Edge Domination in Splitting Graphs

S. K. Vaidya and R. M. Pandit

***Abstract*—For a graph $G = (V, E)$, a subset F of E is called an edge dominating set of G if every edge not in F is adjacent to some edge in F. The edge domination number $\gamma'(G)$ of G is the minimum cardinality of an edge dominating set in G. We investigate edge domination number of splitting graph of some standard graphs.**

***Index Terms*—Edge domination number, splitting graph, edge splitting graph.**

***MSC 2010 Codes* – 05C69, 05C76**

I. Introduction

The Domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it. Many variants of domination models are available in the existing literature: edge domination, total domination, global domination, cycle domination, just to name a few. For a comprehensive bibliography of papers on the concept of domination, the readers are advised to refer Hedetniemi and Laskar [1]. The present work is focused on edge domination in graphs.

We begin with simple, finite, connected and undirected graph $G = (V, E)$ of order n. A set $S \subseteq V$ of vertices in a graph G is called a dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S. A dominating set S is a minimal dominating set(MDS) if no proper subset $S' \subset S$ is a dominating set.

The minimum cardinality of a dominating set of G is called the domination number of G which is denoted by $\gamma(G)$ and the corresponding dominating set is called a γ-set of G.

The open neighborhood $N(v)$ of $v \in V$ is the set of vertices adjacent to v, and the closed neighborhood of v is the set $N[v] = N(v) \cup \{v\}$.

An edge e of a graph G is said to be incident with the vertex v if v is an end vertex of e. In this case, we also say that v is incident with e. Two edges are adjacent, if they have an end vertex in common.

A subset $F \subseteq E$ is an edge dominating set if each edge in E is either in F or is adjacent to an edge in F. An edge dominating set F is called a minimal edge dominating set if no proper subset F' of F is an edge dominating set. The edge domination number $\gamma'(G)$ is minimum cardinality among all minimal edge dominating sets. The concept of edge domination was introduced by Mitchell and Hedetniemi [2] and it was studied by Arumugam and Velammal [3].

Dr. S. K. Vaidya is a Professor in the Department of Mathematics, Saurashtra University, Rajkot -360 005, Gujarat, India. (E-mail: samirkvaidya@yahoo.co.in)

R. M. Pandit is with the Department of Mathematics, A. V. Parekh Technical Institute, Rajkot-360 001, Gujarat, India. (E-mail: pandit.rajesh@ymail.com)

Yannakakis and Gavril [4] proved that Edge dominating set problem for graphs is NP-complete even when restricted to planar or bipartite graphs of maximum degree 3 while bipartite graphs with equal edge domination number and maximum matching cardinality are characterized by Dutton and Klostermeyer [5]. Complementary edge domination in graphs is studied by Kulli and Soner [6] while Jayaram [7] studied the line dominating sets and obtained bounds for the line domination number. Edge domination in graphs of the cube of dimension n is studied by Zelinka [8] while a constructive characterization for trees with equal edge domination and end edge domination numbers is investigated by Muddebihal and Sedamkar [9]. The concept of fractional edge domination in graphs was explored by Arumugam and Jerry [10]. The wheel W_n is defined to be the join $C_{n-1} + K_1$. The vertex corresponding to K_1 is known as apex vertex and the vertices corresponding to cycle are known as rim vertices. The edges corresponding to cycle are known as rim edges and the edges incident with the apex vertex are known as spoke edges.

The complete bipartite graph $K_{1,n}$ is known as the star graph. We identify the vertex of degree n as the apex of $K_{1,n}$.

For any real number n, $\lceil n \rceil$ denotes the smallest integer not less than n and $\lfloor n \rfloor$ denotes the greatest integer not greater than n. Throughout the paper, P_n, C_n, W_n and $K_{1,n}$ will denote the path, the cycle, the wheel and the star graph respectively.

We will provide brief summary of definitions which are useful for the present investigations.

Definition 1.1 For each $e \in E$, $N(e)$ denotes the open neighborhood of e in G. That is, the set of all edges which are adjacent to e in G. Further, $N[e] = N(e) \cup \{e\}$ is the closed neighborhood of e in G.

