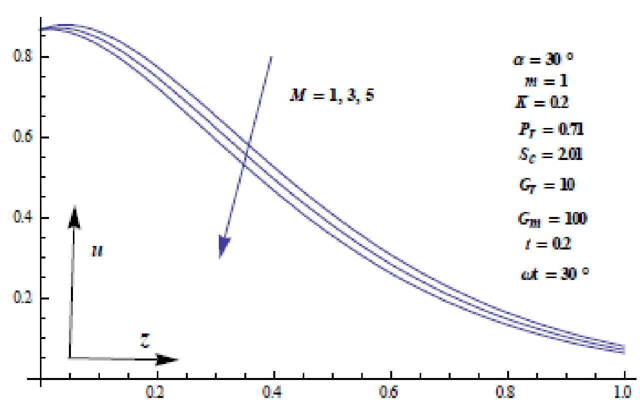

Figure1.5: u v/s z

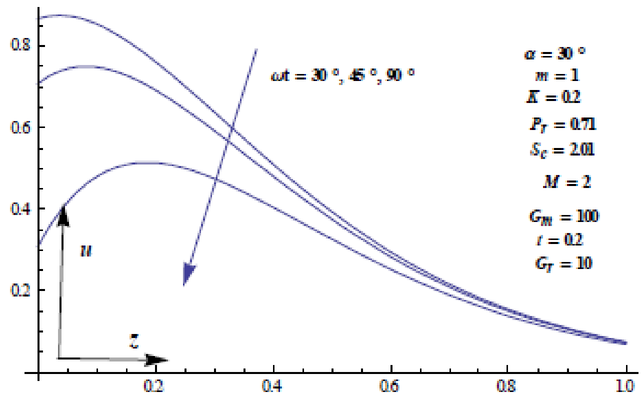

Figure1.6: u v/s z

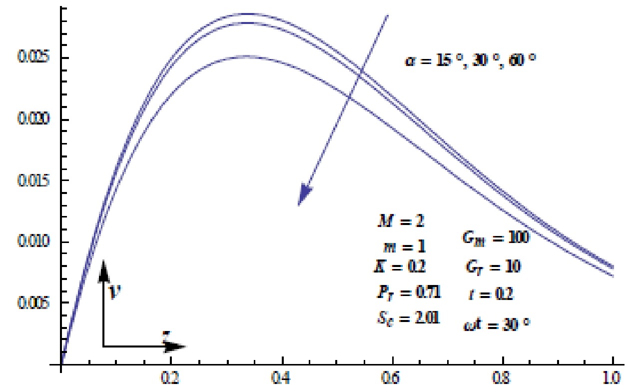

Figure2.0: v v/s z

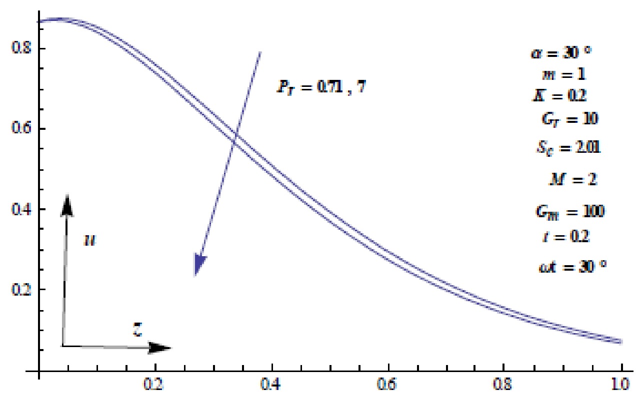

Figure1.7: u v/s z

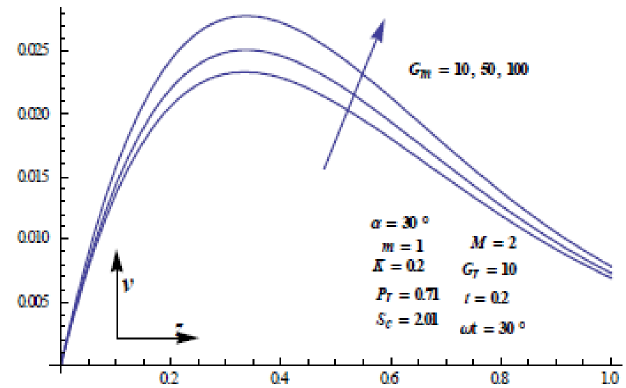

Figure2.1: v v/s z

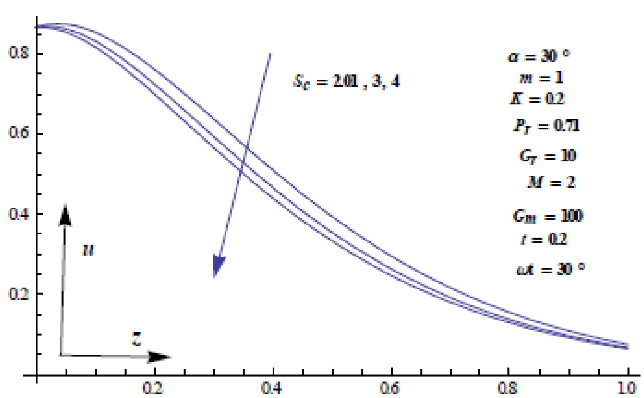

Figure1.8: u v/s z

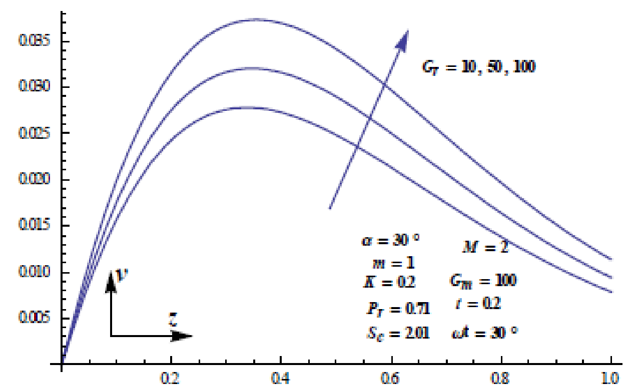

Figure2.2: v v/s z

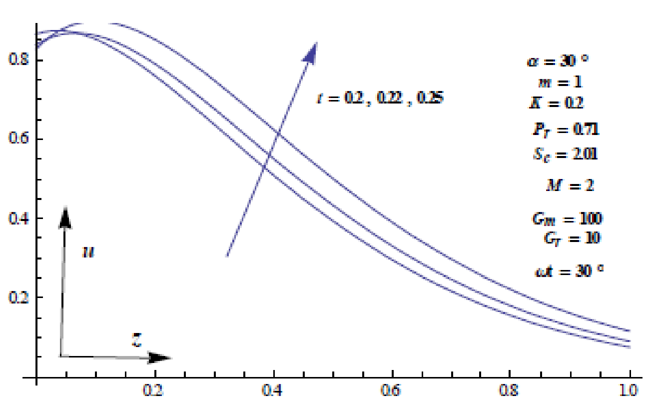

Figure1.9: u v/s z

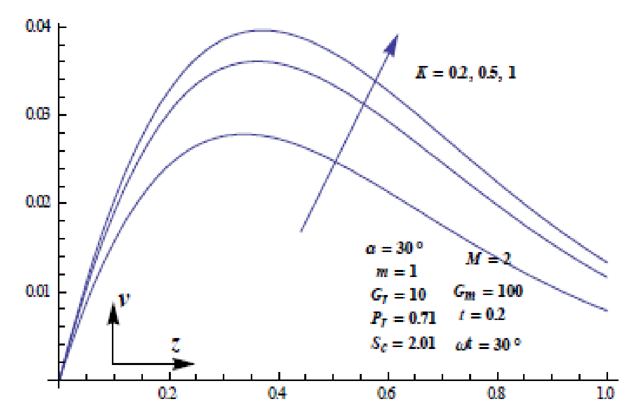

Figure2.3: v v/s z

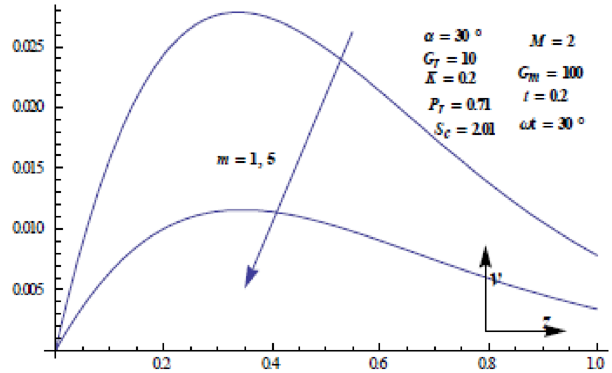

Figure2.4: v v/s z

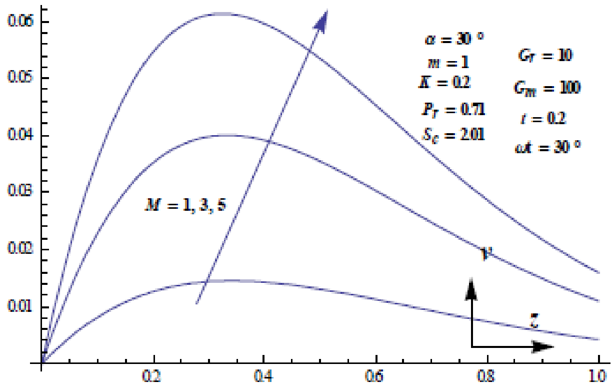

Figure2.5: v v/s z

