Another Form of Bitopological Generalized Functions

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Abstract—The paper introduces (1,2) $\tilde{g}_\alpha$-closed sets in bitopological spaces and establishes the relation between other existing generalised closed sets in bitopological spaces. It derives the basic properties of (1,2) $\tilde{g}_\alpha$-closed sets. As an application a new decomposition of coninuity is introduced.

Index Terms— (1,2) $\tilde{g}_\alpha$-closed set, (1,2) $\tilde{g}_\alpha$-open set, (1,2) $T_{\tilde{g}_\alpha}$-space and (1,2) $^{\#}T_{\tilde{g}_\alpha}$-space .

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I. Introduction

KELLY [2] has introduced the concept of bitopological spaces. Lellis Thivagar [3] has defined (1-2) α-open sets,(1,2)-semi-open sets and (1,2)-pre-open sets in bitopological spaces. Jafari et al.[1] have introduced $\tilde{g}_\alpha$-closed sets in topological spaces and proved that the class of $\tilde{g}_\alpha$-closed sets forms a topology. In this paper, we have introduced (1,2) $\tilde{g}_\alpha$-closed sets. We establihed the relation between other existing generalised closed sets in bitopological spaces. Finally we derived a new decomposition of continuity in bitopological spaces. We also defined (1,2) $T_{\tilde{g}_\alpha}$ and (1,2) $^{\#}T_{\tilde{g}_\alpha}$-spaces in bitopological spaces.

II. Preliminaries

We list some definitions in a topological space (X,τ) which are useful in the following sections. The interior and the closure of a subset A of (X,τ) are denoted by $int(A)$ and $cl(A)$, respectively.

Definition 2.1 A subset A of a topological space (X,τ) is called i) an ω-closed set [9] if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in (X,τ),

(ii) a *g-closed set [10] if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is ω-open in (X,τ) ,

(iii) a $^{\#}$g-semi-closed set(briefly $^{\#}$gs-closed)[12] if $sCl(A)\subseteq U$ whenever $A\subseteq U$ and U is *g-open in (X,τ) and

(iv) a $\tilde{g}_\alpha$-closed set[1] if $\alpha Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is $^{\#}$gs-open in (X,τ)

The complement of ω-closed(resp *g-closed, $^{\#}$gs-closed, $\tilde{g}_\alpha$-closed)set is aid to be ω-open(resp *g-open, $^{\#}$gs-open, $\tilde{g}_\alpha$-open) .

Hereafter throughout our study $(X,\tau_1,\tau_2),(Y,\sigma_1,\sigma_2)$ and $(Z,\eta_1,\eta_2))$ (or simply a space X, Y and Z) will denote the bitopological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.2 A subset A of (X,τ_1,τ_2) is called

(i) $\tau_1\tau_2$-open[3] if $A\in\tau_1\cup\tau_2$

(ii) $\tau_1\tau_2$-closed[3] if $A^c\in\tau_1\cup\tau_2$.

Definition 2.3 (3) Let A be a subset of X. Then $\tau_1\tau_2$-Cl(A) denotes the $\tau_1\tau_2$-closure of A and is defined as the intersection of all $\tau_1\tau_2$-closed sets containing A.

Definition 2.4 (3) Let A be a subset of X. Then $\tau_1\tau_2-Int(A)$ denotes the $\tau_1\tau_2$-interior of A and is defined as the union of all $\tau_1\tau_2$-open sets contained in A.

Definition 2.5 (3) A subset A of X is said to be

(i) (1,2)α-open[3] if $A\subseteq\tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(A)))$.

(ii) (1,2)semi-open[3] if $A\subseteq\tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(A))$

(iii) (1,2)pre-open[3] if $A\subseteq\tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(A))$ and

(iv)(1,2)semi-pre-open[5](briefly (1,2)sp-open) if $A\subseteq\tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(A)))$

For a bitopological space X, the complement of a (1,2)α-open (resp. (1,2)semiopen, and (1,2)pre-open and (1,2)semi-pre-open) set is called a (1,2)α-closed (resp.(1,2)semi-closed, (1,2)pre-closed and (1,2)-semi-pre-closed). The family of all (1,2)α-open (resp. (1,2)α-closed, (1,2)semiopen, (1,2)semi-closed, (1,2)pre-open (1,2)pre-closed, (1,2)-semi-pre-open and (1,2)-semi-pre-closed) sets is denoted by (1,2)$\alpha O(X)$ (resp. (1,2)$\alpha CL(X)$,(1,2)SO(X),(1,2)SCL(X),(1,2)PO(X), (1,2)PCL(X),(1,2)SPO(X) and (1,2)SPCL(X))

Definition 2.6 (3) If A is a subset of the bitopological space X, then the (1,2)semiclosure (resp. (1,2)α-closure ,(1,2)pre-

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closure and (1,2)semi-pre-closure) of A is denoted by (1,2)sCl(A) (resp. $(1,2)\alpha Cl(A)$,(1,2)pCl(A) and (1,2)spcl(A)) and is defined as the intersection of all (1,2)semi-closed sets (resp. $(1,2)\alpha$ -closed sets,(1,2)pre-closed and (1,2)semi-pre-closed sets) containing A.

