# Independent Domination Number in the Context of Switching of a Vertex

S. K. Vaidya and R. M. Pandit

***Abstract*—An independent dominating set of a graph $G = (V(G), E(G))$ is a subset $S$ of $V(G)$ such that every vertex not in $S$ is adjacent to at least one vertex of $S$ and no two vertices in $S$ are adjacent. The independent domination number $i(G)$ of $G$ is the minimum cardinality of an independent dominating set. We investigate the exact value of $i(G)$ for some graphs in the context of switching of a vertex.**

***Index Terms*—Dominating Set, Independent dominating set, Independent domination number, Switching of a vertex.**

***MSC 2010 Codes* – 05C69, 05C76.**

## I. Introduction

THE domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it because it has potential to solve many real life problems involving design and analysis of communication network as well as defence surveillance. Many variants of dominating models are available in the existing literature like Independent domination, Global domination, Total domination, Edge domination, just to name a few. Independent sets play an important role in graph theory in general and in discrete optimization in particular. They also appear in matching theory, coloring of graphs and in theory of trees. The present paper is focused on independent domination in graphs.

By a graph $G$ we mean a simple, finite and undirected graph $G$ with vertex set $V(G)$ and edge set $E(G)$. For a vertex $v \in V(G)$, the open neighborhood of $v$, denoted by $N(v)$, is the set $\{u \in V(G) : uv \in E(G)\}$ and the closed neighborhood of $v$ is the set $N[v] = N(v) \cup \{v\}$. We denote the degree of a vertex $v$ in a graph $G$ by $deg(v)$. The maximum degree among the vertices of $G$ is denoted by $\Delta(G)$. A vertex of degree one is called a pendant vertex.

The set $S \subseteq V(G)$ of vertices in a graph $G$ is called a dominating set if every vertex $v \in V(G)$ is either an element of $S$ or is adjacent to at least one element of $S$. The minimum cardinality of a dominating set in $G$ is called the domination number of $G$ denoted as $\gamma(G)$. An independent set in a graph $G$ is a set of pairwise non-adjacent vertices of $G$. A dominating set which is also independent, is called an independent dominating set. The minimum cardinality of an independent dominating set in $G$ is called the independent domination number $i(G)$ of a graph $G$.

Dr. S. K. Vaidya is a Professor in the Department of Mathematics, Saurashtra University, Rajkot -360 005, Gujarat, India. (E-mail: samirkvaidya@yahoo.co.in)

R. M. Pandit is with the Department of Mathematics, A. V. Parekh Technical Institute, Rajkot-360 001, Gujarat, India. (E-mail: pandit.rajesh@ymail.com)

The theory of independent domination was formalized by Berge [1] and Ore [2] in 1962. The independent domination number and the notation $i(G)$ were introduced by Cockayne and Hedetniemi in [3, 4]. Independent dominating sets and variations of independent dominating sets are now extensively studied in the literature; see for example [5–7] while independent dominating sets in regular graphs are well studied by Goddard *et al.* [8]. The independent domination number of some wheel related graphs is discussed by Vaidya and Pandit [9]. Allan and Laskar [10] have proved that if $G$ is a claw-free graph then $\gamma(G) = i(G)$ while Vaidya and Pandit [11] have found the graphs $G$ containing claw as an induced subgraph with $\gamma(G) = i(G)$. Southey and Henning [12] have considered the ratio of the independent domination number versus the domination number in a cubic graph and also characterized the graphs achieving this ratio of $4/3$.

The wheel $W_n$ is defined to be the join $C_{n-1} + K_1$. The vertex corresponding to $K_1$ is known as apex vertex and the vertices corresponding to cycle are known as rim vertices.

For any real number $n$, $\lceil n \rceil$ denotes the smallest integer not less than $n$ and $\lfloor n \rfloor$ denotes the greatest integer not greater than $n$. We denote the path on $n$ vertices as $P_n$, the cycle on $n$ vertices as $C_n$ and the wheel on $n$ vertices as $W_n$.

For the various graph theoretic notations and terminology we follow West [13] while the terms related to the concept of domination are used in the sense of Haynes *et al.* [14].

In the present paper, we investigate the independent domination number of some graphs in the context of switching of a vertex.

## II. Main Results

**Definition 2.1** The switching of a vertex $v$ of $G$ means removing all the edges incident to $v$ and adding edges joining $v$ to every vertex which is not adjacent to $v$ in $G$. We denote the resultant graph by $\widetilde{G}$.