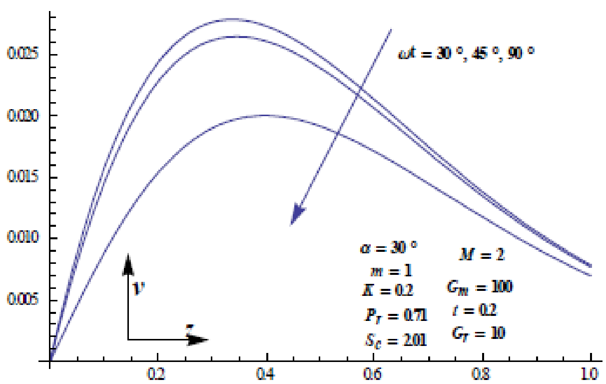

Figure2.6: v v/s z

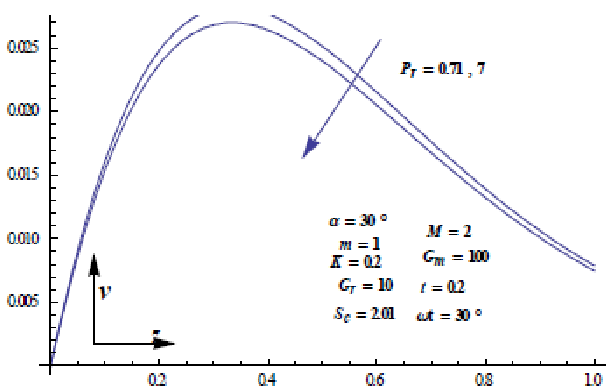

Figure2.7: v v/s z

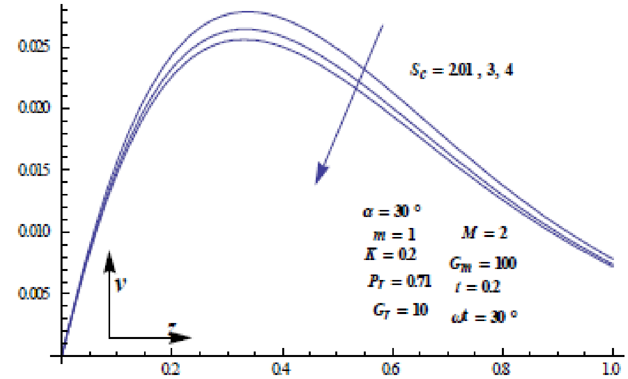

Figure2.8: v v/s z

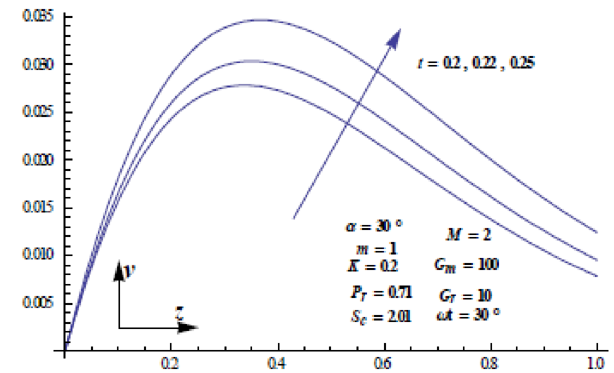

Figure2.9: v v/s z

V. CONCLUSION

The conclusions of the study are as follows:

- Primary Velocity increases with the increase in thermal Grashof number, mass Grashof Number, permeability parameter, Hall current parameter and time.
- Primary Velocity decreases with the angle of inclination of the plate, the magnetic field, phase angle, Prandtl number and Schmidt number.
- Secondary Velocity increases with the increase in thermal Grashof number , mass Grashof Number, permeability parameter, the magnetic field and time.