Remark 2.7 (3) For any subset A of X,

(i) $\tau_1\text{-}Int(A) \subseteq \tau_1\tau_2\text{-}Int(A)$ and $\tau_2\text{-}Int(A) \subseteq \tau_1\tau_2\text{-}Int(A)$,

(ii) $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\text{-}Cl(A)$ and $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_2\text{-}Cl(A)$,

(iii) $\tau_1\tau_2\text{-}Cl(A \cap B) \subseteq \tau_1\tau_2\text{-}Cl(A) \cap \tau_1\tau_2\text{-}Cl(B)$,

(iv) $\tau_1\tau_2\text{-}Int(A) \cup \tau_1\tau_2\text{-}Int(B) \subseteq \tau_1\tau_2 - Int(A \cup B)$,

(v) $(1,2)\alpha O(X) = (1,2)SO(X) \cap (1,2)PO(X)$ and

(vi) $\tau_1\tau_2\text{-}Int(X - A) = X - \tau_1\tau_2\text{-}Cl(A)$.

Definition 2.8 (5) A subset A of X is called a

(i) $(1,2)\alpha$ -generalized closed set (briefly $(1,2)\alpha$ g-closed) if $(1,2)\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)\alpha O(X)$.

(ii) (1,2)semi-generlized-closed set (briefly (1,2)sg-closed) if $(1,2)sCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)SO(X)$.

(iii)(1,2)-generalized-semi-closed set (briefly (1,2)gs-closed) if $(1,2)sCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)\alpha O(X)$.

(iv)(1,2)generalized-semi-pre-closed (briefly (1,2)gsp-closed set) if $(1,2)spCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)\alpha O(X)$.

(v)(1,2)pre-generalized-closed (briefly (1,2)pg-closed) if $(1,2)pCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)PO(X)$.

(vi)(1,2)generalized-pre-closed (briefly (1,2)gp-closed) if $(1,2)pCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)\alpha O(X)$.

The complement of the above mentioned sets are called their respective open sets. The family of all $(1,2)\alpha$ g-closed sets(resp (1,2)sg-closed sets,(1,2)gs-closed sets,(1,2)gsp-closed sets,(1,2)pg-closed sets and (1,2)gp-closed sets) is denoted by $(1,2)\alpha GCL(X)$ (resp (1,2)SGCL(X),(1,2)GSCL(X),(1,2)GSPCL(X),(1,2)PGCL(X) and (1,2)GPCL(X)) The family of all $(1,2)\alpha$ g-open sets(resp (1,2)sg-open sets,(1,2)gs-open sets,(1,2)gsp-open sets,(1,2)pg-open sets and (1,2)gp-open sets) is denoted by $(1,2)\alpha O(X)$ (resp (1,2)SGO(X),(1,2)GSO(X),(1,2)GSPO(X),(1,2)PGO(X) and (1,2)GPO(X))

III. (1,2) $\tilde{g}_\alpha$ -CLOSED SETS

In this scetion, we have defined (1,2) $\tilde{g}_\alpha$ -closed sets using $(1,2)^{\#}gs$ -open sets and establihed the relationship between other existing sets.

Definition 3.1 A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(1,2)\omega$ -closed if $\tau_1\tau_2\text{-}Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)SO(X)$ .The complement of a $(1,2)\omega$ -closed set is called a $(1,2)\omega$ -open set.The family of all $(1,2)\omega$ -closed and the family of all $(1,2)\omega$ -open sets of X are denoted as $(1,2)\omega CL(X)$ and $(1,2)\omega O(X)$ respectively.

Definition 3.2 A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(1,2)^* g$ -closed if $\tau_1\tau_2\text{-}Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)\omega O(X)$ .The complement of a $(1,2)^* g$ -closed set is called a $(1,2)^* g$ -open set.The family of all $(1,2)^* g$ -closed and the family of all $(1,2)^* g$ -open sets of X are denoted as $(1,2)^* GCL(X)$ and $(1,2)^* GOX)$ respectively.

Definition 3.3 A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(1,2)^{\#} gs$ -closed if $(1,2)sCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)^* GO(X)$ .The complement of a $(1,2)^{\#} gs$ -closed set is called a $(1,2)^{\#} gs$ -open set.The family of all $(1,2)^{\#} gs$ -closed and the family of all $(1,2)^{\#} gs$ -open sets of X are denoted as $(1,2)^{\#} GSCL(X)$ and $(1,2)^{\#} GSO(X)$ respectively.

