A New Formulation of Adomian Polynomials

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***Abstract*—A new formulation of Adomian polynomial independent of λ has been discussed. The new formula avoids the parameter which causes the decomposition series to appear to be a perturbation procedure which is an incorrect conclusion. Daftardar-Gejji and Jafari (2006) proposed a new technique, New iterative method (NIM) for solving nonlinear functional equations. They showed that NIM yields better results than the existing Adomian decomposition method. We have shown thatthe NIM technique is nothing but the Adomian decomposition method where nonlinearity is defined by using the new formula of Adomian polynomials. Despite the advantages of NIM proposed by Daftardar-Gejji and Jafari, it still may sometimes be desirable to use ADM. This is illustrated with some examples where it is easy to work with ADM compare to NIM.**

***Index Terms*—Adomian Polynomials; New Iterative Method.**

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I. Introduction

It is well known that the key of the Adomian decomposition method is to decompose the nonlinear term Ny in the equations into a series of polynomials $\sum_{n=0}^{\infty} A_n$, where A_n are the Adomian Polynomials. Adomian [2], [3] formally introduced formulas that can generate Adomian polynomials for all forms of nonlinearity. In order to calculate the A_n, Adomian defines

$$\nu(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n \tag{1}$$

$$N(\nu(\lambda) = \sum_{n=0}^{\infty} \lambda^n A_n \tag{2}$$

Here λ is a parameter introduced for convenience. From (1) and (6.2), one gets

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N(\nu(\lambda)\right]_{\lambda=0}, \quad n = 0, 1, ... \tag{3}$$

Two important observations can be made here. First, A_0 depends only on y_0, A_1 depends only on y_0 and y_1, A_2 depends only on y_0, y_1 and y_2, and so on. Second, the Adomian polynomials introduced above show that the sum of subscripts of each term in A_n is equal to n.

The generation of Adomian polynomials may also be done by simply rearranging the Taylor series expansion of $f(y)$ with respect to a function y_0 as described in [4], where $f(y)$ is the functional form of nonlinear term Ny. Another formulation developed by Adomian [2] for the Adomian polynomial is called as the accelerated Adomian polynomial.

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Adomian defined the accelerated Adomian polynomials in exponential operator notation as,

$$\begin{aligned}
\widehat{A}_0 &= f(y_0)\\
\widehat{A}_1 &= \xi_1 f(y_0)\\
\widehat{A}_2 &= [\xi_2(1+\xi_1)]\, f(y_0)\\
\widehat{A}_3 &= [\xi_3(1+\xi_1+\xi_2(1+\xi_1)]\, f(y_0)\\
\widehat{A}_4 &= [1+\xi_1+\xi_2(1+\xi_1)+\xi_3(1+\xi_1+\xi_2(1+\xi_1)]\, f(y_0)\\
&\vdots
\end{aligned}$$

where ξ_n is an operator defined as

$$\xi_n = e^{u_n d/dy_0} - 1 \tag{4}$$

Recently, some feasible methods for the calculation of Adomian polynomials in a simple way without any need for the formulas introduced by Adomian have been studied by many authors [9], [11], [13]. In this paper we will discuss the new definition of Adomian polynomials independent of λ. In 2006, Daftardar-Gejji and Jafari [7] proposed a new technique called new iterative method (NIM) for solving nonlinear functional equations. They have shown that NIM yields better results than the existing Adomian decomposition method (ADM). We have shown that the NIM technique is nothing but the Adomian decomposition method where nonlinearity is defined by using the new definition of Adomian polynomials. Some examples are given and the method is compared with the ADM.

II. A New Formula of Adomian Polynomials

Adomian polynomials A_n are not unique. Certain terms can be moved forward or back to the component u_n for higher or lower n [2]. This affects the convergence rate since n-term approximations are sought. For example, if $Nu = u^2$, we have

$$\begin{aligned}
A_0 &= u_0^2\\
A_1 &= 2u_0u_1\\
A_2 &= u_1^2 + 2u_0u_2
\end{aligned}$$

If the u_1^2 term were moved from A_2 to A_1, the solution $u = \sum_{n=0}^{\infty} u_n$ would be unaffected, but an approximate solution which only went as far as the u_2 term would be different, closer to the final result.