- Secondary Velocity decreases with the angle of inclination of plate, Hall current parameter, Prandtl number and Schmidt number.
- τ_x increases with the increase in Gr, Gm, m and t, and it decreases with angle of inclination of plate, ωt, M, Pr and Sc.
- τ_y increases with the increase in Gr, Gm, M and t, and it decreases with angle of inclination of the plate, ωt, m, Pr and Sc.

APPENDEX

$$A_1 = -1 - e^{2\sqrt{a}z} - A_{16} + e^{2\sqrt{a}z} A_{17},$$

$$A_2 = -1 - e^{2\sqrt{a}z} + A_{16} + e^{2\sqrt{a}z} A_{17},$$

$$A_3 = -1 - A_{18} + erf[\frac{z - 2t\sqrt{\frac{aP_r}{-1+P_r}}}{2t}] + A_{18} erf[\frac{z + 2t\sqrt{\frac{aP_r}{-1+P_r}}}{2t}],$$

$$A_4 = 1 + A_{18} + erf[\frac{2t\sqrt{\frac{a}{-1+P_r}} - z\sqrt{P_r}}{2t}], - A_{18} erf[\frac{2t\sqrt{\frac{a}{-1+P_r}} + z\sqrt{P_r}}{2t}]$$

$$A_8 = -A_4,$$

$$A_5 = -1 - A_{19} + erf[\frac{z - 2t\sqrt{\frac{aS_c}{-1+S_c}}}{2t}] + A_{19} erf[\frac{z + 2t\sqrt{\frac{aS_c}{-1+S_c}}}{2t}],$$

$$A_6 = -1 - A_{19} - erf[\frac{2t\sqrt{\frac{a}{-1+S_c}} - 2\sqrt{S_c}}{2t}] - A_{19} erf[\frac{2t\sqrt{\frac{a}{-1+S_c}} + 2\sqrt{S_c}}{2t}],$$

$$A_7 = -A_6, A_9 = \frac{2e^{-\sqrt{a}z} A_1}{z}(1-at),$$

$$A_{10} = (2e^{\frac{-z^2 P_r}{4t}}\sqrt{t} + \sqrt{\pi} z A_{11})\sqrt{P_r}, A_{11} = -1 + erf[\frac{z\sqrt{P_r}}{2\sqrt{t}}],$$