Definition 3.4 A subset A of a bitopological space (X, τ_1, τ_2) is said to be (1,2) $\tilde{g}_\alpha$ -closed if $(1,2)\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in (1,2)^{\#} GSO(X)$ .The complement of a (1,2) $\tilde{g}_\alpha$ -closed set is called a (1,2) $\tilde{g}_\alpha$ -open set. The family of all (1,2) $\tilde{g}_\alpha$ -closed and the family of all (1,2) $\tilde{g}_\alpha$ -open sets of X are denoted as $(1,2)\tilde{G}_\alpha CL(X)$ and $(1,2)\tilde{G}_\alpha O(X)$ respectively.

Example 3.5 Let $X = \{a,b,c\}, \tau_1 = \{\phi, X, \{a\}, \{a,b\}\}, \tau_2 = \{\phi, X, \{b,c\}\}$.

$\tau_1\tau_2$ O(X)={ ϕ , X,{a},{a,b},{b,c}},

$\tau_1\tau_2$ CL(X)={ ϕ ,X,{a},{c},{b,c}}

(1,2) ω CL(X)={ ϕ ,X,{a},{c},{a,c},{b,c}}

(1,2) ω O(X)={ ϕ ,X,{a},{b},{a,b},{b,c}}

$(1,2)^*$GCL(X)={ ϕ ,X,{a},{c},{a,c},{b,c}}

$(1,2)^*$O(X)={ ϕ ,X,{a},{b},{a,b},{b,c}}

$(1,2)^{\#}$GSCL(X)={ ϕ ,X,{c},{a,c},{b,c}}

$(1,2)^{\#}$GSO(X)={ ϕ ,X,{a},{b},{a,b}}

(1,2) $\tilde{G}_\alpha$ CL(X)={ ϕ ,X,{c},{a,c},{b,c}}

(1,2) $\tilde{G}_\alpha$ O(X)={ ϕ ,X,{a},{b},{a,b}}

Proposition 3.6 Every $(1,2)\alpha$ -closed set is (1,2) $\tilde{g}_\alpha$ -closed.

Proof. Let A be a $(1,2)\alpha$ -closed set and U be any $(1,2)^{\#} gs$ -open set containing A. Then $(1,2)\alpha Cl(A) = A \subseteq U$. Hence A is (1,2) $\tilde{g}_\alpha$ -closed.

Remark 3.7 The converse of the proposition 3.6 is not true.

Example 3.8 In example 3.5 $\{a, c\}$ is (1,2) $\tilde{g}_\alpha$ -closed but not $(1,2)\alpha$ -closed.

Proposition 3.9 Any (1,2)semi-closed set is $(1,2)^{\#}gs$-closed.

Proof. Let A be any (1,2)semi-closed set and U be any $(1,2)^{\#}gs$-open set containing A, then $(1,2)sCl(A) = A \subseteq U$. Hence A is $(1,2)^{\#}gs$-closed.

Proposition 3.10 Every $(1,2)\,\tilde{g}_{\alpha}$-closed set is $(1,2)gs$-closed.

Proof. Let A be a $(1,2)\,\tilde{g}_{\alpha}$- closed set and U be any $(1,2)\alpha$-open set containing A. Since any $(1,2)\alpha$-open set is (1,2)-semi-open and any (1,2)semi-open set is $(1,2)^{\#}gs$-open, we have $(1,2)sCl(A) \subseteq (1,2)\alpha Cl(A) \subseteq U$. Hence A is (1,2)gs-closed.

Proposition 3.11 Every $(1,2)\,\tilde{g}_{\alpha}$-closed set is $(1,2)sg$-closed.

Proof. Let A be a $(1,2)\,\tilde{g}_{\alpha}$-closed set and U be any $(1,2)semi$-open set containing A. Since any $(1,2)semi$-open set is $(1,2)^{\#}gs$-open, we have $(1,2)sCl(A) \subseteq (1,2)\alpha Cl(A) \subseteq U$. Hence A is (1,2)sg-closed.

Remark 3.12 The converse of the propositions 3.10 and 3.11 are not true.

Example 3.13 Let $X = \{a,b,c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{b\}\}$. The set $\{b\}$ is (1,2)gs-closed and (1,2)sg-closed but not $(1,2)\,\tilde{g}_{\alpha}$-closed.

Proposition 3.14 Every $(1,2)\,\tilde{g}_{\alpha}$-closed set is $(1,2)gsp$-closed.

Proof. Let A be a $(1,2)\,\tilde{g}_{\alpha}$-closed set and U be any $(1,2)\alpha$-open set containing A. Since any $(1,2)\alpha$-open set is (1,2)-semi-open and any (1,2)semi-open set is $(1,2)^{\#}gs$-open, we have $(1,2)spCl(A) \subseteq (1,2)\alpha Cl(A) \subseteq U$. Hence A is (1,2)gs-closed.