**Proposition 2.1** [15] $i(P_n) = i(C_n) = \lceil \frac{n}{3} \rceil$.

**Theorem 2.1** If $\widetilde{P_n}$ is the graph obtained by switching of an internal vertex of path $P_n$ $(n \leq 6)$ then

$$i(\widetilde{P_n}) = \begin{cases} 3 & \text{if } n = 3, \\ 2 & \text{if } n = 4, 5, \\ 2 \text{ or } 3 & \text{if } n = 6. \end{cases}$$

***Proof***: Let $\widetilde{P_n}$ denote the graph obtained by switching of an internal vertex $v$ of $P_n$ $(n \leq 6)$.

**Case: I.** $n = 3$.

In this case, the graph $\widetilde{P_3}$ will become a null graph with three vertices which implies that $i(\widetilde{P_3}) = 3$.

**Case: II.** $n = 4, 5$.

Since $N[v_1] \neq V(\widetilde{P_n})$ for any vertex $v_1 \in V(\widetilde{P_n})$, it follows that $i(\widetilde{P_n}) > 1$. Moreover, two non-adjacent vertices of $\widetilde{P_n}$ can dominate all the vertices of $\widetilde{P_n}$. Therefore, $i(\widetilde{P_n}) = 2$ for $n = 4, 5$.

**Case: III.** $n = 6$.

**Subcase: I.** $d(u, v) > 1$ where $u$ is a pendant vertex of $P_6$.

Here, $\widetilde{P_6}$ has two pendant vertices. In order to dominate these two pendant vertices, at least two vertices are essential. Moreover, it is also possible to take two non-adjacent vertices which can dominate all the vertices of $\widetilde{P_6}$. Hence, $i(\widetilde{P_6}) = 2$.

**Subcase: II.** $d(u, v) = 1$ where $u$ is a pendant vertex of $P_6$.

Since $\widetilde{P_6}$ has an isolated vertex $u$, every dominating set of $\widetilde{P_6}$ must contain the isolated vertex. Now, $N[v_1] \neq V(\widetilde{P_6}) - \{u\}$ for any $v_1 \in V(\widetilde{P_6})$. Hence, $i(\widetilde{P_6}) > 2$. Moreover, three pairwise non-adjacent vertices can dominate all the vertices of $\widetilde{P_6}$. Therefore, $i(\widetilde{P_6}) = 3$.

Thus, for $n \leq 6$, we have proved that

$$i(\widetilde{P_n}) = \begin{cases} 3 & \text{if } n = 3, \\ 2 & \text{if } n = 4, 5, \\ 2 \text{ or } 3 & \text{if } n = 6. \end{cases}$$

**Theorem 2.2** If $\widetilde{P_n}$ is the graph obtained by switching of an arbitrary vertex $v$ of path $P_n$ then

$$i(\widetilde{P_n}) = \begin{cases} 2 & \text{if } v \text{ is a pendant vertex of path } P_n (n \neq 3), \\ 3 & \text{if } v \text{ is an internal vertex of path } P_n (n > 6). \end{cases}$$

***Proof***: Let $\widetilde{P_n}$ denote the graph obtained by switching of a vertex $v$ of $P_n$.

**Case: I.** Let the switched vertex be a pendant vertex of $P_n\,(n \neq 3)$.

For $P_2$, the graph $\widetilde{P_2}$ will become a null graph with two vertices and hence, $i(\widetilde{P_2}) = 2$.

For $P_n\,(n > 3)$, the graph $\widetilde{P_n}$ has a pendant vertex. In order to dominate this pendant vertex, at least one vertex of $\widetilde{P_n}$ is required. Now, by definition of switching of a vertex, the switched vertex of $\widetilde{P_n}$ is adjacent to all the vertices of $\widetilde{P_n}$ except the pendant vertex of $\widetilde{P_n}$. Hence, at least two non-adjacent vertices, namely, the switched vertex and the pendant vertex of $\widetilde{P_n}$ are enough to dominate all the vertices of $\widetilde{P_n}$. Thus, $i(\widetilde{P_n}) = 2$.

**Case: II.** Let the switched vertex be an internal vertex $v$ of $P_n\,(n > 6)$.

**Subcase: I.** $d(u, v) = 1$ where $u$ is a pendant vertex of $P_n$.

In this case, $\widetilde{P_n}$ has an isolated vertex and a pendant vertex. Therefore, every independent dominating set of $\widetilde{P_n}$ must contain the isolated vertex of $\widetilde{P_n}$. Now, by arguing as in above Case: I, at least two non-adjacent vertices are essential to dominate the vertices of $\widetilde{P_n}$ except the isolated vertex. Hence, for any independent dominating set $S$ of $\widetilde{P_n}$, $|\ S\ | \geq 3$. This implies that $i(\widetilde{P_n}) = 3$.