$$A_{12} = -1 + erf[\frac{z\sqrt{S_c}}{2\sqrt{t}}],$$

$$A_{13} = e^{\frac{at}{-1+P_r} - z\sqrt{\frac{aP_r}{-1+P_r}}}, A_{14} = e^{\frac{at}{-1+S_c} - z\sqrt{\frac{aS_c}{-1+S_c}}},$$

$$A_{15} = A_{20} + A_{21} - A_{22} - A_{23}, A_{16} = erf[\frac{2\sqrt{at} - z}{2\sqrt{t}}],$$

$$A_{17} = erf[\frac{2\sqrt{at} + z}{2\sqrt{t}}], A_{18} = e^{-2z\sqrt{\frac{aP_r}{-1+P_r}}},$$

$$A_{19} = e^{-2z\sqrt{\frac{aS_c}{-1+S_c}}}, A_{20} = e^{-z\sqrt{a-iw}} + e^{z\sqrt{a-iw}},$$

$$A_{21} = e^{-z\sqrt{a+iw}+2itw} + e^{z\sqrt{a+iw}+2itw},$$

$$A_{22} = e^{-z\sqrt{a-iw}} erf[\frac{z - 2t\sqrt{a-iw}}{2\sqrt{t}}] + e^{z\sqrt{a-iw}} erf[\frac{z + 2t\sqrt{a-iw}}{2\sqrt{t}}],$$

$$A_{23} = e^{-z\sqrt{a+iw}+2itw} erf[\frac{z - 2t\sqrt{a+iw}}{2\sqrt{t}}] + e^{z\sqrt{a+iw}+2itw} erf[\frac{z + 2t\sqrt{a+iw}}{2\sqrt{t}}].$$

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Table 1: Skin friction for different parameters

α	M	m	Pr	Sc	Gm	Gr	ωt	K	t	τ_x	τ_y
15°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.781138	0.202454
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
45°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.038846	0.192066
60°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.555084	0.183752
30°	1	1	0.71	2.01	100	10	30°	0.2	0.2	0.598808	0.101803
30°	3	1	0.71	2.01	100	10	30°	0.2	0.2	0.39095	0.2902
30°	5	1	0.71	2.01	100	10	30°	0.2	0.2	0.185782	0.460118
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
30°	2	2	0.71	2.01	100	10	30°	0.2	0.2	0.617181	0.163663
30°	2	3	0.71	2.01	100	10	30°	0.2	0.2	0.659752	0.124041
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
30°	2	1	7.00	2.01	100	10	30°	0.2	0.2	0.363946	0.194891
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
30°	2	1	0.71	3.00	100	10	30°	0.2	0.2	0.261186	0.192338
30°	2	1	0.71	4.00	100	10	30°	0.2	0.2	0.0979578	0.188503
30°	2	1	0.71	2.01	10	10	30°	0.2	0.2	-1.4875	0.171593
30°	2	1	0.71	2.01	50	10	30°	0.2	0.2	-0.606601	0.183527
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
30°	2	1	0.71	2.01	100	50	30°	0.2	0.2	1.6194	0.21815
30°	2	1	0.71	2.01	100	100	30°	0.2	0.2	3.02356	0.242782
30°	2	1	0.71	2.01	100	10	90°	0.2	0.2	3.82199	0.0935567
30°	2	1	0.71	2.01	100	10	45°	0.2	0.2	1.13483	0.179833
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
30°	2	1	0.71	2.01	100	10	30°	0.5	0.2	1.1378	0.232909
30°	2	1	0.71	2.01	100	10	30°	1.0	0.2	1.37713	0.247056
30°	2	1	0.71	2.01	100	10	30°	0.2	0.2	0.494635	0.198445
30°	2	1	0.71	2.01	100	10	30°	0.2	0.3	2.38197	0.258214
30°	2	1	0.71	2.01	100	10	30°	0.2	0.4	4.48292	0.331068