Consider the general nonlinear functional equation

$$y = N(y) + f \tag{5}$$

where N is a nonlinear continuous function. Abbaoui and Cherruault [1] have defined the Adomian polynomials

$$N(\sum_{i=0}^{\infty} y_i) = \sum_{i=0}^{\infty} A_i \tag{6}$$

and by introducing an artificial parameter λ as follows

$$N(\sum_{i=0}^{\infty}\lambda^i y_i) = \sum_{i=0}^{\infty}\lambda^i A_i \tag{7}$$

We now discuss the new formulation that is independent of parameter λ.
Consider the relation

$$N(\sum_{i=0}^{\infty} y_i) = \sum_{i=0}^{\infty}\widehat{A}_i \tag{8}$$

Then $\widehat{A}_i$ can be listed directly by writing

$$N(\sum_{i=0}^{n} y_i) = \sum_{i=0}^{n}\widehat{A}_i \tag{9}$$

By induction we have

$$\begin{aligned}
\widehat{A}_0 &= N(y_0)\\
\widehat{A}_1 &= N(y_0+y_1) - N(y_0)\\
&\vdots\\
\widehat{A}_n &= N(y_0+y_1+...+y_n) - N(y_0++y_1+...+y_{n-1})
\end{aligned} \tag{10}$$

We can see that A_n are the approximations of $\widehat{A}_n$. The formula presented above is the re-discovery of the accelerated Adomian polynomials [2]. The principal reason for developing this form was not the improvement in an already fast convergence, but the avoidance of the parameterization which causes the decomposition series to appear to be a perturbation procedure which is an incorrect conclusion.

III. Comparison of Decomposition Method with New Iterative Method (NIM)

NIM consists of finding the solutions of nonlinear functional equation (5) as an infinite series

$$y = \sum_{i=0}^{\infty} y_i \tag{11}$$

According to NIM the nonlinear operator N is decomposed as

$$N\left(\sum_{i=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\} \tag{12}$$

From (11) and (12), (5) is equivalent to

$$\sum_{i=0}^{\infty} y_i = f + N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\} \tag{13}$$

Hence

$$\begin{cases} y_0 = f\\ y_1 = N(y_0)\\ y_{m+1} = N(y_0+y_1+...+y_m) - N(y_0+y_1+...+y_{m-1}) \end{cases} \tag{14}$$

Then

$$(y_1+...+y_m) = N(y_0+...+y_m), \quad m = 1,2,... \tag{15}$$

$$y = f + \sum_{i=0}^{\infty} y_i \tag{16}$$

If N is a contraction, i.e. $\|N(x)-N(y)\| \le K \parallel x-y \parallel$ for $0<K<1$, then

$$\begin{aligned}
\parallel y_{m+1} \parallel &= \parallel N(y_0+....+y_m) - N(y_0+....+y_{m-1}) \parallel\\
&\le K \parallel y_m \parallel \le K^m \parallel y_0 \parallel, \; m = 0,1,2...
\end{aligned} \tag{17}$$

and the series $y = \sum_{i=0}^{\infty} y_i$ absolutely and uniformly converges to a solution of equation (5) [6], which is unique, in view of the Banach fixed point theorem [8]. Later a new proof of convergence was given by Adomian and Cherruault [5].

We can see that NIM technique is using the same recurrence relation as the Adomian decomposition method. The nonlinear term $N(y)$ is represented as the difference of the partial sums which are nothing but the newly formulated Adomian polynomials. Furthermore, the new definition of Adomian polynomials is the rearrangement of the terms in the old Adomian polynomials. So we can say that NIM technique is same as the Adomian decomposition method where nonlinearity is expressed in terms of the newly defined Adomian polynomials.