Definition 1.2 The degree of an edge $e = uv$ of G is defined by $deg(e) = deg(u) + deg(v) - 2$, that is, the number of edges adjacent to it. The maximum degree of an edge in G is denoted by $\triangle'(G)$.

For the various graph theoretic notations and terminology, we follow West [11] while the terms related to the concept of domination are used in the sense of Haynes *et al.* [12].

The present work is aimed to investigate some new results on the concept of edge domination in graphs.

II. Edge Domination in Splitting Graphs

Definition 2.1 Duplication of a vertex v of graph G produces a new graph G' by adding a vertex v' with $N(v') = N(v)$. In other words, a vertex v is said to be duplication of v if all the vertices which are adjacent to v are now adjacent to v' also.

Definition 2.2 If the vertices of graph G are duplicated altogether then the resultant graph is known as splitting graph of G, which is denoted as $S'(G)$.

Theorem 2.1 [7] An edge dominating set S is minimal if and only if for each $e \in S$, one of the following two conditions holds:
(a) $N(e) \cap S = \emptyset$.
(b) there exists an edge $f \in E - S$, such that $N(f) \cap S = \{e\}$.

Theorem 2.2 Let $S'(P_n)$ be the splitting graph of path P_n. Then $\gamma'(S'(P_n)) = \lceil \frac{n}{2} \rceil$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n which are duplicated by the vertices $v'_1, v'_2, \ldots, v'_n$ respectively and let $e_1, e_2, \ldots, e_{n-1}$ be the edges of P_n. Then the resultant graph $S'(P_n)$ will have $2n$ vertices and $3(n-1)$ edges.
Now, we construct an edge set of $S'(P_n)$ as follows:

$$F = \begin{cases} \{e_1, e_3, e_5, \ldots, e_{2i+1}\} & \text{if } n \text{ is even} \\ \\ \{e_1, e_3, e_5, \ldots, e_{2i+1}\} \cup \{e_{n-1}\} & \text{if n is odd} \end{cases}$$
for $0 \leq i \leq \lfloor \frac{n-2}{2} \rfloor$ with $\mid F \mid = \lceil \frac{n}{2} \rceil$.

Since each edge in $E(S'(P_n))$ is either in F or adjacent to an edge in F, the above set F is an edge dominating set of $S'(P_n)$.

Moreover, for each edge $e \in F$, there exists an edge $f \in E(S'(P_n)) - F$ for which $N(f) \cap F = \{e\}$. Therefore, by Theorem 2.1., the set F is a minimal edge dominating set of $S'(P_n)$. Now, $deg(e_{2i+1}) = 6 = \triangle'(S'(P_n))$ for $1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor, n \geq 5$, and $deg(e_1) = 4 = deg(e_{n-1}) = \triangle'(S'(P_n)) - 2$ and the edges e_{2i+1} for $1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor$ being non adjacent to each other, will dominate maximum distinct edges of $S'(P_n)$. Therefore, the above set F is of minimum cardinality $\lceil \frac{n}{2} \rceil$. Hence, the above set F is a minimal edge dominating set with minimum cardinality among all minimal edge dominating sets of $S'(P_n)$. Thus, $\gamma'(S'(P_n)) = \lceil \frac{n}{2} \rceil$. □

Theorem 2.3 Let $S'(C_n)$ be the splitting graph of cycle C_n. Then $\gamma'(S'(C_n)) = \lceil \frac{n}{2} \rceil$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of cycle C_n which are duplicated by the vertices $v'_1, v'_2, \ldots, v'_n$ respectively and let $e_1, e_2, \ldots, e_n$ be the edges of C_n. Then the resultant graph $S'(C_n)$ will have $2n$ vertices and $3n$ edges.

Now, we construct an edge set of $S'(C_n)$ as follows:
$F = \{e_1, e_3, e_5, \ldots, e_{2i+1}\}$, where $0 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$.