**Subcase: II.** $d(u, v) > 1$ where $u$ is a pendant vertex of $P_n$.

Here, $\widetilde{P_n}$ has two pendant vertices. In order to dominate these pendant vertices, at least two vertices are required. Since $N[v_1] \cup N[v_2] \neq V(\widetilde{P_n})$ for any $v_1, v_2 \in V(\widetilde{P_n})$, it follows that two vertices are not enough to dominate all the vertices of $\widetilde{P_n}$. Moreover, the switched vertex $v$ dominates all the vertices of $\widetilde{P_n}$ except the pendant vertices of $\widetilde{P_n}$. Hence, the three pairwise non-adjacent vertices, namely, the switched vertex and two pendant vertices, dominate all the vertices of $\widetilde{P_n}$. Thus, every independent dominating set of $\widetilde{P_n}$ must have at least three vertices of $\widetilde{P_n}$ which implies that $i(\widetilde{P_n}) = 3$.

Hence, we have proved that

$$i(\widetilde{P_n}) = \begin{cases} 2 & \text{if } v \text{ is a pendant vertex of path } P_n (n \neq 3), \\ 3 & \text{if } v \text{ is an internal vertex of path } P_n (n > 6). \end{cases}$$

**Remark 2.1** Switching of a pendant vertex of $P_3$ is again $P_3$ and by Proposition 2.1, $i(P_3) = 1$. Therefore, $i(\widetilde{P_3}) = 1$.

**Illustration 2.1** Switching of a pendant vertex $v_1$ of $P_7$ and its independent dominating set is shown in Figure. 1 while switching of an internal vertex $v_4$ of $P_7$ and its independent dominating set is shown in Figure. 2.

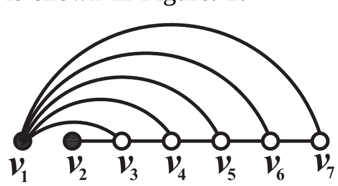


Figure. 1.

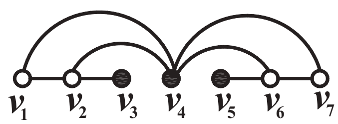


Figure. 2.

**Theorem 2.3** If $\widetilde{C_n}$ is the graph obtained by switching of an arbitrary vertex of cycle $C_n\,(n \leq 7)$ then

$$i(\widetilde{C_n}) = \begin{cases} 1 & \text{if } n = 4, \\ 2 & \text{if } n \neq 4. \end{cases}$$

***Proof***: Let $\widetilde{C_n}$ denote the graph obtained by switching of a vertex $v$ of $C_n\,(n \leq 7)$. Then, $|\ V(\widetilde{C_n})\ | = n$.

**Case: I.** $n = 4$.

Since there exists a vertex $u \in V(\widetilde{C_n})$ such that $N[u] = V(\widetilde{C_n})$, it follows that $i(\widetilde{C_n}) = 1$.

**Case: II.** $n \neq 4$.

For $C_3$, the graph $\widetilde{C_3}$ has an isolated vertex and hence, it is clear that $i(\widetilde{C_3}) = 2$.

For $n > 4$, the graph $\widetilde{C_n}$ has two pendant vertices. In order to dominate these two pendant vertices of $\widetilde{C_n}$, at least two vertices are required. Moreover, it is possible to take two non-adjacent vertices of $\widetilde{C_n}$ which can dominate all the vertices of $\widetilde{C_n}$. Hence, $i(\widetilde{C_n}) = 2$.

Thus, we have proved that

$$i(\widetilde{C_n}) = \begin{cases} 1 & \text{if } n = 4, \\ 2 & \text{if } n \neq 4. \end{cases}$$

**Theorem 2.4** If $\widetilde{C_n}$ is the graph obtained by switching of an arbitrary vertex of cycle $C_n\,(n > 7)$ then $i(\widetilde{C_n}) = 3$.

***Proof***: Let $\widetilde{C_n}$ denote the graph obtained by switching of a vertex $v$ of $C_n\,(n > 7)$. Then, $\widetilde{C_n}$ has two pendant vertices.