For example, consider the nonlinear integral equation

$$y(x) = g(x) + \int_0^x k(x,t,y(t))dt \tag{18}$$

where k is a nonlinear continuous function and g is given in R.

NIM and ADM both consists of calculating the solution as an infinite series

$$y = \sum_{n=0}^{\infty} y_n \tag{19}$$

Using NIM, we have

$$\begin{aligned}
y_0(x) &= g(x)\\
y_1(x) &= \int_0^x k(x,t,y_0(t))dt\\
y_{m+1}(x) &= \int_0^x \left(k(x,t,y_0+...+y_m) - k(x,t,y_0+...+y_{m-1})\right)dt
\end{aligned}$$

Using decomposition method, the Adomian polynomials for the nonlinear k are

$$\begin{aligned}
\widehat{A}_0 &= k(x,t,y_0)\\
\widehat{A}_1 &= k(x,t,y_0+y_1) - k(x,t,y_0)\\
\widehat{A}_m &= k(x,t,\sum_{i=0}^{m} y_i) - k(x,t,\sum_{i=0}^{m-1} y_i)
\end{aligned}$$

Hence,

$$y_0(x) = g(x)$$
$$y_1(x) = \int_0^x \widehat{A}_0 dt = \int_0^x k(x,t,y_0(t))dt$$
$$y_{m+1}(x) = \int_0^x \widehat{A}_m dt = \int_0^x \big(k(x,t,y_0+...+y_m) - k(x,t,y_0+...+y_{m-1})\big)dt$$

which is same as the NIM technique.

IV. Comparison of Classic Adomian Polynomials with Newly Formulated Polynomials

The newly formulated alias the accelerated Adomian polynomials provide the fastest rate-of-convergence of decomposition series for the nonlinear function $N(y)$. But usually, we choose to employ the ordinary or classic Adomian polynomials due to their sufficiently rapid rate of convergence, and with fewer summands per Adomian polynomial. Sometimes it is easier to see the sum to which the approximation series converges when using the A_n. Also in some problems involving differential equations, integrability difficulties arise with the $\widehat{A}_n$. Classic Adomian polynomials make our calculation by recursion easier. Despite the advantages of NIM proposed by Daftardar-Gejji and Jafari [7], it may still be desirable to use ADM sometimes. This is illustrated with some examples.

Example 4.1

Consider the nonlinear singular boundary value problem

$$y'' + \frac{1}{x}y' + e^{y(x)} = 0, \qquad 0 < x \le 1$$
$$y'(0) = 0, \quad y(1) = 0 \tag{20}$$

The exact solutions are $y(x) = 2\ln((B+1)/(Bx^2+1))$ where $B = 3 - 2\sqrt{2}$.

Using ADM:

$$L = x^{-1}\frac{d}{dx}(x^1)\frac{d}{dx}$$
$$e^y = \sum_{n=0}^{\infty} A_n$$
$$L^{-1}(.) = \int_0^x t^{-1}\int_0^t s^1(.)ds\ dt$$

Then the recurrent scheme of ADM is

$$y_0 = \beta$$
$$y_1 = -\int_0^x t^{-1}\int_0^t s^1 A_0 ds\ dt = -\int_0^x t^{-1}\int_0^t s^1(e^{y_0})ds\ dt = \frac{-1}{4}e^{\beta}x^2$$
$$y_2 = -\int_0^x t^{-1}\int_0^t s^1 A_1 ds\ dt = -\int_0^x t^{-1}\int_0^t s^1(e^{y_0}y_1)ds\ dt = \frac{1}{64}e^{2\beta}x^4$$
$$y_3 = -\int_0^x t^{-1}\int_0^t s^1 A_2 ds\ dt = -\int_0^x t^{-1}\int_0^t s^1(e^{y_0}y_2 + \frac{1}{2}e^{y_0}y_1^2)ds\ dt = \frac{-1}{768}e^{3\beta}x^6$$
$$y_4 = -\int_0^x t^{-1}\int_0^t s^1 A_3 ds\ dt = -\int_0^x t^{-1}\int_0^t s^1(e^{y_0}y_3 + e^{y_0}y_1y_2 + \frac{1}{6}e^{y_0}y_1^3)ds\ dt = \frac{1}{8192}e^{4\beta}x^8$$

To achieve maximum absolute error of 3.34×10^{-12} we have taken fourteen terms of the ADM series

$$\phi_{14} = \sum_{k=0}^{13} y_k \tag{21}$$

The constant β is found by applying the condition $y(1) = 0$. Further accuracy can be achieved by taking more of the series terms.