The above set F is an edge dominating set of $S'(C_n)$ because each edge in $E(S'(C_n))$ is either in F or adjacent to an edge in F. Moreover, for each edge $e \in F$, there exists an edge $f \in E(S'(C_n)) - F$ for which $N(f) \cap F = \{e\}$. Therefore, by Theorem 2.1., the set F is a minimal edge dominating set of $S'(C_n)$. Now, each edge in the set F is of maximum degree in $S'(C_n)$ and the edges e_{2i+1} for $0 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$ being non adjacent to each other, will dominate maximum distinct edges of $S'(C_n)$. Therefore, the above set F is of minimum cardinality $\lceil \frac{n}{2} \rceil$. Hence, the above set F is a minimal edge dominating set with minimum cardinality among all minimal edge dominating sets of $S'(C_n)$. Thus, $\gamma'(S'(C_n)) = \mid F \mid = \lceil \frac{n}{2} \rceil$ as required. □

Theorem 2.4 Let $S'(W_n)$ be the splitting graph of wheel W_n. Then $\gamma'(S'(W_n)) = \lfloor \frac{n}{2} \rfloor$.

Proof: Let $v_1, v_2, \ldots, v_{n-1}$ be the rim vertices of wheel W_n which are duplicated by the vertices $v'_1, v'_2, \ldots, v'_{n-1}$ respectively and let c denotes the apex vertex of W_n which is duplicated by the vertex c'. Let $e_1, e_2, \ldots, e_{n-1}$ be the rim edges of W_n. Then the resultant graph $S'(W_n)$ will have $2n$ vertices and $5(n-1)$ edges.

First, we construct an edge set of $S'(W_n)$ as follows:
$F = \{e_1, e_3, e_5, \ldots, e_{2j+1}\}$, where $0 \leq j \leq \lfloor \frac{n-2}{2} \rfloor$.

Since each edge in $E(S'(W_n))$ is either in F or adjacent to an edge in F, the above set F is an edge dominating set of $S'(W_n)$.

Moreover, for each edge $e \in F$, there exists an edge $f \in E(S'(W_n)) - F$ for which $N(f) \cap F = \{e\}$. Therefore, by Theorem 2.1., the set F is a minimal edge dominating set of $S'(W_n)$. Now, $deg(e_{2j+1}) = 10$ for $0 \leq j \leq \lfloor \frac{n-2}{2} \rfloor$ and from the nature of the graph $S'(W_n)$, it is easy to observe that the edges in F being non adjacent to each other, will dominate maximum distinct edges of $S'(W_n)$. Therefore, the above set F is of minimum cardinality $\lfloor \frac{n}{2} \rfloor$. Hence, the above set F is a minimal edge dominating set with minimum cardinality among all minimal edge dominating sets of $S'(W_n)$. Thus, $\gamma'(S'(W_n)) = \lfloor \frac{n}{2} \rfloor$. □

Theorem 2.5 Let $S'(K_{1,n})$ be the splitting graph of star $K_{1,n}$. Then $\gamma'(S'(K_{1,n})) = 2$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the pendant vertices of star $K_{1,n}$ which are duplicated by the vertices $v'_1, v'_2, \ldots, v'_n$ respectively and let c be the apex vertex of $K_{1,n}$ which is duplicated by the vertex c'. Then the resultant graph $S'(K_{1,n})$ will have $2(n+1)$ vertices and $3n$ edges.

Now, in $S'(K_{1,n})$, an edge incident with the vertex c and an edge incident with the vertex c' will dominate all the edges incident with the vertex c and with the vertex c' respectively. Therefore, two such edges are enough to dominate all the edges of $S'(K_{1,n})$. Therefore, any edge dominating set F of $S'(K_{1,n})$ must have atleast two such edges for its minimum cardinality. Hence, $\mid F \mid \geq 2$ which implies that $\gamma'(S'(K_{1,n})) = 2$ as required. □

III. Edge Domination in Edge Splitting Graphs

Definition 3.1 Duplication of an edge $e = uv$ of graph G produces a new graph G' by adding an edge $e' = u'v'$ such that with $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.