In order to dominate two pendant vertices of $\widetilde{C_n}$, at least two vertices are required. Since $|\ N[u]\ | \leq 4$ where $u$ is a vertex of $\widetilde{C_n}$ which dominates a pendant vertex, it follows that two vertices are not enough to dominate $n$ vertices of

$\widetilde{C_n}$. Now, the switched vertex $v$ dominates all the vertices of $\widetilde{C_n}$ except the pendant vertices. Hence, at least three vertices are essential to dominate all the vertices of $\widetilde{C_n}$. Moreover, it is also possible to take three pairwise non-adjacent vertices which can dominate all the vertices of $\widetilde{C_n}$. Therefore, for any independent dominating set $S$ of $\widetilde{C_n}$, $\mid S \mid \geq 3$ implying that $i(\widetilde{C_n}) = 3$.

**Illustration 2.2** Switching of a vertex $v_1$ of $C_8$ and its independent dominating set is shown in Figure. 3.

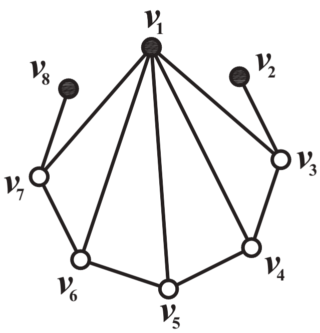


Figure. 3.

**Definition 2.2** The flower graph $Fl_n$ is the graph obtained from the wheel $W_n$ by attaching a pendant edge to each rim vertex of $W_n$ and then by joining each pendant vertex of pendant edge to the apex vertex of $W_n$. We call the apex vertex of wheel as the apex vertex of flower graph $Fl_n$.

**Theorem 2.5** If $\widetilde{Fl_n}$ is the graph obtained by switching of an arbitrary vertex $v$ of flower graph $Fl_n$ then

$$i(\widetilde{Fl_n}) = \begin{cases} n & \text{if } v \text{ is the apex vertex ,} \\ 2 & \text{otherwise.} \end{cases}$$

***Proof***: Let $\widetilde{Fl_n}$ denote the graph obtained by switching of a vertex $v$ of $Fl_n$. Then, $\mid V(\widetilde{Fl_n}) \mid = 2n-1$.

**Case: I.** Let the switched vertex be the apex vertex of $Fl_n$.

In this case, the apex vertex $c$ of $\widetilde{Fl_n}$ will become the isolated vertex of $\widetilde{Fl_n}$ and the graph $\widetilde{Fl_n}$ has $n-1$ pendant vertices. Now, $c$ being an isolated vertex, every dominating set must contain $c$ and in order to dominate $n-1$ pendant vertices of $\widetilde{Fl_n}$, at least $n-1$ vertices of $\widetilde{Fl_n}$ are required. Moreover, it is also possible to take $n-1$ pairwise non-adjacent vertices which can dominate the pendant vertices as well as the remaining vertices of $\widetilde{Fl_n}$ except the isolated vertex. Hence, any independent dominating set of $\widetilde{Fl_n}$ must have at least $(n-1)+1 = n$ vertices which implies that $i(\widetilde{Fl_n}) = n$.

**Case: II.** Let the switched vertex $v$ be a vertex other than the apex vertex of $Fl_n$.

Since there exists no vertex $u \in V(\widetilde{Fl_n})$ such that $N[u] = V(\widetilde{Fl_n})$, every dominating set of $\widetilde{Fl_n}$ must have more than one element. Now, $N[v] \cup N[c] = V(\widetilde{Fl_n})$. Hence, the two non-adjacent vertices of $\widetilde{Fl_n}$, namely, the switched vertex $v$ and the apex vertex $c$, dominate all the vertices of $\widetilde{Fl_n}$. Therefore, for any independent dominating set $S$ of $\widetilde{Fl_n}$, $\mid S \mid \geq 2$ implying that $i(\widetilde{Fl_n}) = 2$.

Hence, we have proved that

$$i(\widetilde{Fl_n}) = \begin{cases} n & \text{if } v \text{ is the apex vertex ,} \\ 2 & \text{otherwise.} \end{cases}$$

**Definition 2.3** A gear graph $G_n$ is obtained from the wheel $W_n$ by adding a vertex between every pair of adjacent vertices of the cycle $C_{n-1}$ corresponding to $W_n$. We call the apex vertex of $W_n$ as the apex vertex of $G_n$.