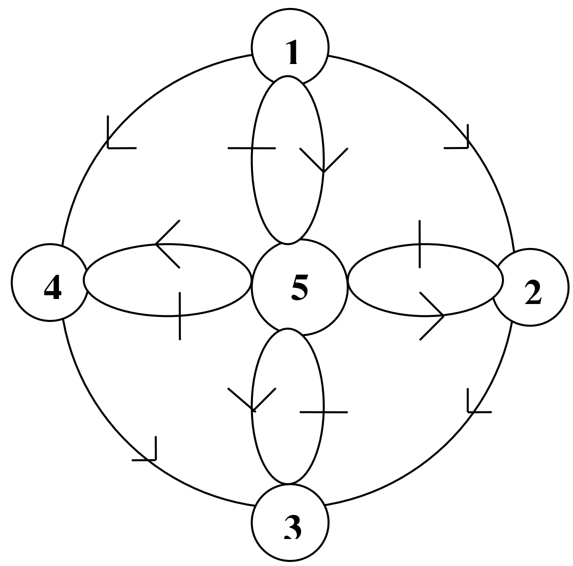
Proposition 3.15 Every $(1,2)\,\tilde{g}_{\alpha}$-closed set is $(1,2)gp$-closed.

Proof. Let A be a $(1,2)\,\tilde{g}_{\alpha}$-closed set and U be any $(1,2)\alpha$-open set containing A. Since any $(1,2)\alpha$-open set is (1,2)-semi-open and any (1,2)semi-open set is $(1,2)^{\#}gs$-open, we have $(1,2)pCl(A) \subseteq (1,2)\alpha Cl(A) \subseteq U$. Hence A is (1,2)gp-closed.

Example 3.16 In Example 3.13 the set $\{a,b\}$ is (1,2)gp-closed and (1,2)gsp-closed but not $(1,2)\,\tilde{g}_{\alpha}$-closed.

Remark 3.17 From the above dicussions we have the following figure which gives the retionship between the different generalized closed sets in bitopological spaces.

1. $(1,2)\alpha$ g-closed sets. 2. $(1,2)gs$-closed sets. 3. $(1,2)gsp$-closed sets. 4. $(1,2)gp$-closed sets. 5. $(1,2)\,\tilde{g}_{\alpha}$-closed sets.



IV. PROPERTIES OF $(1,2)\ \tilde{g}_{\alpha}$-CLOSED SETS

Theorem 4.1 If a subset A of a bitopological space X is $(1,2)\,\tilde{g}_{\alpha}$-closed then $(1,2)\alpha Cl(A) - A$ contains no nonempty $(1,2)^{\#}gs$-closed set.

Proof. Let A be a $(1,2)\,\tilde{g}_{\alpha}$-closed set and U be a $(1,2)^{\#}gs$-open set containing A.Then $(1,2)\alpha Cl(A) \subseteq U$. Let F be a nonempty $(1,2)^{\#}gs$-closed set such that $F \subseteq (1,2)\alpha Cl(A) - A$ .Then $F^c \supseteq [(1,2)\alpha Cl(A) - A]^c$. which implies that $A \subseteq F^c$. Hence $(1,2)\alpha Cl(A) \subseteq F^c$ and so $F \subseteq ((1,2)\alpha Cl(A))^c$. So $F \subseteq (1,2)\alpha Cl(A) \cap ((1,2)\alpha Cl(A))^c$. Hence $F = \phi$.

Theorem 4.2 If A is $(1,2)^{\#}$gs-open and $(1,2)\,\tilde{g}_{\alpha}$-closed subset of X then A is a $(1,2)\alpha$-closed subset of X.

Proof. Since A is $(1,2)^{\#}$gs-open and $(1,2)\,\tilde{g}_{\alpha}$-closed, $(1,2)\alpha Cl(A) \subseteq A$ .Hence A is $(1,2)\alpha$-closed.

Theorem 4.3 Let A be a $(1,2)\,\tilde{g}_{\alpha}$-closed subset of X.If $A \subseteq B \subseteq (1,2)\alpha Cl(A)$ then B is also a $(1,2)\,\tilde{g}_{\alpha}$-closed subset of X.

Proof. Let U be a $(1,2)^{\#}$gs-open set of X such that $B \subseteq U$ .Then $A \subseteq U$ .Since A is an $(1,2)\,\tilde{g}_{\alpha}$-closed set $(1,2)\alpha Cl(A) \subseteq U$ .Also

$B \subseteq (1,2)\alpha Cl(A), (1,2)\alpha Cl(B) \subseteq (1,2)\alpha Cl(A) \subseteq U$. Hence B is also a (1,2) $\tilde{g}_\alpha$ -closed subset of X.

Theorem 4.4 A (1,2) $\tilde{g}_\alpha$-closed subset of X is $(1,2)\alpha$ -closed if and only if $(1,2)\alpha Cl(A) - A$ is $(1,2)\alpha$ -closed.

Proof. Let A be $(1,2)\alpha$ -closed then $(1,2)\alpha Cl(A) - A = \phi$ which is (1,2) $\tilde{g}_\alpha$ -closed.