Definition 3.2 If the edges of graph G are duplicated altogether then the resultant graph is known as edge splitting graph of G, which is denoted as $S'_e(G)$.

Theorem 3.1 Let $S'_e(P_n)$ be the edge splitting graph of path P_n. Then

$$\gamma'(S'_e(P_n)) = \begin{cases} n-1 & \text{if n} \equiv 0 \; or \; 1 \,(\text{mod}\, 3) \\ \\ n & \text{otherwise.} \end{cases}$$

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n and let $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n which are duplicated by the edges $e'_1, e'_2, \ldots, e'_{n-1}$ respectively. Then the graph $S'_e(P_n)$ will have $(3n-2)$ vertices and $(4n-6)$ edges.

Let the vertices $v_i, v'_i, v''_j \in V(S'_e(P_n))$ and the edges $e_k, e'_k, f_l, f'_j \in E(S'_e(P_n))$ for $1 \leq i \leq n,\ 2 \leq j \leq n-1,\ 1 \leq k \leq n-1$ and $1 \leq l \leq n-2$ where $e_k = v_k v_{k+1}$, $e'_1 = v'_1 v'_2$, $e'_k = v''_k v'_{k+1}$ $(2 \leq k \leq n-1)$, $f_l = v_l v''_{l+1}$ and $f'_j = v'_j v_{j+1}$.

Case-I $n \equiv 0\,(mod\,3)$.

For $n = 3$, we claim that $F = \{f_1, f_2'\}$ is the only minimal edge dominating set of P_3 with minimum cardinality because for any $e \in F$, the set $F - \{e\}$ will not be an edge dominating set and $deg(f_1) = deg(f_2') = 2 = \triangle'(S_e'(P_3))$ and all the edges of $S_e'(P_3)$ dominated by the set F are distinct. Therefore, $\gamma'(S_e'(P_3)) = 2$ that is, $\gamma'(S_e'(P_n)) = n - 1$ for $n = 3$.

For $n > 3$, first we construct an edge set of $S_e'(P_n)$ as follows:

$$F = \{f_1, f_4, \ldots, f_{3r+1}, f_2', f_5', \ldots, f_{3r+2}', e_3', e_6', \ldots, e_{3t}'\},$$

$$\text{where } 0 \leq r \leq \tfrac{n-3}{3},\ 1 \leq t \leq \tfrac{n-3}{3}.$$

The above set F is an edge dominating set of $S_e'(P_n)$ as each edge in $E(S_e'(P_n))$ is either in F or adjacent to an edge in F.

Moreover, for each edge $e \in F$, there exists an edge $f \in E(S_e'(P_n)) - F$ for which $N(f) \cap F = \{e\}$. Therefore, by Theorem 2.1., the set F is a minimal edge dominating set of $S_e'(P_n)$. Further, we claim that at least $(n-1)$ distinct edges are required to dominate the duplicated edges $e_1', e_2', \ldots, e_{n-1}'$ of $S_e'(P_n)$ because any two of the duplicated edges are not adjacent to each other and there is no edge which is adjacent to any two of the duplicated edges. Therefore, any edge dominating set of $S_e'(P_n)$ must have at least $(n-1)$ distinct edges. Now, the above set F is a minimal edge dominating set of $S_e'(P_n)$ and $\mid F \mid = 2(\frac{n-3}{3} + 1) + \frac{n-3}{3} = \frac{2n-6+6+n-3}{3} = \frac{3n-3}{3} = n - 1$. Hence, F is a minimal edge dominating set with minimum cardinality among all minimal edge dominating sets of $S_e'(P_n)$. Thus, we have proved that $\gamma'(S_e'(P_n)) = \mid F \mid = n - 1$ for $n \equiv 0\,(mod\,3)$.

Case-II $n \equiv 1\,(mod\,3)$.

We construct an edge set of $S_e'(P_n)$ as follows:

$$F = \{f_2, f_5, \ldots, f_{3r+2}, f_2', f_5', \ldots, f_{3r+2}', e_2', e_5', \ldots, e_{3r+2}'\},$$

$$\text{where } 0 \leq r \leq \tfrac{n-4}{3}.$$

Here, $\mid F \mid = 3\left(\frac{n-4}{3} + 1\right) = 3\left(\frac{n-1}{3}\right) = n - 1$.

