Robustness of Triallel Cross Experiments using NBIB Mating Design against Interchange of a Cross

R. Shunmugathai and M.R. Srinivasan

***Abstract*—Mating Designs are the study of progenies developed through various methods like Triallel Cross Plans which are subjected to Incomplete Block Designs. The concept of robustness in designs has been studied and available in the literature. The effects of missing blocks on Triallel Cross Plans are examined in this study. A-efficiencies based on non -zero eigenvalues suggest that these designs are fairly robust. The investigation shows that Nested Balanced Incomplete Block Designs are fairly robust in terms of efficiency. In this paper, the robustness of Nested Balanced Incomplete Block Design for Triallel Crosses against interchange of any two crosses between any two blocks.**

***Index Terms*—Nested Balanced Incomplete Block Design; Efficiency of residual design; Mating Design; Youden Square Design; Latin Square Design. *MSC 2010 Codes* – 45B05, 51E05.**

I. Introduction

TRIALLEL crosses form an important class of Mating Designs, under breeding experiments, used for studying the genetic properties of a set of inbred lines. Rawlings and Cockerham [29] introduced Triallel Crosses or three - way crosses or Triallel Mating Design as a set of all possible three- way hybrids based on a given set of lines constitutes the Triallel Crosses or Triallel Mating Design.If v is the number of lines then there would be $v^* = v(v-1)(v-2)/2$ distinct three-way crosses constituting the Triallel Mating Design.Most of the common commercial hybrids in corn are either three - way hybrids or double cross - hybrids.The experience gained in corn and maize breeding has helped the cause of Triallel in other plants and animal breeding such as swine breeding, silk worm breeding and chicken breeding.Triallel has helped the breeders to improve the quantitative traits of economic and nutritional importance in crops and animals.It has been established that the three way hybrids are more stable than the pure lines and single cross hybrids and exhibit individual as well as population buffering mechanisms because of the broad genetic base.

Hinkelmann [18] introduced the Partial Triallel Cross (PTC) and formulated method of construction of Partial Triallel Cross (PTC) and its analysis.Hinkelmann [18]suggested an alternative model for Triallel crosses and Ponnuswamy [26] developed the analysis based on this alternative model. All possible three - way hybrids of the type $(AB)C$ where AB is the F_1 hybrid and C is an unrelated parent crossed to it to get the three - way hybrid. A and B are called as grand parents or half parents and C is called the parent. Thus given v lines there will be in total $v^* = v(v-1)(v-2)/2$ distinct three - way hybrids, without including reciprocal crosses.For example, with 5 lines there will be v^*=30 crosses and they are:

(1× 2)3	(1 × 2)4	(1 × 2)5	(1 × 3)2	(1 × 3)4	(1 × 3)5
(1 × 4)2	(1 × 4)3	(1 × 4)5	(1 × 5)2	(1 × 5)3	(1 × 5)4
(2 × 3)1	(2 × 3)4	(2 × 3)5	(2 × 4)1	(2 × 4)3	(2 × 4)5
(2 × 5)1	(2 × 5)3	(2 × 5)4	(3 × 4)1	(3 × 4)2	(3 × 4)5
(3 × 5)1	(3 × 5)2	(3 × 5)4	(4 × 5)1	(4 × 5)2	(4 × 5)3

Characteristics of Triallel Mating Design:

1) Each line would appear in $r_H = (n-1)(n-2)$ three-way crosses as grand-parent and in $r_F = (n-1)(n-2)/2$ three -way crosses as parent.
2) Each pair of lines will occur in $r_d = (n-2)$ crosses both of them as grand - parents, and in $r_s = (n-2)$ of the crosses one of them as parent and the other as grand-parent or vice-versa.
3) Each triplet of lines is involved in three distinct crosses, two of them as grand-parent and one of them as parent.

The Quadratic Unbiased Estimators proposed by Ponnuswamy [25] have fared better than the mathematically elegant Quadratic Least Square Estimators for Triallel Mating Design for the estimation of design and genetic components of variance.Ponnuswamy and Srinivasan [24] have made substantial contributions to the theory of Partial Triallel Crosses (PTC).Srinivasan [32] has developed Quadratic Least Square estimators for design and genetic components of variance for Triallel Mating Design.Subbarayan [31] has studied on some methods of construction of Partial Triallel Matting Designs for estimation of genetic components of variance. He has studied the estimation of design and genetic components of variance for Partial Triallel Cross Mating Design based on Self - Orthogonal Latin Squares, BIB Design, Partially Doubly Balanced Incomplete Block Design and Partially Balanced Incomplete Block Design.

Arora and Aggarwal [2], Ceranka, Chudzik, Dobek and Kielczewska [7], and Ponnuswamy and Srinivasan [24] have discussed the construction of Triallel Crosses. Customarily, Triallel crosses have been conducted using a Completely Randomized Design or a Randomised Complete Block Design involving n_c treatments.Thus, if p is a large adoption of an unblocked design or a Complete Block Design, it is not appropriate unless the experimental units are extremely homogeneous. Gupta and Kageyama [13], Dey and Midha [10] and Das and Gupta [9] start with number of lines as p, rather than n_c, the total number of distinct crosses in the experiment.