Using NIM:

The boundary value problem (20) is first converted to the integral equation

$$y(x) = \beta + \int_0^x t^{-1}\int_0^t s^1(-e^y)ds\ dt \tag{22}$$

Let $N(y) = \int_0^x t^{-1}\int_0^t s^1(-e^y)$ and $y(0) = \beta$. In view of algorithm (14)

$$y_0 = \beta$$
$$y_1 = N(y_0) = -\int_0^x t^{-1}\int_0^t s^1(e^{y_0})ds\ dt = -\frac{1}{4}e^{\beta}x^2$$

$$y_2 = N(y_0+y_1) - N(y_0) = -\int_0^x t^{-1}\int_0^t s^1(e^{(y_0+y_1)} - e^{y_0})ds\ dt = -EulerGamma + \frac{e^{\beta}x^2}{4} - Gamma[0, \frac{e^{\beta}x^2}{4}] - \log[\frac{e^{\beta}x^2}{4}]$$
$$y_3 = N(y_0+y_1+y_2) - N(y_0+y_1)$$

where $EulerGamma$ is Euler's constant γ, with numerical value $\simeq 0.577216$ and $Gamma[a,z]$ is the incomplete gamma function $\Gamma(a,z)$. The incomplete gamma function satisfies $\Gamma(a,z) = \int_z^{\infty} t^{a-1}e^{-t}dt$. We can see that the calculations become too complicated while evaluating y_3 and Mathematica fails to obtain the solution. Only series expansion could be the solution but that too will be complex to perform. So in this case NIM fails to obtain the solution while ADM produces the solution with great accuracy.

Example 4.2

Consider the system of inhomogeneous partial differential equations

$$u_t + vu_x + u = 1$$
$$v_t - uv_x - v = 1$$
$$u(x,0) = e^x, \quad v(x,0) = e^{-x}$$

Using ADM:

$$L = \frac{d}{dt}, \quad vu_x = \sum_{n=0}^{\infty} E_n, \quad uv_x = \sum_{n=0}^{\infty} F_n, \quad L^{-1}(\cdot) = \int_0^t (\cdot)dt$$

where E_n and F_n are the Adomian polynomials that can be generated for any form of nonlinearity according to specific algorithm [2], [9].

Operating with L_t^{-1} and using the initial data we obtain

$$u_0(x,t) = e^x + t$$
$$v_0(x,t) = e^{-x} + t$$

$$u_1(x,t) = -t - e^x t - \frac{t^2}{2} - \frac{e^x t^2}{2}$$
$$v_1(x,t) = -t + e^{-x} t + \frac{t^2}{2} - \frac{e^{-x} t^2}{2}$$
$$u_2(x,t) = \frac{t^2}{2} + e^x t^2 + \frac{t^3}{2} + \frac{e^x t^3}{3} + \frac{e^x t^4}{8}$$
$$v_2(x,t) = -\frac{t^2}{2} + e^{-x} t^2 + \frac{t^3}{2} - \frac{e^{-x} t^3}{3} + \frac{e^{-x} t^4}{8}$$
$$u_3(x,t) = -\frac{t^3}{2} - \frac{e^x t^3}{2} - \frac{t^4}{8} - \frac{11 e^x t^4}{24} - \frac{t^5}{10} - \frac{e^x t^5}{24} - \frac{e^x t^6}{48}$$
$$v_3(x,t) = -\frac{t^3}{2} + \frac{e^{-x} t^3}{2} + \frac{t^4}{8} - \frac{11 e^{-x} t^4}{24} - \frac{t^5}{10} + \frac{e^{-x} t^5}{24} - \frac{e^{-x} t^6}{48}$$