Conversely $(1,2)\alpha Cl(A) - A$ itself is a subset of it. By Theorem 4.1 it is equal to ϕ . Hence A is $(1,2)\alpha$ -closed.

Remarks 4.5 Union of two (1,2) $\tilde{g}_\alpha$ -closed sets need not be a (1,2) $\tilde{g}_\alpha$ -closed set.

Example 4.6 Let X={a,b,c,d}

τ_1={ ϕ,X,{a},{b},{a,b},{b,c,d}}, τ_2={ ϕ,X,{a,d},{a,c,d}}

(1,2) $\tilde{G}_\alpha$ CL(X)={ ϕ,X,{a},{c},{d},{c,d},{a,c,d},{b,c,d}}

The sets $\{a\}$ and $\{c\}$ are (1,2) $\tilde{g}_\alpha$-closed sets. But their union $\{a, c\}$ is not (1,2) $\tilde{g}_\alpha$ -closed.

Definition 4.7 The intersection of all $(1,2)^{\#} gs$ -open subsets of X containing A is called the $(1,2)^{\#} gs$ -kernel of A and is denoted by $(1,2)^{\#}$ gs-ker(A).

Lemma 4.8 A subset A of X is (1,2) $\tilde{g}_\alpha$-closed if and only if $(1,2)\alpha Cl(A) \subseteq (1,2)^{\#} gs - ker(A)$.

Proof. Suppose that A is (1,2) $\tilde{g}_\alpha$-closed in X. Then $(1,2)\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\#}$ gs-open in X. Let $x \in (1,2)\alpha Cl(A)$. If $x \notin (1,2)^{\#} gs - ker(A)$ then there is a $(1,2)^{\#}$ gs-open set U such that $x \notin U$. Since U is a $(1,2)^{\#}$ gs-open set containing A, we have $x \notin (1,2)\alpha Cl(A)$,a contradiction.

Conversely let $(1,2)\alpha Cl(A) \subseteq (1,2)^{\#} gs - ker(A)$. If U is any $(1,2)^{\#}$ gs-open set containing A, then $(1,2)\alpha Cl(A) \subseteq (1,2)^{\#} gs - ker(A) \subseteq U$. Therefore A is (1,2) $\tilde{g}_\alpha$ -closed.

Theorem 4.9 (5) For a subset A of X $(1,2)\alpha Cl(A^c) = ((1,2)\alpha Int(A))^c$

Theorem 4.10 A subset A of X is (1,2) $\tilde{g}_\alpha$ -open if and only if $F \subseteq (1,2)\alpha Int(A)$ whenever F is $(1,2)^{\#}$ gs - closed and $F \subseteq A$.

Proof. Necessity Let A be a (1,2) $\tilde{g}_\alpha$ -open set in X. Let F be a $(1,2)^{\#}$ gs-closed such that $F \subseteq A$ .Then $A^c \subseteq F^c$ where F^c is $(1,2)^{\#}$ gs-open. A^c is (1,2) $\tilde{g}_\alpha$-closed implies that $(1,2)\alpha Cl(A^c) \subseteq F^c$ i.e $((1,2)\alpha Int(A))^c \subseteq F^c$ .That is $F \subseteq (1,2)\alpha Int(A)$

Sufficiency. Suppose F is $(1,2)^{\#}$ gs-closed and $F \subseteq A$ .Also $F \subseteq (1,2)\alpha Int(A)$ .Let $A^c \subseteq U$ where U is $(1,2)^{\#}$ gs-open.Then $U^c \subseteq A$ where U^c is $(1,2)^{\#}$ gs-closed.By hypothesis $U^c \subseteq (1,2)\alpha Int(A)$ .That is $((1,2)\alpha Int(A))^c \subseteq U$ .i.e. $(1,2)\alpha Cl(A^c) \subseteq U$. This implies that A^c is (1,2) $\tilde{g}_\alpha$-closed. Hence A is (1,2) $\tilde{g}_\alpha$ -open.

Theorem 4.11 If $(1,2)\alpha Int(A) \subseteq B \subseteq A$ and A is (1,2) $\tilde{g}_\alpha$-open then B is (1,2) $\tilde{g}_\alpha$ -open.

Proof. $(1,2)\alpha Int(A) \subseteq B \subseteq A$ implies $A^c \subseteq B^c \subseteq ((1,2)\alpha int A)^c$ i.e. $A^c \subseteq B^c \subseteq ((1,2)\alpha Cl(A^c)$ and A^c is (1,2) $\tilde{g}_\alpha$ -closed. By theorem 4.3 B^c is (1,2) $\tilde{g}_\alpha$ -closed. Hence B is (1,2) $\tilde{g}_\alpha$ -open.

V. APPLICATIONS

Definition 5.1 A space (X, τ_1, τ_2) is said to be a $(1,2)T_{\tilde{g}_\alpha}$ -space if every (1,2) $\tilde{g}_\alpha$ -closed set in X is $(1,2)\alpha$ -closed.