R. Shunmugathai is a Research Scholar in the Department of Statistics, University of Madras, Chennai-600 005, India (e-mail: shunmuga_77@yahoo.co.in)

Dr. M.R. Srinivasan is a Professor in the Department of Statistics, University of Madras, Chennai-600 005, India (e-mail: mrsvasan8@hotmail.com)

Discussions on Design of comparative Experiments are studied by Hinkelmann and Kempthorne [17], Atkinson, Donev, and Tobias [1], Hinkelmann and Kempthorne [18] and Bailey [4]. Eric W. Weisstein and [12] discussed the Balanced Incomplete Block Design (BIBD) as a well studied experimental design with desirable features from a statistical perspective. Preece [27] has introduced the case of two - way elimination of heterogeneity, one nested within the other known as Nested Balanced Incomplete Block Design.

An arrangement of v treatments each replicated r times in two system of block is said to be a Nested Balanced Block Design with parameter
$(v, r, b_1, k_1, \lambda_1, b_2, k_2, \lambda_2, m)$ if:

1) Second system is nested within the first, with each block from the first system containing m blocks from the second system (sub blocks).
2) Ignoring the second system leaves a Balanced Block Design with b_1 blocks each of k_1 units with λ_1 concurrence.
3) Ignoring the first system leaves a Balanced Block Design with b_2 blocks each of k_2 units with λ_2 concurrences.

The Parametric relationships are given as:

1) $vr = b_1k_1 = b_1k_2m = b_2k_2$
2) $\lambda_1(v-1) = r(k_1-1) = (v-1)\lambda_2 = r(k_2-1)$
3) $(\lambda_1 - m\lambda_2)(v-1) = r(m-1)$

Robustness has been investigated using the connectedness criterion Ghosh [14] and the efficiency criterion John [19]. Das and Kageyama [11] showed that Balanced Block Designs and extended Balanced Block Designs are fairly robust against the unavailability of $s(s \leq k)$ observations in any block, while any Youden design and Latin Square Design are found to be fairly robust against the loss of any one column. Robustness of experimental designs for a certain type of Triallel Cross experiments is investigated. Nested Balanced Block Designs are introduced and it is shown how these designs give rise to optimal designs for Triallel Crosses.

Das and Gupta [9] used Nested Balanced Block Designs for Triallel Crosses.
The constructions of Triallel Crosses start with p, the number of lines rather than n_c the total number of distinct crosses in the experiment.They have developed the procedures for optimal designs involving n experimental units where $n \leq n_c$.They obtained optimal Mating Designs using Nested Balanced Incomplete Block Design with sub-blocks of size 3 each. Kleczkouski[20] devised a form of Nested Balanced Incomplete Block Design with $v = 8$ treatments for a series of experiment in which bean plants, in two primary leaves stage, were inoculated with sap from tobacco plants infected with tobacco necrosis virus. The treatments were eight different virus concentrations. Each leaf had two inoculations,one for each half- leaf.Ignoring the leaf positions, plants and leaves were, respectively, the blocks and sub blocks of a Nested Balanced Incomplete Block Design.

The experimental design theory developed in the above investigations assumes the absence of disturbances like missing observations, outlying observations or inadequacy of assumed model, etc.These assumptions may, however, be violated in real life; thus rendering even an optimal design poor.Consequently, the design that is efficient for estimating various treatment contrasts may no longer remain efficient after it undergoes a disturbance.Interchange of a pair of treatments (crosses) is one such aberration that needs attention during the execution of an experiment.Interchange of a pair of crosses is said to have occurred,if two experimental units belonging to different blocks receive the crosses originally designated for the other.Such discrepancy may occur due to the following reasons:

1) Due to interchange of tags or labels attached to the seed packets of different crosses that could not be detected before the application of crosses to the experimental units,
2) Human errors in the preparation of field layout plan, where each of a pair of blocks accommodates a cross originally designated for the other.

These types of disturbances were first reported by Pearce [28] in a Randomized Block Design set up.These disturbances have been termed as mechanical errors by Gomez and Gomez [15] whereas Pearce [23] called these as errors in the application of the treatments. The properties of the original design may be affected in presence of such discrepancy. Therefore, there is a need to address the problem by knowing the designs that are insensitive to such types of disturbances. Batra Sreenath and Parsad[6] studied the robustness of block designs against interchange of a pair of treatments. Panda, Sharma and Parsad [21] investigated the robustness of optimal block designs for Triallel Cross experiments against exchange of a cross. Here, an attempt has been made to investigate the robustness of optimal block designs for Triallel Crosses against interchange of any two crosses between any two blocks. Brandon Ogbunugafor, James B. Pease, and Paul E. Turner, [3] obtained the environmental robustness are phenotypic constancy in the face of environmental variation, where epistasis may be uninvolved.