It can easily be observed that the noise terms are appearing between the components $u_0, u_1, u_2...$ and $v_0, v_1, v_2...$, and the cancellations occur in every other term with solution converging smoothly to $(u,v) = (e^{x-t}, e^{-x+t})$. The four term approximation gives

$$(u,v) = \Big(e^x \left(1 - t + \frac{t^2}{2} - \frac{t^3}{6} + ...\right), \; e^{-x}\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + ...\right)\Big) \qquad (23)$$

Using NIM:

The initial value problem (27) is equivalent to the following integral equation

$$u(x,t) = e^x + t - L_t^{-1}(vu_x + u) \qquad (24)$$
$$v(x,t) = e^{-x} + t - L_t^{-1}(uv_x + v) \qquad (25)$$

Let $N(u) = L^{-1}(vu_x)$ and $N(v) = L^{-1}(uv_x)$. In view of algorithm (14)

$$u_0(x,t) = e^x + t$$
$$v_0(x,t) = e^{-x} + t$$

$$u_1(x,t) = -t - e^x t - \frac{t^2}{2} - \frac{e^x t^2}{2}$$
$$v_1(x,t) = -t + e^{-x} t + \frac{t^2}{2} - \frac{e^{-x} t^2}{2}$$
$$u_2(x,t) = \frac{t^2}{2} + e^x t^2 + \frac{5t^3}{6} + \frac{e^x t^4}{8} - \frac{t^5}{20} + \frac{e^x t^5}{20}$$
$$v_2(x,t) = -\frac{t^2}{2} + e^{-x} t^2 + \frac{5t^3}{6} + \frac{e^{-x} t^4}{8} - \frac{t^5}{20} - \frac{e^{-x} t^5}{20}$$
$$u_3(x,t) = -\frac{5t^3}{6} - \frac{e^x t^3}{6} - \frac{5t^4}{24} - \frac{e^x t^4}{3} - \frac{t^5}{20} + \frac{11 e^x t^5}{120} + \frac{t^6}{120} - \frac{5 e^x t^6}{72} - \frac{9t^7}{280} - \frac{e^x t^7}{140} - \frac{19 e^x t^8}{1920} - \frac{t^9}{576} - \frac{e^x t^9}{216} + \frac{e^x t^{10}}{1600} + \frac{t^{11}}{4400} + \frac{e^x t^{11}}{4400}$$
$$v_3(x,t) = -\frac{5t^3}{6} + \frac{e^{-x} t^3}{6} + \frac{5t^4}{24} - \frac{e^{-x} t^4}{3} - \frac{t^5}{20} - \frac{11 e^{-x} t^5}{120} - \frac{t^6}{120} - \frac{5 e^{-x} t^6}{72} - \frac{9t^7}{280} + \frac{e^{-x} t^7}{140} - \frac{19 e^{-x} t^8}{1920} - \frac{t^9}{576} + \frac{e^{-x} t^9}{216} + \frac{e^{-x} t^{10}}{1600} + \frac{t^{11}}{4400} - \frac{e^{-x} t^{11}}{4400}$$

The four term approximation gives

$$(u,v) = \left(e^x \left(1 - t + \frac{t^2}{2} - \frac{t^3}{6} + ...\right), e^{-x}\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + ...\right)\right) \qquad (26)$$

Comparing the results obtained by ADM and NIM it can be concluded that both the series converges smoothly to the exact solution. NIM contains more series term than the ADM but the four term approximation of both the series is correct to same order t^3. However, in NIM because of the rapid increase in the noise terms it is difficult to judge the sum to which the approximation series is converging while in ADM number of terms are increasing slowly, making it easier to see the sum to which the series is converging. Also this rapid increase in noise terms makes NIM difficult to proceed further as calculations becomes complex rapidly.