**Theorem 2.6** If $\widetilde{G_n}$ is the graph obtained by switching of a rim vertex of $W_n$ in $G_n$ $(n \leq 6)$ then

$$i(\widetilde{G_n}) = \begin{cases} 2 & \text{if } n = 4, \\ 3 & \text{if } n = 5, 6. \end{cases}$$

***Proof***: Let $\widetilde{G_n}$ denote the graph obtained by switching of a rim vertex $v$ of $W_n$ in $G_n$ $(n \leq 6)$.

**Case: I.** $n = 4$.

Since there exists no vertex $u \in V(\widetilde{G_4})$ such that $N[u] = V(\widetilde{G_4})$, it follows that $i(\widetilde{G_4}) > 1$. But there exists two non-adjacent vertices of $\widetilde{G_4}$ which can dominate all the vertices of $\widetilde{G_4}$. Therefore, $i(\widetilde{G_4}) = 2$.

**Case: II.** $n = 5, 6$.

In this case, $N[v_1] \cup N[v_2] \neq V(\widetilde{G_n})$ for any two vertices $v_1, v_2 \in V(\widetilde{G_n})$. Therefore, two vertices are not enough to dominate all the vertices of $\widetilde{G_n}$. But, it is possible to take three pairwise non-adjacent vertices of $\widetilde{G_n}$ which can dominate all the vertices of $\widetilde{G_n}$. This implies that $i(\widetilde{G_n}) = 3$. Thus, we have proved that

$$i(\widetilde{G_n}) = \begin{cases} 2 & \text{if } n = 4, \\ 3 & \text{if } n = 5, 6. \end{cases}$$

**Theorem 2.7** If $\widetilde{G_n}$ is the graph obtained by switching of an arbitrary vertex $v$ of gear graph $G_n$ then

$$i(\widetilde{G_n}) = \begin{cases} 4 & \text{if } v \text{ is a rim vertex of } W_n \text{ in } G_n\ (n > 6), \\ \left\lceil \frac{2(n-1)}{3} \right\rceil & \text{if } v \text{ is the apex vertex ,} \\ 3 & \text{otherwise} (n \neq 4). \end{cases}$$

***Proof***: Let $\widetilde{G_n}$ denote the graph obtained by switching of a vertex $v$ of $G_n$. Then, $\mid V(\widetilde{G_n}) \mid = 2n-1$.

**Case: I.** Let the switched vertex $v$ be a rim vertex of $W_n$ in $G_n$ $(n > 6)$.

In this case, the graph $\widetilde{G_n}$ has five types of vertices, namely, the switched vertex $v$ such that $deg(v) = 2n-5 = \triangle(\widetilde{G_n})$, the apex vertex $c$ with $deg(c) = n-2$, $n-3$ vertices with degree 3, $n-2$ vertices with degree 4 and two pendant vertices. In order to dominate two pendant vertices of $\widetilde{G_n}$, at least two vertices are required. Since $N[v_1] \cup N[v_2] \neq V(\widetilde{G_n})$, it follows that two vertices are not enough to dominate all the vertices of $\widetilde{G_n}$. Moreover, as $deg(u) \leq 4$ for all $u \in V(\widetilde{G_n}) - \{v, c\}$, three pairwise non-adjacent vertices are not enough to dominate $\mid V(\widetilde{G_n}) \mid = 2n-1$ vertices. Since any independent set of $G$ containing a vertex of maximum degree $\triangle(G)$ contains at most $n - \triangle(G)$ vertices where $n = o(G)$, $i(G) \leq n - \triangle(G)$. Hence, $i(\widetilde{G_n}) \leq (2n-1) - (2n-5) = 4$. Now, four pairwise non-adjacent vertices, namely, the switched vertex, the apex vertex and two pendant vertices, dominate all the vertices of $\widetilde{G_n}$. Therefore, any independent dominating set of $\widetilde{G_n}$ must have at least four vertices of $\widetilde{G_n}$ implying that $i(\widetilde{G_n}) = 4$.

**Case: II.** Let the switched vertex be the apex vertex $c$ of $G_n$.

In this case, $V(\widetilde{G_n}) = V(C_{2(n-1)}) \cup \{c\}$. Now, by Proposition 2.1, $i(C_{2(n-1)}) = \left\lceil \frac{2(n-1)}{3} \right\rceil$. Moreover, it is also possible to take $\left\lceil \frac{2(n-1)}{3} \right\rceil$ vertices which can also dominate the apex vertex in $\widetilde{G_n}$. Therefore, at least $\left\lceil \frac{2(n-1)}{3} \right\rceil$ pairwise non-adjacent vertices are essential to dominate all the vertices of $\widetilde{G_n}$. Hence, $i(\widetilde{G_n}) = \left\lceil \frac{2(n-1)}{3} \right\rceil$.