In this paper, it has been noticed that Triallel Cross Plan is fairly robust against the interchange of two crosses between two blocks.Since number of common lines between any two cross which are changed are different, and hence number of common lines between any two crosses are at most two depending upon the crosses, two cases arises.Corresponding C^* matrices and their non-zero eigen-values with multiplicities are computed for each set of parameters and it appears that the Triallel Cross Plan is fairly robust against the unavailability of two blocks. All other cases can be obtained in the usual way.It has been seen that design is robust against interchange of any two crosses between any two blocks. Here, the efficiencies of 36 Triallel Cross Plan was worked out.In fact, all the design satisfies $e(s) \geq 0.90$.Thus it seems that design is fairly robust against loss of two blocks.

Most of the robustness criteria against the unavailability of data are:

1) To get the connectedness of the residual design ;
2) To have the variance balance of the residual design;
3) To consider the A-efficiency of residual design for the robustness study.

In the present investigation, consider a connected plan D.Let D^* be the residual design obtained when one cross is lost and two crosses are interchange between two blocks.Assume D^* to be connected.In this case, the criterion of robustness against the interchange of two crosses between two blocks is the overall A-efficiency of the residual design D^* given by,

$$e(s) = \frac{\text{Sum of reciprocals of non-zero eigenvalues of } C}{\text{Sum of reciprocals of non-zero eigenvalues of } C^*}$$

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \quad (1)$$

II. C - Matrix of Triallel Cross Plan

We know that for any block design C matrix can be defined as,

$$C = rI_v - \frac{NN'}{k_1^{-1}}$$

Since for Triallel Cross plan C matrix can be given by,

$$C_t = G_{ti} - \frac{NN'}{k_1^{-1}},$$

where

$$G_{ti} = \begin{pmatrix} w_{ti} & g'_{ii} \\ - & w_{t'i} \end{pmatrix}, \quad w_{ti} = w_{t'i} = r(k_1 - 1), \text{ and } g_{ii} = 1$$

$$NN' = \begin{bmatrix} \sum n_{1j}^2 & \sum n_{1j}n_{2j} & \sum n_{1j}n_{vj} \\ \sum n_{2j}n_{1j} & \sum n_{2j}^2 & \sum n_{2j}n_{vj} \\ \sum n_{vj}n_{1j} & \sum n_{vj}n_{2j} & \sum n_{vj}^2 \end{bmatrix}$$

Thus it is obvious that for this Triallel Cross Plan,

$$\sum n_{vj}^2 = r(k_1 - 1)^2$$

$$\sum n_{ij}n_{mj} = \lambda_1(k_1 - 1)^2$$

$$k = \frac{k_1(k_1 - 1)}{3}$$

Now C matrix is given as,

$$C = \begin{bmatrix} r(k_1-1) & \lambda_1 & \lambda_1 \\ \lambda_1 & r(k_1-1) & \lambda_1 \\ \lambda_1 & \lambda_1 & r(k_1-1) \end{bmatrix} - \frac{\begin{bmatrix} r(k_1-1)^2 & \lambda_1(k_1-1) & \lambda_1(k_1-1) \\ \lambda_1(k_1-1) & r(k_1-1)^2 & \lambda_1(k_1-1) \\ \lambda_1(k_1-1) & \lambda_1(k_1-1) & r(k_1-1)^2 \end{bmatrix}}{\frac{k_1(k_1-1)}{3}}$$

$$C = \theta(I_v - \frac{E_{vv}}{v})$$

The non - zero eigenvalues of C matrix and its corresponding multiplicity of Triallel Cross Plan can be given by,

$$\theta = \frac{\lambda_1 v(k_1 - 2)}{k_1}$$

with multiplicity $(v - 1)$.

III. Robustness of Triallel Crosses Plan using NBIBD against the Interchange of two crosses

Triallel Cross Plan was constructed from Nested Balanced Incomplete Block Design with parameters $v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2, m$. Now consider treatment of the Nested Balanced Incomplete Block Designs as lines and cross them between the lines in each block.This results in Triallel Cross Plan that involves p line with $p(p-1)(p-2)/3$ crosses. Without loss of generality one cross is interchanged with another cross from Triallel Cross Plan in two blocks.Call this design as an original design and assume that the original design C^* is connected design.

This situation can be treated by separating into two cases are considered:

(i) Between two affected crosses no line is common and after interchange of crosses, one cross is being repeated in one block and other cross does not get repeated in another block.

(ii) Between two affected crosses one line is common and after interchange of crosses both the affected crosses are repeated.

All the two cases are depending upon the common number of lines between two interchanged crosses and the affected blocks from two crosses interchanged in a Nested Balanced Incomplete Block Design.The efficiency factor depends upon the common number of lines between two lost blocks.The efficiency for all the two cases when the common number of lines between two lost blocks are $0, 1, 2, 3, \ldots, (k_1 - 1)$, k_1 are respectively studied.Here,the robustness criterion of Nested Balanced Incomplete Block Design is further discussed for the different value of common number of lines between two blocks.

Case i: Between two affected crosses no line is common and after interchange of crosses, one cross is being repeated in one block and other cross does not get repeated in another block.