Example 4.3

Consider the Duffing's equation

$$\frac{d^2y}{dx^2} + 3y - 2y^3 = g(x) = \cos x \sin 2x$$
$$y(0) = 0, \quad y'(0) = 1 \qquad (27)$$

The analytic solution of this equation is $y(x) = \sin x$

Since the complicated excitation term $g(x)$ can cause difficult integrations and proliferation of terms, we can express $g(x)$ in Taylor series at $x_0 = 0$, which is truncated for simplification. We replace $g(x)$ by

$$\tilde{g}(x) = 2x - \frac{7}{3}x^3 + \frac{61}{60}x^5 - \frac{547}{2520}x^7, \qquad (28)$$

then equation (27) becomes

$$\frac{d^2y}{dx^2} + 3y - 2y^3 = \widetilde{g}(x) \tag{29}$$

Using ADM:

Let

$$L = \frac{d^2}{dx^2},\ y^3 = \sum_{n=0}^{\infty} A_n,$$

$$L^{-1} = \int_0^x \int_0^{x1} [.] dx_2 dx_1$$

Then the recurrent scheme of ADM is

$$y_0 = x + L^{-1}\widetilde{g},$$
$$y_{n+1} = 2L^{-1}A_n - 3L^{-1}y_n, \quad n \geq 0, \tag{30}$$

The partial sum $\phi_5 = \sum_{n=0}^{4} y_n$ is calculated. We get a series given by

$$\begin{aligned} y(x) = {} & x - 0.166667x^3 + 0.00833333x^5 - 0.000198413x^7 + \\ & 2.75573 \times 10^{-6}x^9 - 0.0002405x^{11} - 0.00127712x^{13} + \\ & 0.00215905x^{15} - 0.000162307x^{17} - 0.000373187x^{19} + \\ & 0.0000601042x^{21} + 0.0000221355x^{23} - 7.58525 \times 10^{-6}x^{25} \\ & + \cdots\cdots\cdots + 4.33636 \times 10^{-26}x^{79} - \\ & 2.305121 \times 10^{-27}x^{81} + 9.86811 \times 10^{-29}x^{83} - \\ & 3.21500 \times 10^{-30}x^{85} + 7.14154 \times 10^{-32}x^{87} - \\ & 8.15219 \times 10^{-34}x^{89}. \end{aligned} \tag{31}$$

Because of the truncation of the excitation term $g(x)$, we get a truncated series $\widetilde{\phi}_5$ to order x^9

$$\widetilde{\phi}_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \tag{32}$$

which coincides with the first five terms in $\phi_5(x)$, and is a partial sum of the Taylor's series of the solution $y(x) = \sin x$ at $x_0 = 0$. Venktarangan and Rajalakshmi [12] presented a technique for oscillatory equations. Because (27) form an oscillatory system, based on their technique we apply the Laplace transform and Padé approximant to deal with the truncated series. Applying Laplace transformation to $\widetilde{\phi}_5(x)$

$$\mathcal{L}[\widetilde{\phi}_5(x)] = \frac{1}{s^2} - \frac{1}{s^4} + \frac{1}{s^6} - \frac{1}{s^8} + \frac{1}{s^{10}} \tag{33}$$

For simplicity, let $s = 1/t$, then

$$\mathcal{L}[\widetilde{\phi}_5(x)] = t^2 - t^4 + t^6 - t^8 + t^{10}. \tag{34}$$

[L/M] Padé approximant of (34) with $L \geq 2$, $M \geq 2$, and $L + M \leq 10$ yields

$$\left[\frac{L}{M}\right] = \frac{t^2}{1+t^2} \tag{35}$$

Obtain [L/M] in terms of s

$$\left[\frac{L}{M}\right] = \frac{1}{1+s^2} \tag{36}$$

By using inverse Laplace transformation to [L/M], we obtain the true solution $y(x) = \sin x$.