Theorem 5.2 (5) For any subset A of X, $x \in (1,2)\alpha Cl(A)$ if and only if every $(1,2)\alpha$ -open set U containing x intersects A.

Theorem 5.3 For a space X the following conditions are equivalent.

(i) X is a $(1,2)T_{\tilde{g}_\alpha}$ -space.

(ii) Every singleton of X is either $(1,2)^{\#} g$ s-closed or $(1,2)\alpha$ -open.

Proof. $(i) \Rightarrow ii)$ Let $x \in X$ suppose that $\{x\}$ is not a $(1,2)^{\#}$ gs-closed set of X. Then $X - \{x\}$ is not a $(1,2)^{\#}$ gs-open set. So X is the only $(1,2)^{\#}$ gs-open set containing $X - \{x\}$. Then $X - \{x\}$ is a (1,2) $\tilde{g}_\alpha$ -closed set of X. Since X is a $(1,2)T_{\tilde{g}_\alpha}$ -space $X - \{x\}$ is a $(1,2)\alpha$ -closed set of X and hence $\{x\}$ is a $(1,2)\alpha$ -open set of X.

$ii) \Rightarrow i)$ Let A be a (1,2) $\tilde{g}_\alpha$-closed set of X. $A \subseteq (1,2)\alpha Cl(A)$ Let $x \in (1,2)\alpha Cl(A)$ by ii) $\{x\}$ is either $(1,2)^{\#}$ gs- closed or $(1,2)\alpha$ - open.

Case i) suppose that $\{x\}$ is $(1,2)^{\#}$ gs- closed.If $x \notin A$, $(1,2)\alpha Cl(A) - A$ contains a non empty $(1,2)^{\#}$ gs-closed set $\{x\}$ .By Theorem 4.1 we arrive at a contradiction. Thus $x \in A$.

Case ii) suppose that $\{x\}$ is $(1,2)\alpha$ -open.Since $x \in (1,2)\alpha Cl(A), \{x\} \cap A \neq \phi$ .This implies $x \in A$ .Thus in any case $x \in A$, So $(1,2)\alpha Cl(A) \subseteq A$. Therefore $(1,2)\alpha Cl(A) = A$ or equivalently A is $(1,2)\alpha$ -closed. Hence X is a $(1,2)T_{\tilde{g}_\alpha}$ -space.

Definition 5.4(6) A space (X, τ_1, τ_2) is called an Ultra-$T_{1/2}$ -space if every $(1,2)\alpha$ g-closed set in it is $(1,2)\alpha$ -closed.

Definition 5.5 (6) A space (X, τ_1, τ_2) is called an Ultra semi $T_{1/2}$-space if every $(1,2)sg$ -closed set in it is $(1,2)semi$ -closed.

Definition 5.6 A space (X,τ_1,τ_2) is called a $(1,2)^{\#}T_{\tilde{g}_\alpha}$ -space if every (1,2) $\tilde{g}_\alpha$ - set in it is $(1,2)semi$ -closed.

Proposition 5.7 Every Ultra-$T_{1/2}$-space is a (1,2) $T_{\tilde{g}_\alpha}$ -space.

Proof. Let X be an Ultra-$T_{1/2}$-space and U be a (1,2) $\tilde{g}_\alpha$ -closed set in it. Since every (1,2) $\tilde{g}_\alpha$ - closed set is $(1,2)\alpha$ g-closed, U is $(1,2)\alpha$ g-closed. Since X is an Ultra-$T_{1/2}$-space U is a $(1,2)\alpha$ -closed set. Hence X is a $(1,2)T_{\tilde{g}_\alpha}$ -space.

Propostion 5.8 Every Ultra semi $T_{1/2}$-space is a $(1,2)^{\#}T_{\tilde{g}_\alpha}$ -space.

Proof. Let X be an Ultra semi $T_{1/2}$-space and U be a (1,2) $\tilde{g}_\alpha$ - closed set in it. Since every (1,2) $\tilde{g}_\alpha$ - closed set is $(1,2)\,sg$ -closed U is $(1,2)\,sg$ -closed. Since X is an Ultra semi $T_{1/2}$-space U is a (1,2) semi-closed set. Hence X is a $(1,2)^{\#}T_{\tilde{g}_\alpha}$ -space.

Remark 5.9 The converse of the propositions 5.7 and 5.8 need not be true.

Example 5.10 Let $X=\{a,b,c\},\tau_1=\{\phi,X,\{a\}\},\tau_2=\{\phi,X,\{b\}\}$. The space X is a $(1,2)^{\#}T_{\tilde{g}_\alpha}$ -space but not an Ultra semi $T_{1/2}$-space.

Example 5.11 Let $X=\{a,b,c\},\tau_1=\{\phi,X,\{a\}\},\tau_2=\{\phi,X,\{b,c\}\}$. The space X is a (1,2) $T_{\tilde{g}_\alpha}$ -space but not an Ultra-$T_{1/2}$-space.