Consider a Nested Balanced Incomplete Block Design D with parameters $v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2, m$.It follows that C matrix of design D is always given by,

$$C^* = R^* - N^* K_1^{-1^*} N'^*$$

Further C^* - matrix of the residual design can be rewritten as,

$$k_1(k_1 - 3)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & \varepsilon_{14} & \varepsilon_{15} & \varepsilon_{16} & \varepsilon_{17} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & \varepsilon_{24} & \varepsilon_{25} & \varepsilon_{26} & \varepsilon_{27} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & \varepsilon_{34} & \varepsilon_{35} & \varepsilon_{36} & \varepsilon_{37} \\ \varepsilon_{41} & \varepsilon_{42} & \varepsilon_{43} & \varepsilon_{44} & \varepsilon_{45} & \varepsilon_{46} & \varepsilon_{47} \\ \varepsilon_{51} & \varepsilon_{52} & \varepsilon_{53} & \varepsilon_{54} & \varepsilon_{55} & \varepsilon_{56} & \varepsilon_{57} \\ \varepsilon_{61} & \varepsilon_{62} & \varepsilon_{63} & \varepsilon_{64} & \varepsilon_{65} & \varepsilon_{66} & \varepsilon_{67} \\ \varepsilon_{71} & \varepsilon_{72} & \varepsilon_{73} & \varepsilon_{74} & \varepsilon_{75} & \varepsilon_{76} & \varepsilon_{77} \end{bmatrix}$$

where

$$\begin{aligned}
\varepsilon_{11} &= \varepsilon_{44}=\varepsilon_{55}=\varepsilon_{66}=\varepsilon_{77}\\
&= \lambda_1(v-1)(k_1-1)(k_1-3)\\
\varepsilon_{12} &= \varepsilon_{13}=\varepsilon_{14}=\varepsilon_{15}=\varepsilon_{16}\\
&= \varepsilon_{17}=\varepsilon_{21}=\varepsilon_{31}=\varepsilon_{41}=-\lambda_1(k_1-1)(k_1-3)\\
\varepsilon_{45} &= \varepsilon_{51}=\varepsilon_{54}=\varepsilon_{61}=\varepsilon_{67}=\varepsilon_{71}\\
&= \varepsilon_{76}=-\lambda_1(k_1-1)(k_1-3)]\\
\varepsilon_{22} &= \varepsilon_{33}=\lambda_1(v-1)(k_1-1)(k_1-3)-9\\
\varepsilon_{23} &= \varepsilon_{32}=-\lambda_1(k_1-1)(k_1-3)-9\\
\varepsilon_{24} &= \varepsilon_{26}=\varepsilon_{35}=\varepsilon_{36}=\varepsilon_{42}\\
&= -\lambda_1(k_1-1)(k_1-3)-3(k_1-1)\\
\varepsilon_{47} &= \varepsilon_{53}=\varepsilon_{57}=\varepsilon_{62}=\varepsilon_{63}=\varepsilon_{74}\\
&= \varepsilon_{75}=-\lambda_1(k_1-1)(k_1-3)-3(k_1-1)\\
\varepsilon_{25} &= \varepsilon_{27}=\varepsilon_{34}=\varepsilon_{37}=\varepsilon_{43}=\varepsilon_{46}\\
&= -\lambda_1(k_1-1)(k_1-3)+3(k_1-1)\\
\varepsilon_{52} &= \varepsilon_{56}=\varepsilon_{64}=\varepsilon_{65}=\varepsilon_{72}=\varepsilon_{73}\\
&= -\lambda_1(k_1-1)(k_1-3)+3(k_1-1)
\end{aligned}$$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $(\lambda_1 ab+w)+(27+9(9+3a^2)^1/2)$ with multiplicity 1.
2) $(\lambda_1 ab+w)-(27-9(9+3a^2)^1/2)$ with multiplicity 1.
3) $(\lambda_1 ab+w)$ with multiplicity $(v-3)$

For simplification, we set

$$a=k_1-1 \quad \text{and} \quad b=k_1-3$$

Theorem 1: Triallel Cross Plan obtained from a Nested Balanced Incomplete Block Designs with parameters $v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2$, are fairly robust against the interchange of one cross with another cross between two blocks of a Triallel Cross Plan between two blocks.Between two affected crosses no line is common, provided the overall efficiency of the residual design is given by,

$$e(s)=\frac{(v-1)(\alpha+D)(\alpha-E)}{(\alpha(\alpha-E)+\alpha(\alpha+D)+(v-3)(\alpha+D)(\alpha-E)}$$

Where,

$$\begin{aligned}
W &= \lambda_1(v-1)(k_1-1)(k_1-3)=\lambda_1(v-1)ab\\
\alpha &= \lambda_1 v(k_1-1)(k_1-3)\\
E &= [27+9(9+3a^2)^1/2]\\
D &= [27-9(9+3a^2)^1/2]
\end{aligned}$$