**Case: III.** Let the switched vertex $v$ be neither a rim vertex nor the apex vertex of $W_n$ in $G_n$ $(n \neq 4)$.

Since $N[v_1] \cup N[v_2] \neq V(\widetilde{G_n})$ for any two non-adjacent vertices $v_1$ and $v_2$ of $\widetilde{G_n}$, it follows that any independent dominating set of $\widetilde{G_n}$ must have more than two vertices of $\widetilde{G_n}$. Therefore, $i(\widetilde{G_n}) > 2$. Now, $i(G) \leq n - \triangle(G)$. Hence, $i(\widetilde{G_n}) \leq (2n-1) - (2n-4) = 3$. This implies that $i(\widetilde{G_n}) = 3$.

Thus, we have proved that

$$i(\widetilde{G_n}) = \begin{cases} 4 & \text{if } v \text{ is a rim vertex of } W_n \text{ in } G_n \ (n > 6), \\ \left\lceil \frac{2(n-1)}{3} \right\rceil & \text{if } v \text{ is the apex vertex ,} \\ 3 & \text{otherwise}(n \neq 4). \end{cases}$$

**Remark 2.2** Let $v$ be neither a rim vertex of $W_4$ in $G_4$ nor the apex vertex of $G_4$. In this case, $N[v_1] \neq V(\widetilde{G_4})$ for $v_1 \in V(\widetilde{G_4})$ and there exists $v_1, v_2 \in V(\widetilde{G_4})$ such that $N[v_1] \cup N[v_2] = V(\widetilde{G_4})$. Hence, $i(\widetilde{G_4}) = 2$.

## III. Conclusions

Here, we have taken up a problem to determine the independent domination number for some graphs in the context of switching of a vertex. To derive similar results in the context of other variants of domination is an open area of research.

## Acknowledgment

The authors are highly thankful to the anonymous referees for their valuable suggestions.

## References

[1] C. Berge, *Theory of Graphs and its Applications*, Methuen, London, 1962.
[2] O. Ore, "Theory of graphs", *Amer. Math. Soc. Transl.*, vol. 38, pp. 206-212, 1962.
[3] E. J. Cockayne and S. T. Hedetniemi, "Independent graphs", *Congr. Numer.*, vol. 10, pp. 471-491, 1974.
[4] E. J. Cockayne and S. T. Hedetniemi, "Towards a theory of domination in graphs", *Networks*, vol. 7, pp. 247-261, 1977.
[5] S. Ao, E. J. Cockayne, G. MacGillivray and C. M. Mynhardt, "Domination critical graphs with higher independent domination numbers", *J. Graph Theory*, vol. 22, pp. 9-14, 1996.
[6] P. Haxell, B. Seamone and J. Verstraëte,"Independent dominating sets and Hamiltonian cycles", *J. Graph Theory*, vol. 54, pp. 233-244, 2007.
[7] W. C. Shiu, X. Chen and W. H. Chan, "Triangle-free graphs with large independent domination number", *Discrete Optim.*, vol. 7, pp. 86-92, 2010.
[8] W. Goddard, M. Henning, J. Lyle and J. Southey, "On the independent domination number of regular graphs", *Ann. Comb.*, vol. 16, pp. 719-732, 2012.
[9] S. K. Vaidya and R. M. Pandit, "Independent domination number of some wheel related graphs", Communicated, 2014.
[10] R. B. Allan and R. Laskar, "On domination and independent domination numbers of a graph", *Discrete Math.*, vol. 23, pp. 73-76, 1978.
[11] S. K. Vaidya and R. M. Pandit, "Graphs with equal domination and independent domination numbers", *TWMS J. App. Eng. Math.*, vol. 5, 2015, (In Press).
[12] J. Southey and M. A. Henning, "Domination versus independent domination in cubic graphs", *Discrete Math.*, vol. 313, pp. 1212-1220, 2013.
[13] D. B. West, *Introduction to Graph Theory*, Prentice Hall of India, New Delhi, 2003.
[14] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, New York, 1998.
[15] W. Goddard and M. Henning, "Independent domination in graphs: A survey and recent results", *Discrete Math.*, vol. 313, pp. 839-854, 2013.