Definition 5.12 (7) A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be

(i) $(1,2)\alpha$ -continuous if the inverse image of every $(1,2)\alpha$ -closed set in Y is $(1,2)\alpha$ -closed in X.

(ii) $(1,2)\,sg$ -continuous if the inverse image of every $(1,2)\alpha$ -closed set in Y is $(1,2)\,sg$ -closed in X.

(iii) $(1,2)\,gsp$ -continuous if the inverse image of every $(1,2)\alpha$ -closed set in Y is $(1,2)\,gsp$ -closed in X.

(iv) $(1,2)\alpha$ g-irresolute if the inverse image of every $(1,2)\alpha$ g-closed set in Y is $(1,2)\alpha$ g-closed in X.

(v) $(1,2)\alpha$ g-continuous if the inverse image of every $(1,2)\alpha$ -closed set in Y is $(1,2)\alpha$ g-closed in X.

Definition 5.13 A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (1,2) $\tilde{g}_\alpha$ - continuous if the inverse image of every $(1,2)\alpha$ -closed set in Y is (1,2) $\tilde{g}_\alpha$ - closed in X.

Proposition 5.14 Every $(1,2)\alpha$ -continuous function is (1,2) $\tilde{g}_\alpha$ - continuous.

Proof. Let U be a $(1,2)\alpha$ -closed set in Y. Then $f^{-1}(V)$ is a $(1,2)\alpha$ -closed set in X. Since every $(1,2)\alpha$ -closed set is (1,2) $\tilde{g}_\alpha$ - closed, f is (1,2) $\tilde{g}_\alpha$ - continuous.

Rermark 5.15 The converse of the proposition 5.14 need not be true.

Example 5.16 Let $X=\{a,b,c,d\},\tau_1=\{\phi,X,\{a,b\}\},\tau_2=\{\phi,X,\{b,c\}\}$.
Y={a,b,c,d}, σ_1={ ϕ ,Y,{a},{a,b},{a,b,c},{a,b,d}}, σ_2={ ϕ ,Y,{c},{b,c},{c,d},{b,c,d},{a,c,d}}
The function f is defined as $f(a)=a,f(b)=b,f(c)=c,f(d)=d$. The function f is (1,2) $\tilde{g}_\alpha$ - -continuous but not $(1,2)\alpha$ -continuous. Since $f^{-1}(\{a\})=\{a\}$ is not $(1,2)\alpha$ -open in X.

Proposition 5.17 Every (1,2) $\tilde{g}_\alpha$ - continuous function is $(1,2)\alpha$ g-continuous(resp (1,2)sg and (1,2)gsp-continuous).

Proof. Let U be a $(1,2)\alpha$ -closed set in Y. Then $f^{-1}(U)$ is a (1,2) $\tilde{g}_\alpha$ - closed set in X. Since every (1,2) $\tilde{g}_\alpha$ - closed set is $(1,2)\alpha$ g-closed (resp (1,2)sg and (1,2)gsp-closed). Hence f is $(1,2)\alpha$ -continuous resp((1,2)sg-continuous and (1,2)gsp-continuous).

Remark 5.18 T he converse of the proposition 5.17 need not be true.

Example 5.19 Let $X=\{a,b,c\}=Y,\tau_1=\{\phi,X,\{a\}\},\tau_2=\{\phi,X,\{b\}\}$.
$\sigma_1=\{\phi,\{a\},Y\},\sigma_2=\{\phi,Y,\{b,c\}\}$.
The function f is defined as $f(a)=b,f(b)=c,f(c)=a$. The function f is $(1,2)\alpha$ -continuous (resp (1,2)sg and (1,2)gsp-continuous) but not (1,2) $\tilde{g}_\alpha$ -continuous. Since $f^{-1}(\{b,c\})=\{a,b\}$ is $(1,2)\alpha$ g-closed,(1,2)sg and (1,2)gsp-closed but not (1,2) $\tilde{g}_\alpha$ - closed in X.

Remark 5.20 The composition of two (1,2) $\tilde{g}_\alpha$ - conitnuous functions need not be a (1,2) $\tilde{g}_\alpha$ - continuous function.

Example 5.21 Let X={a,b,c,d}=Y=Z
τ_1={ ϕ ,X,{a},{b},{a,b},{b,c,d}}, τ_2={ ϕ ,X,{a,d},{a,c,d}}
$\sigma_1=\{\phi,\{a\},Y\},\sigma_2=\{\phi,Y,\{b\}\}$.
$\eta_1=\{\phi,Z,\{d\}\},\eta_2=\{\phi,Z,\{b,c\},\{b,c,d\},\{a,b,c\}\}$.
The functions $f:X\to Y,g:Y\to Z$ are identity functions. f and g are (1,2) $\tilde{g}_\alpha$ - conituous but gof is not (1,2) $\tilde{g}_\alpha$ -conituous. Since $f^{-1}(g^{-1}(\{a,b,c\})=\{a,b,c\}$ is not (1,2) $\tilde{g}_\alpha$ -closed in X.