Proof: Without loss of generality, if one cross is interchange with another cross between two blocks of Triallel Cross Plan, C^* matrix of the residual design is given by,

$$k_1(k_1-3)C^*=\begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & \varepsilon_{14} & \varepsilon_{15} & \varepsilon_{16} & \varepsilon_{17}\\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & \varepsilon_{24} & \varepsilon_{25} & \varepsilon_{26} & \varepsilon_{27}\\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & \varepsilon_{34} & \varepsilon_{35} & \varepsilon_{36} & \varepsilon_{37}\\
\varepsilon_{41} & \varepsilon_{42} & \varepsilon_{43} & \varepsilon_{44} & \varepsilon_{45} & \varepsilon_{46} & \varepsilon_{47}\\
\varepsilon_{51} & \varepsilon_{52} & \varepsilon_{53} & \varepsilon_{54} & \varepsilon_{55} & \varepsilon_{56} & \varepsilon_{57}\\
\varepsilon_{61} & \varepsilon_{62} & \varepsilon_{63} & \varepsilon_{64} & \varepsilon_{65} & \varepsilon_{66} & \varepsilon_{67}\\
\varepsilon_{71} & \varepsilon_{72} & \varepsilon_{73} & \varepsilon_{74} & \varepsilon_{75} & \varepsilon_{76} & \varepsilon_{77}
\end{bmatrix}$$

where

$\varepsilon_{11}=\varepsilon_{44}=\varepsilon_{55}=\varepsilon_{66}=\varepsilon_{77}=\lambda_1(v-1)(k_1-1)(k_1-3)$
$\varepsilon_{12}=\varepsilon_{13}=\varepsilon_{14}=\varepsilon_{15}=\varepsilon_{16}=\varepsilon_{17}=\varepsilon_{21}=\varepsilon_{31}=\varepsilon_{41}= -\lambda_1(k_1-1)(k_1-3)$
$\varepsilon_{45}=\varepsilon_{51}=\varepsilon_{54}=\varepsilon_{61}=\varepsilon_{67}=\varepsilon_{71}=\varepsilon_{76}= -\lambda_1(k_1-1)(k_1-3)]$
$\varepsilon_{22}=\varepsilon_{33}=\lambda_1(v-1)(k_1-1)(k_1-3)-9$
$\varepsilon_{23}=\varepsilon_{32}=-\lambda_1(k_1-1)(k_1-3)-9$
$\varepsilon_{24}=\varepsilon_{26}=\varepsilon_{35}=\varepsilon_{36}=\varepsilon_{42}= -\lambda_1(k_1-1)(k_1-3)-3(k_1-1)$
$\varepsilon_{47}=\varepsilon_{53}=\varepsilon_{57}=\varepsilon_{62}=\varepsilon_{63}=\varepsilon_{74}=\varepsilon_{75}= -\lambda_1(k_1-1)(k_1-3)-3(k_1-1)$
$\varepsilon_{25}=\varepsilon_{27}=\varepsilon_{34}=\varepsilon_{37}=\varepsilon_{43}=\varepsilon_{46}= -\lambda_1(k_1-1)(k_1-3)+3(k_1-1)$
$\varepsilon_{52}=\varepsilon_{56}=\varepsilon_{64}=\varepsilon_{65}=\varepsilon_{72}=\varepsilon_{73}= -\lambda_1(k_1-1)(k_1-3)+3(k_1-1)$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $(\lambda_1 ab+w)+(27+9(9+3a^2)^1/2)$ with multiplicity 1.
2) $(\lambda_1 ab+w)-(27-9(9+3a^2)^1/2)$ with multiplicity 1.
3) $(\lambda_1 ab+w)$ with multiplicity $(v-3)$

Further, overall A-efficiency is calculated as,

$$e(s)=\frac{\text{Sum of reciprocals of non-zero eigenvalues of } C}{\text{Sum of reciprocals of non-zero eigenvalues of } C^*}$$

$$e(s)=\frac{\phi_2(s)}{\phi_1(s)}$$

That is,

$$\phi_2(s)=\frac{(v-1)}{(\alpha)} \tag{2}$$

$$\phi_1(s)=\frac{\alpha(\alpha-E)+\alpha(\alpha+D)+(v-3)(\alpha+D)(\alpha-E)}{\alpha(\alpha+D)(\alpha-E)}$$

Finally, A- efficiency is given by,

$$e(s)=\frac{(v-1)(\alpha+D)(\alpha-E)}{\alpha(\alpha-E)+\alpha(\alpha+D)+(v-3)(\alpha+D)(\alpha-E)}$$

Example 1: Let D represent the Nested Balanced Incomplete Block Design with parameters v = 7, b_1= 7, b_2 = 14, r = 6, k_1 = 6, k_2 = 3, λ_1 = 5. Design D is given by Table 1.

Let us interchange the two crosses (5 × 6 × 7) and (1 × 2 × 4) from block 1 and block 7 where no line is common. The C^* matrix of the residual design is given by,

$$30C^*=\begin{bmatrix}
450 & -75 & -75 & -75 & -75 & -75 & -75\\
-75 & 441 & -66 & -60 & -90 & -60 & -90\\
-75 & -66 & 441 & -90 & -60 & -60 & -90\\
-75 & -60 & -90 & 450 & -75 & -90 & -60\\
-75 & -90 & -60 & -75 & 450 & -90 & -60\\
-75 & -60 & -60 & -90 & -90 & 450 & -75\\
-75 & -90 & -90 & -60 & -60 & -75 & 450
\end{bmatrix}$$

The non zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) 482.573, with multiplicity 1.
2) 484.679, with multiplicity 1.
3) .525, with multiplicity 4.