Using NIM:

The initial value problem (27) is equivalent to the following integral equation

$$y = x + L^{-1}\widetilde{g} + L^{-1}(2y^3 - 3y) \tag{37}$$

Let $N(y) = L^{-1}(2y^3 - 3y)$. In view of algorithm (14)

$$\begin{aligned} y_0 &= x + L^{-1}\widetilde{g} \\ y_1 &= N(y_0) = L^{-1}(2y_0^3 - 3y_0) \\ y_2 &= N(y_0 + y_1) - N(y_0) \\ &= L^{-1}[2(y_0+y_1)^3 - 3(y_0+y_1)] - L^{-1}[2y_0^3 - 3y_0] \end{aligned}$$

The calculation of ϕ_n becomes complex rapidly. The five-term approximate solution is calculated and we get a series in x given by

$$\begin{aligned} y(x) = {} & x - 0.166667x^3 + 0.00833333x^5 - 0.000198413x^7 + \\ & 2.75573 \times 10^{-6}x^9 - 0.0002405x^{11} - 2.72683 \times 10^{-7}x^{13} + \\ & 0.0000178959x^{15} - 0.000017204x^{17} + 8.77695 \times 10^{-6}x^{19} - \\ & 2.67825 \times 10^{-6}x^{21} + 4.32343 \times 10^{-7}x^{23} + 3.12779 \times 10^{-8}x^{25} - \\ & 4.69177 \times 10^{-8}x^{27} + 1.89516 \times 10^{-8}x^{29} - 5.39105 \times 10^{-9}x^{31} + \\ & 1.06786 \times 10^{-9}x^{33} - 4.64413 \times 10^{-11}x^{35} - 7.48963 \times 10^{-11}x^{37} + \\ & 4.08446 \times 10^{-11}x^{39} - 1.35596 \times 10^{-11}x^{41} + 3.15534 \times 10^{-12}x^{43} \\ & - 3.03963 \times 10^{-13}x^{45} - 1.82318 \times 10^{-13}x^{47} + \\ & 1.30143 \times 10^{-13}x^{49} - 4.55242 \times 10^{-14}x^{51} + 8.21051 \times 10^{-15}x^{53} \\ & + \cdots\cdots + 2.12465 \times 10^{-314}x^{799} - 1.30669 \times 10^{-316}x^{801} + \\ & 6.44769 \times 10^{-319}x^{803} - 2.39322 \times 10^{-321}x^{805} + \\ & 5.94003 \times 10^{-324}x^{807} - 7.39451 \times 10^{-327}x^{809}. \end{aligned} \tag{38}$$

Because of the truncation of the excitation term $g(x)$, we get a truncated series $\widetilde{\phi}_5$ to order x^9

$$\widetilde{\phi}_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \tag{39}$$

We see that the terms involving y_n arrive later in the A_n that is their appearance in the $\widehat{A}_n$ is accelerated over that of the A_n. Hence more and more terms are moved forward in the $\widehat{A}_n$ and so an n-term approximation using $\widehat{A}_n$ contain more of the series. NIM technique is based on the accelerated Adomian polynomials. Comparing the series solution obtained by NIM and ADM we can see that because of the truncation of the excitation term $g(x)$, both the techniques produce the solution correct to the same order x^9. So it is worthless to compute more of the series containing higher powers of x as the solution depends upon the truncated Taylor series of $g(x)$. Also it requires more computational time to evaluate the higher order terms. Further improvement on the accuracy level can be made by taking more terms in the Taylor expansion for the $g(x)$ but at the expense of an increased amount of computational complexity. This problem appears more promptly in the case of NIM as more and more terms are moved forward in each iterate as compare to ADM. Also we can see that the higher powers of series terms are getting subsequently small and small to the order 10^{-327} in case of NIM. In such case the removal of higher power terms strategy or stepwise Adomian as mentioned in [10] can be used to obtain the solution. Above said strategy makes the method more accurate by avoiding the calculation complexity, and underflows, for $x < 1$.

Based on the above examples it can be concluded that despite the advantages of new iterative method it may still be sometimes desirable to use decomposition method for solving the nonlinear equations.

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