By the arguments similar to Case-I, we reach to $\gamma'(S_e'(P_n)) = \mid F \mid = n - 1$ for $n \equiv 1\,(mod\,3)$.

Case-III $n \equiv 2\,(mod\,3)$.

In $S_e'(P_n)$, any two of the edges $e_1', e_2', \ldots, e_{n-1}'$ are not adjacent to each other and also no edge is adjacent to any two of these edges. Therefore, any edge dominating set F of $S_e'(P_n)$ must contain $(n-1)$ distinct edges to dominate these $(n-1)$ duplicated edges. Moreover, from the nature of $S_e'(P_n)$ where $n \equiv 2\,(mod\,3)$, it can be seen that only $(n-1)$ distinct edges are not enough to dominate all the edges of $S_e'(P_n)$ which implies that $\mid F \mid > n - 1$.

Now, we construct an edge set of $S_e'(P_n)$ as follows:

$$F_1 = \{f_2, f_5, \ldots, f_{3r+2}, f_2', f_5', \ldots, f_{3r+2}', e_2', e_5', \ldots, e_{3r+2}', e_{n-1}', e_{n-1}\},$$

$$\text{where } 0 \leq r \leq \tfrac{n-5}{3}.$$

Here, $\mid F_1 \mid = 3\left(\frac{n-5}{3} + 1\right) + 2 = n - 5 + 3 + 2 = n$.

By the arguments similar to Case-I, F_1 is a minimal edge dominating set of $S_e'(P_n)$. Since $\mid F \mid > n - 1$ and here $\mid F_1 \mid = n$, it follows that $\gamma'(S_e'(P_n)) = \mid F_1 \mid = n$ for $n \equiv 2\,(mod\,3)$.

Thus, we have proved that

$$\gamma'(S_e'(P_n)) = \begin{cases} n-1 & \text{if n} \equiv 0 \text{ } or \text{ } 1\,(\text{mod}3) \\ \\ n & \text{otherwise.} \end{cases}$$

Theorem 3.2 Let $S_e'(C_n)$ be the edge splitting graph of the cycle C_n. Then $\gamma'(S_e'(C_n)) = n$.

***Proof*:** Let $v_1, v_2, \ldots, v_n$ be the vertices of cycle C_n and let $e_1, e_2, \ldots, e_n$ be the edges of cycle C_n which are duplicated by the edges $e_1', e_2', \ldots, e_n'$ respectively. Then the graph $S_e'(C_n)$ will have $3n$ vertices and $4n$ edges.

Let the vertices $v_i, v_i', v_i'' \in V(S_e'(C_n))$ and the edges $e_i, e_i', f_i, f_i' \in E(S_e'(C_n))$ for $i = 1, 2, \ldots, n$ where $f_j = v_j v_{j+1}''$ for $1 \leq j \leq n-1$, $f_n = v_n v_1'$, $f_j' = v_{j+1} v_j'$ for $2 \leq j \leq n-1$, $f_1' = v_2 v_1''$, $f_n' = v_1 v_n'$.

Now, we construct an edge set $F = \{f_1, f_2', f_3', \ldots, f_{n-1}', f_n'\}$.

Since each edge in $E(S_e'(C_n))$ is either in F or adjacent to an edge in F, the above set F is an edge dominating set of $S_e'(C_n)$.

Moreover, for each edge $e \in F$, there exists an edge $f \in E(S_e'(C_n)) - F$ for which $N(f) \cap F = \{e\}$. Therefore, by Theorem 2.1., the set F is a minimal edge dominating set of $S_e'(C_n)$. Further, we claim that at least n distinct edges are required to dominate the duplicated edges $e_1', e_2', \ldots, e_n'$ of $S_e'(C_n)$ because any two of the duplicated edges are not adjacent to each other and also no edge is adjacent to any two duplicated edges. Therefore, any edge dominating set of $S_e'(C_n)$ must have at least n distinct edges. Now, the above set F is a minimal edge dominating set of $S_e'(C_n)$ and $\mid F \mid = n$. Hence, F is a minimal edge dominating set with minimum cardinality among all minimal edge dominating sets of $S_e'(C_n)$. Thus, $\gamma'(S_e'(C_n)) = n$ as required.