The overall A- efficiency of the design is

$$e(s)=0.940192$$

TABLE I
NESTED BALANCED INCOMPLETE BLOCK DESIGN

Block	NBIBD						Crosses in the NBIBD	
1	2	3	4	5	6	7	2 × 3 × 5	**4**× **6** × **7**
2	1	3	4	5	6	7	3 × 4 × 6	1 × 5 × 7
3	1	2	4	5	6	7	4 × 5 × 7	1 × 2 × 6
4	1	2	3	5	6	7	1 × 5 × 6	2 × 3 × 7
5	1	2	3	4	6	7	2 × 6 × 7	1 × 3 × 4
6	1	2	3	4	5	7	1 × 3 × 7	2 × 4 × 5
7	1	2	3	4	5	6	**1** × **2** × **4**	3 × 5 × 6

Case ii: Between two affected crosses one line is common and after interchange of crosses both the affected crosses are repeated.

Consider a Nested Balanced Incomplete Block Design D with parameters $v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2, m$. Without loss of generality one cross is interchanged with another cross between two blocks of Triallel Cross Plan. Let the blocks be bi and b_j. Between two affected crosses one line is common and after interchange of crosses both the affected crosses are repeated in respective blocks. Call this design as a residual design and assume that the residual design D^* is connected. Let C^* be the information matrix of the residual design is given as,

$$C^* = R^* - N^* K_1^{-1^*} N'^*$$

Further C^* - matrix of the residual design can be rewritten as,

$$k_1(k_1-3)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & \varepsilon_{14} & \varepsilon_{15} & \varepsilon_{16} & \varepsilon_{17} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & \varepsilon_{24} & \varepsilon_{25} & \varepsilon_{26} & \varepsilon_{27} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & \varepsilon_{34} & \varepsilon_{35} & \varepsilon_{36} & \varepsilon_{37} \\ \varepsilon_{41} & \varepsilon_{42} & \varepsilon_{43} & \varepsilon_{44} & \varepsilon_{45} & \varepsilon_{46} & \varepsilon_{47} \\ \varepsilon_{51} & \varepsilon_{52} & \varepsilon_{53} & \varepsilon_{54} & \varepsilon_{55} & \varepsilon_{56} & \varepsilon_{57} \\ \varepsilon_{61} & \varepsilon_{62} & \varepsilon_{63} & \varepsilon_{64} & \varepsilon_{65} & \varepsilon_{66} & \varepsilon_{67} \\ \varepsilon_{71} & \varepsilon_{72} & \varepsilon_{73} & \varepsilon_{74} & \varepsilon_{75} & \varepsilon_{76} & \varepsilon_{77} \end{bmatrix}$$

where
$\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = \varepsilon_{55} = \lambda_1(v-1)(k_1-1)(k_1-3) - 9$
$\varepsilon_{12} = \varepsilon_{21} = \varepsilon_{31} = \varepsilon_{32} = \varepsilon_{35} = \varepsilon_{53} = -\lambda_1(k_1-1)(k_1-3) + 9$
$\varepsilon_{13} = \varepsilon_{15} = \varepsilon_{23} = \varepsilon_{25} = \varepsilon_{51} = \varepsilon_{52} = -\lambda_1(k_1-1)(k_1-3) - 9$
$\varepsilon_{14} = \varepsilon_{24} = \varepsilon_{34} = \varepsilon_{41} = \varepsilon_{42} = \varepsilon_{43} = \varepsilon_{45} = \varepsilon_{46} = \varepsilon_{47} = \varepsilon_{54} = \varepsilon_{64} = \varepsilon_{67} = \varepsilon_{74} = \varepsilon_{76} = -\lambda_1(k_1-1)(k_1-3)$
$\varepsilon_{16} = \varepsilon_{26} = \varepsilon_{37} = \varepsilon_{57} = \varepsilon_{61} = \varepsilon_{62} = \varepsilon_{73} = \varepsilon_{75} = -\lambda_1(k_1-1)(k_1-3) - 3(k_1-1)$
$\varepsilon_{17} = \varepsilon_{27} = \varepsilon_{36} = \varepsilon_{56} = \varepsilon_{63} = \varepsilon_{65} = \varepsilon_{71} = \varepsilon_{72} = -\lambda_1(k_1-1)(k_1-3) + 3(k_1-1)$
$\varepsilon_{44} = \varepsilon_{66} = \varepsilon_{77} = \lambda_1(v-1)(k_1-1)(k_1-3)$

The non-zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $(\lambda_1 ab + w) + (9 - 9(1+a^2)^1/2)$ with multiplicity 1.
2) $(\lambda_1 ab + w) - (9 + 9(1+a^2)^1/2)$ with multiplicity 1.
3) $(\lambda_1 ab + w)$ with multiplicity $(v-3)$