Theorem 5.22 If $f:X\to Y$ is (1,2) $\tilde{g}_\alpha$ - continuous and $g:Y\to Z$ is $(1,2)\alpha$ -continuous then gof is (1,2) $\tilde{g}_\alpha$ -continuous.

Proof. Let U be a $(1,2)\alpha$ -closed set in Z. Then $g^{-1}(U)$ is $(1,2)\alpha$ -closed in Y. Since f is (1,2) $\tilde{g}_\alpha$ -continuous $f^{-1}(g^{-1}(U))=(gof)^{-1}(U)$ is (1,2) $\tilde{g}_\alpha$ -closed in X. Hence gof is (1,2) $\tilde{g}_\alpha$ -continuous.

Theorem 5.23 If $f:X\to Y$ and $g:Y\to Z$ are (1,2) $\tilde{g}_\alpha$ -continuous and Y is a $(1,2)T_{\tilde{g}_\alpha}$ -space then gof is (1,2) $\tilde{g}_\alpha$ -continuous.

Proof. Let U be a $(1,2)\alpha$ -closed set in Z. Then $g^{-1}(U)$ is a (1,2) $\tilde{g}_\alpha$ - closed in Y. Since Y is a $(1,2)T_{\tilde{g}_\alpha}$ -space $g^{-1}(U)$ is a $(1,2)\alpha$ -closed set in Y. Since f is (1,2) $\tilde{g}_\alpha$ - -continuous

$f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is (1,2) $\tilde{g}_\alpha$-closed in X. Hence gof is (1,2) $\tilde{g}_\alpha$-continuous. function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is (1,2) $\tilde{g}_\alpha$- continuous if and only if $f^{-1}(U)$ is (1,2) $\tilde{g}_\alpha$- -open in X for every $(1,2)\alpha$ -open set U in Y.

Proof. Let f be (1,2) $\tilde{g}_\alpha$- continuous and U be a $(1,2)\alpha$ -open set in Y then $f^{-1}(U^c)$ is (1,2) $\tilde{g}_\alpha$- closed in X.Also $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is (1,2) $\tilde{g}_\alpha$- open in X. Conversely let U be a $(1,2)\alpha$ -closed set in Y then U^c is $(1,2)\alpha$ -open in Y.By hypothesis $f^{-1}(U^c)$ is (1,2) $\tilde{g}_\alpha$- open in X. Again $f^{-1}(U^c) = (f^{-1}(U))^c$ Thus $f^{-1}(U)$ is (1,2) $\tilde{g}_\alpha$- closed in X.Therefore f is (1,2) $\tilde{g}_\alpha$- continuous. function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (1,2) $\tilde{g}_\alpha$- irresolute if the inverse image of every (1,2) $\tilde{g}_\alpha$- closed set in Y is (1,2) $\tilde{g}_\alpha$- closed in X.

Proposition 5.26 Every (1,2) $\tilde{g}_\alpha$- irresolute function is (1,2) $\tilde{g}_\alpha$- continuous.

Proof. Let U be a $(1,2)\alpha$ -closed set in Y then it is (1,2) $\tilde{g}_\alpha$- closed in Y. Then $f^{-1}(U)$ is a (1,2) $\tilde{g}_\alpha$- closed set in X. (Since every $(1,2)\alpha$ -closed set is (1,2) $\tilde{g}_\alpha$- closed). Hence f is (1,2) $\tilde{g}_\alpha$- continuous.

Remark 5.27 The converse need not be true.

Example 5.28 Let τ_1={ ϕ,X,{a},{b},{a,b},{b,c,d}}, τ_2={ ϕ ,X,{a,d},{a,c,d}}
. $\sigma_1 = \{\phi,\{a\},Y\}, \sigma_2 = \{\phi, Y, \{b\}\}$.

The function f is the identity map. The function f is (1,2) $\tilde{g}_\alpha$- continuous but not (1,2) $\tilde{g}_\alpha$- irresolute. Since $f^{-1}(\{a,c\}) = \{a,c\}$ is not (1,2) $\tilde{g}_\alpha$- closed in X.

Proposition 5.29 Composition of two(1,2) $\tilde{g}_\alpha$- irresolute functions is (1,2) $\tilde{g}_\alpha$- irresolute.

Proof. Let the functions $f: X\to Y, g: Y\to Z$ are (1,2) $\tilde{g}_\alpha$- irresolute and U be any (1,2) $\tilde{g}_\alpha$- closed set in Z. Then $g^{-1}(U)$ is (1,2) $\tilde{g}_\alpha$- closed in Y. Since f is also (1,2) $\tilde{g}_\alpha$- irresolute, $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is (1,2) $\tilde{g}_\alpha$- closed in X. Hence gof is (1,2) $\tilde{g}_\alpha$- irresolute.

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