Theorem 3.3 Let $S_e'(W_n)$ be the edge splitting graph of wheel W_n. Then $\gamma'(S_e'(W_n)) = 2(n-1)$.

Proof: Let $v_1, v_2, \ldots, v_{n-1}$ be the rim vertices of W_n and c be the apex vertex of W_n. Let $e_1, e_2, \ldots, e_{n-1}$ be the rim edges of W_n which are duplicated by the edges $e_1', e_2', \ldots, e_{n-1}'$ respectively and let $f_1, f_2, \ldots, f_{n-1}$ be the spoke edges of W_n which are duplicated by the edges $f_1', f_2', \ldots, f_{n-1}'$ respectively.

Now, from the nature of $S_e'(W_n)$, we observe that at least $2(n-1)$ distinct edges are required to dominate the duplicated edges of $S_e'(W_n)$ because

(i) none of the edges $e_1', e_2', \ldots, e_{n-1}'$ of $S_e'(W_n)$ are adjacent to each other.

(ii) none of the edges $f_1', f_2', \ldots, f_{n-1}'$ of $S_e'(W_n)$ are adjacent to each other.

(iii) there is no edge which is adjacent to any two of the edges $e_1', e_2', \ldots, e_{n-1}'$ of $S_e'(W_n)$.

(iv) there is no edge which is adjacent to any two of the edges $f_1', f_2', \ldots, f_{n-1}'$ of $S_e'(W_n)$.

(v) there are total $2(n-1)$ duplicated edges of $S_e'(W_n)$.

Since these $2(n-1)$ edges can also dominate the remaining edges of $S'_e(W_n)$, it follows that any edge dominating set F of $S'_e(W_n)$ must have at least $2(n-1)$ distinct edges of $S'_e(W_n)$. Hence, $\mid F \mid \geq 2(n-1)$ which implies that $\gamma'(S'_e(W_n)) = 2(n-1)$ as required.

Theorem 3.4 Let $S'_e(K_{1,n})$ be the edge splitting graph of star $K_{1,n}$. Then $\gamma'(S'_e(K_{1,n})) = n$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the pendant vertices of star $K_{1,n}$ and let c denotes the apex vertex of $K_{1,n}$. Let $e_1, e_2, \ldots, e_n$ be the pendant edges of $K_{1,n}$ which are duplicated by the edges $e'_1, e'_2, \ldots, e'_n$ respectively. Then the resultant graph $S'_e(K_{1,n})$ will have $(3n+1)$ vertices and $n(n+1)$ edges.

Now, from the nature of $S'_e(K_{1,n})$, we observe that at least n distinct edges are required to dominate the duplicated edges of $S'_e(K_{1,n})$ because

(i) none of the edges $e'_1, e'_2, \ldots, e'_n$ of $S'_e(K_{1,n})$ are adjacent to each other.

(ii) there is no edge which is adjacent to any two of the edges $e'_1, e'_2, \ldots, e'_n$ of $S'_e(K_{1,n})$.

(iii) there are total n duplicated edges of $S'_e(K_{1,n})$.

Since these n edges can also dominate the remaining edges of $S'_e(K_{1,n})$, it follows that any edge dominating set F of $S'_e(K_{1,n})$ must have at least n distinct edges of $S'_e(W_n)$. Hence, $\mid F \mid \geq n$ implying that $\gamma'(S'_e(K_{1,n})) = n$.

IV. Concluding Remarks

The discussion on global domination number in the context of some graph operations was held by Vaidya and Pandit [13], [14] while here we contribute some new results for edge domination in graphs and determine edge domination number for the larger graphs obtained by means of graph operations on some standard graphs.

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