For simplification, we set

$$a = k_1 - 1 \quad \text{and} \quad b = k_1 - 3$$

Proof: Without loss of generality, if one cross is interchange with another cross between two blocks of Triallel Cross Plan, C^* matrix of the residual design is given by,

$$k_1(k_1-3)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & \varepsilon_{14} & \varepsilon_{15} & \varepsilon_{16} & \varepsilon_{17} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & \varepsilon_{24} & \varepsilon_{25} & \varepsilon_{26} & \varepsilon_{27} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & \varepsilon_{34} & \varepsilon_{35} & \varepsilon_{36} & \varepsilon_{37} \\ \varepsilon_{41} & \varepsilon_{42} & \varepsilon_{43} & \varepsilon_{44} & \varepsilon_{45} & \varepsilon_{46} & \varepsilon_{47} \\ \varepsilon_{51} & \varepsilon_{52} & \varepsilon_{53} & \varepsilon_{54} & \varepsilon_{55} & \varepsilon_{56} & \varepsilon_{57} \\ \varepsilon_{61} & \varepsilon_{62} & \varepsilon_{63} & \varepsilon_{64} & \varepsilon_{65} & \varepsilon_{66} & \varepsilon_{67} \\ \varepsilon_{71} & \varepsilon_{72} & \varepsilon_{73} & \varepsilon_{74} & \varepsilon_{75} & \varepsilon_{76} & \varepsilon_{77} \end{bmatrix}$$

where
$\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = \varepsilon_{55} = \lambda_1(v-1)(k_1-1)(k_1-3) - 9$
$\varepsilon_{12} = \varepsilon_{21} = \varepsilon_{31} = \varepsilon_{32} = \varepsilon_{35} = \varepsilon_{53} = -\lambda_1(k_1-1)(k_1-3) + 9$
$\varepsilon_{13} = \varepsilon_{15} = \varepsilon_{23} = \varepsilon_{25} = \varepsilon_{51} = \varepsilon_{52} = -\lambda_1(k_1-1)(k_1-3) - 9$
$\varepsilon_{14} = \varepsilon_{24} = \varepsilon_{34} = \varepsilon_{41} = \varepsilon_{42} = \varepsilon_{43} = \varepsilon_{45} = \varepsilon_{46} = \varepsilon_{47} = \varepsilon_{54} = \varepsilon_{64} = \varepsilon_{67} = \varepsilon_{74} = \varepsilon_{76} = -\lambda_1(k_1-1)(k_1-3)$
$\varepsilon_{16} = \varepsilon_{26} = \varepsilon_{37} = \varepsilon_{57} = \varepsilon_{61} = \varepsilon_{62} = \varepsilon_{73} = \varepsilon_{75} = -\lambda_1(k_1-1)(k_1-3) - 3(k_1-1)$
$\varepsilon_{17} = \varepsilon_{27} = \varepsilon_{36} = \varepsilon_{56} = \varepsilon_{63} = \varepsilon_{65} = \varepsilon_{71} = \varepsilon_{72} = -\lambda_1(k_1-1)(k_1-3) + 3(k_1-1)$
$\varepsilon_{44} = \varepsilon_{66} = \varepsilon_{77} = \lambda_1(v-1)(k_1-1)(k_1-3)$

The non zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $(\lambda_1 ab + w) + (9 - 9(1+a^2)^1/2)$ with multiplicity 1.
2) $(\lambda_1 ab + w) - (9 + 9(1+a^2)^1/2)$ with multiplicity 1.
3) $(\lambda_1 ab + w)$ with multiplicity $(v-3)$.

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{3}$$

That is,

$$\phi_2(s) = \frac{(v-1)}{(\alpha)} \tag{4}$$

$$\phi_1(s) = \frac{\alpha(\alpha - Z) + \alpha(\alpha + Y) + (v-3)(\alpha + Y)(\alpha - Z)}{\alpha(\alpha + Y)(\alpha - Z)} \tag{5}$$

Finally, A-efficiency is given by,

$$e(s) = \frac{(v-1)(\alpha + Y)(\alpha - Z)}{\alpha(\alpha - Z) + \alpha(\alpha + Y) + (v-3)(\alpha + Y)(\alpha - Z)} \tag{6}$$

Example 2: Let D represent the Nested Balanced Incomplete Block Design with parameters v = 7, b_1= 7, b_2 = 14, r = 6, k_1 = 6, k_2 = 3, λ_1 = 5. Design D is given by Table 2.

Let us interchange the two crosses (2 × 3 × 5) and (1 × 2 × 6) from block 1 and block 3 where one line is common.

TABLE II
NESTED BALANCED INCOMPLETE BLOCK DESIGN

Block	NBIBD						Crosses in the NBIBD	
1	2	3	4	5	6	7	$\mathbf{2} \times \mathbf{3} \times \mathbf{5}$	$4\times 6 \times 7$
2	1	3	4	5	6	7	$3 \times 4 \times 6$	$1 \times 5 \times 7$
3	1	2	4	5	6	7	$4 \times 5 \times 7$	$\mathbf{1} \times \mathbf{2} \times \mathbf{6}$
4	1	2	3	5	6	7	$1 \times 5 \times 6$	$2 \times 3 \times 7$
5	1	2	3	4	6	7	$2 \times 6 \times 7$	$1 \times 3 \times 4$
6	1	2	3	4	5	7	$1 \times 3 \times 7$	$2 \times 4 \times 5$
7	1	2	3	4	5	6	$1 \times 2 \times 4$	$3 \times 5 \times 6$

The C^* matrix of the residual design is given by

$$30C^* = \begin{bmatrix} 441 & -84 & -66 & -75 & -66 & -60 & -90 \\ -84 & 441 & -66 & -75 & -66 & -60 & -90 \\ -66 & -66 & 441 & -75 & -84 & -90 & -60 \\ -75 & -75 & -75 & 450 & -75 & -75 & -75 \\ -66 & -66 & -84 & -75 & 441 & -90 & -60 \\ -60 & -60 & -90 & -75 & -90 & 450 & -75 \\ -90 & -90 & -60 & -75 & -60 & -75 & 450 \end{bmatrix}$$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) 460.9131, with multiplicity 1.
2) 553.086, with multiplicity 1.
3) .525, with multiplicity 4.

The overall A- efficiency of the design is,

$$e(s) = 0.968939$$

IV. CONCLUSION

Various methods like Triallel Cross Plans which are subjected to Incomplete Block Designs are developed for Mating Designs. The literature provides the analysis of such plans, namely the estimation of variance components, design and genetics. To examine the robustness of various Mating Designs as it depends on the underlying experimental design is carried out in this study. Robustness of Triallel Cross Plan is examined using Nested Balanced Incomplete Block Design. There are varying Nested Balanced Incomplete Block Designs for different parametric values with interchange of two crosses between two blocks. This paper dealt with a class of Nested Balanced Incomplete Block Design for varying parametric values with interchange of two crosses between two blocks and efficiencies are calculated for each case. The variances in efficiencies depend on the following:

1) the number of common lines between two blocks is zero
2) the number of common lines between two blocks is one

Results have shown that the efficiencies of case (ii) will be more than case (i) with the same parameters. The efficiency is obtained and the cases are compared to determine the robustness of the Nested Balanced Incomplete Block Designs. It appears that Nested Balanced Incomplete Block Designs are fairly robust against the interchange of two crosses between two blocks. It may be observed that the Nested Balanced Incomplete Block Mating Design is fairly robust against the interchange of two crosses between two blocks. Efficiency of designs depends on the parameters $(v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2)$ in the case of Nested Balanced Incomplete Block Design. It has been observed that efficiency increases with the increase of number of treatment v, but not on b_1, b_2, r, k_1 and λ_1. However for the same set of parameter efficiency gets reduced for case (i) to case (ii) in all situations that as observations gets increased. But in all cases it has been found to be highly robust.

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TABLE III
EFFICIENCY TABLE WHEN INTERCHANGE OF TWO CROSSES BETWEEN TWO BLOCKS FROM A NESTED BALANCED INCOMPLETE BLOCK DESIGN

D.N	$v = p$	b_1	b_2	r	k_1	k_2	λ	Case i	Case ii
1	7	7	21	6	6	2	5	0.940192	0.968939
2	7	7	14	6	6	3	5	0.940192	0.968939
3	9	12	24	8	6	2	5	0.965814	0.982037
4	9	9	36	8	8	2	7	0.986706	0.992621
5	9	9	13	8	8	4	7	0.986706	0.992621
6	10	15	45	9	6	2	5	0.972886	0.985686
7	10	15	45	9	6	3	5	0.972886	0.985686
8	10	10	30	9	9	3	8	0.992408	0.995737
9	11	11	55	10	10	2	9	0.995329	0.997359
10	11	11	22	10	10	2	9	0.995329	0.997359
11	12	22	66	11	6	2	5	0.981774	0.990305
12	12	22	44	11	6	3	5	0.981774	0.990305
13	13	26	78	12	6	2	3	0.972839	0.985962
14	13	26	52	12	6	3	3	0.972839	0.985962
15	13	13	78	12	12	4	11	0.997935	0.998823
16	13	13	52	12	12	3	11	0.997935	0.998823
17	13	13	49	12	12	4	11	0.997935	0.998823
18	13	13	26	12	12	4	11	0.997935	0.998823
19	15	35	105	14	6	2	3	0.980175	0.989656
20	15	35	70	14	6	3	3	0.980175	0.989656
21	15	21	105	14	10	2	9	0.997564	0.99862
22	15	21	42	14	10	2	9	0.997564	0.99862
23	15	15	105	14	14	2	13	0.998946	0.999397
24	15	15	30	14	14	2	13	0.998946	0.999397
25	16	60	120	15	4	2	3	0.908184	0.967251
26	16	40	120	15	6	2	5	0.990172	0.994718
27	16	40	80	15	6	3	5	0.990172	0.994718
28	16	30	60	15	8	4	7	0.996072	0.997804
29	16	24	120	15	10	2	9	0.99787	0.998793
30	16	24	48	15	10	5	9	0.99787	0.998793
31	16	20	120	15	12	2	11	0.99866	0.999235
32	16	20	80	15	12	3	11	0.99866	0.999235
33	16	20	60	15	12	4	11	0.99866	0.999235
34	16	20	40	15	12	6	11	0.99866	0.999235
35	16	16	80	15	15	3	14	0.999217	0.999551
36	16	16	48	15	15	5	14	0.999217	0.999551

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