Another Form of Bitopological Generalized Functions

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Abstract—The paper introduces (1,2) $\sim g_{\alpha}$-closed sets in bitopological spaces and establishes the relation between other existing generalised closed sets in bitopological spaces. It derives the basic properties of (1,2) $\sim g_{\alpha}$-closed sets. As an application a new decomposition of continuity is introduced.

Index Terms—(1,2) $\sim g_{\alpha}$-closed set, (1,2) $\sim g_{\alpha}$-open set, (1,2) $T_{\sim g_{\alpha}}$-space and (1,2) $\#T_{\sim g_{\alpha}}$-space.

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I. INTRODUCTION

KELLY [2] has introduced the concept of bitopological spaces. Lellis Thivagar [3] has defined (1-2) $\alpha$-open sets, (1,2)-semi-open sets and (1,2)-pre-open sets in bitopological spaces. Jafari et al.[1] have introduced $\sim g_{\alpha}$-closed sets in topological spaces and proved that the class of $\sim g_{\alpha}$-closed sets forms a topology. In this paper, we have introduced (1,2) $\sim g_{\alpha}$-closed sets. We established the relation between other existing generalised closed sets in bitopological spaces. Finally we derived a new decomposition of continuity in bitopological spaces. We also defined (1,2) $T_{\sim g_{\alpha}}$ and (1,2) $\#T_{\sim g_{\alpha}}$-spaces in bitopological spaces.

II. PRELIMINARIES

We list some definitions in a topological space $(X, \tau)$ which are useful in the following sections. The interior and the closure of a subset $A$ of $(X, \tau)$ are denoted by $\text{Int}(A)$ and $\text{cl}(A)$, respectively.

Definition 2.1 A subset $A$ of a topological space $(X, \tau)$ is called (i) an $\omega$-closed set [9] if $\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$, 
(ii) a $g$-closed set [10] if $\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\omega$-open in $(X, \tau)$, 
(iii) a $g$-semi-closed set(briefly $gs$-closed)[12] if $s\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g$-open in $(X, \tau)$ and 
(iv) a $\sim g_{\alpha}$-closed set[1] if $\alpha\text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\sim g_{\alpha}$-open in $(X, \tau)$.

The complement of $\omega$-closed(resp $g$-closed, $gs$-closed, $\sim g_{\alpha}$-closed) set is aid to be $\omega$-open(resp $g$-open, $gs$-open, $\sim g_{\alpha}$-open).

Hereafter throughout our study $(X, \tau_1, \tau_2), (Y, \sigma_1, \sigma_2)$ and $(Z, \eta_1, \eta_2)$ (or simply a space $X$, $Y$ and $Z$) will denote the bitopological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.2 A subset $A$ of $(X, \tau_1, \tau_2)$ is called 
(i) $\tau_1\tau_2$-open[3] if $A \in \tau_1 \cup \tau_2$
(ii) $\tau_1\tau_2$-closed[3] if $A^c \in \tau_1 \cup \tau_2$.

Definition 2.3 (3) Let $A$ be a subset of $X$. Then $\tau_1\tau_2$-$\text{Cl}(A)$ denotes the $\tau_1\tau_2$-closure of $A$ and is defined as the intersection of all $\tau_1\tau_2$-closed sets containing $A$.

Definition 2.4 (3) Let $A$ be a subset of $X$. Then $\tau_1\tau_2$-$\text{Int}(A)$ denotes the $\tau_1\tau_2$-interior of $A$ and is defined as the union of all $\tau_1\tau_2$-open sets contained in $A$.

Definition 2.5 (3) A subset $A$ of $X$ is said to be 
(i) $(1,2)\alpha$-open[3] if $A \subseteq \tau_1\tau_2$-$\text{Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2$-$\text{Int}(A)))$.
(ii) $(1,2)$semi-open[3] if $A \subseteq \tau_1\tau_2$-$\text{Cl}(\tau_1\tau_2$-$\text{Int}(A))$
(iii) $(1,2)$pre-open[3] if $A \subseteq \tau_1\tau_2$-$\text{Int}(\tau_1\tau_2\text{-Cl}(A))$ and 
(iv)$(1,2)$semi-pre-open[5](briefly $(1,2)$sp-open) if $A \subseteq \tau_1\tau_2$-$\text{Cl}(\tau_1\tau_2$-$\text{Int}(\tau_1\tau_2\text{-Cl}(A)))$.

For a bitopological space $X$, the complement of a $(1,2)\alpha$-open (resp. $(1,2)$semiopen, and $(1,2)$pre-open and (1,2)semi-pre-open) set is called a $(1,2)\alpha$-closed (resp.(1,2)semi-closed, (1,2)pre-closed and (1,2)semi-pre-closed). The family of all $(1,2)\alpha$-open (resp. $(1,2)\alpha$-closed, $(1,2)$semi-closed, $(1,2)$pre-closed, $(1,2)$semi-pre-closed) sets is denoted by $(1,2)\alpha\text{O}(X)$ (resp. $(1,2)\alpha\text{Cl}(X)$, $(1,2)\text{SO}(X),(1,2)\text{SCL}(X),(1,2)\text{PO}(X), (1,2)\text{PCL}(X),(1,2)\text{SPO}(X)$ and $(1,2)\text{SPCL}(X)$)

Definition 2.6 (3) If $A$ is a subset of the bitopological space $X$, then the (1,2)semiclosure (resp. (1,2)$\alpha$-closure ,$(1,2)$pre-
(1,2)α-open sets of X are denoted as (1,2)αO(X) and (1,2)α-open sets of X are denoted as (1,2)αO(X).

Definition 3.2 A subset A of a bitopological space \((X, τ_1, τ_2)\) is said to be \((1,2)^*g\) -closed if \(τ_1 τ_2 - Cl(A) ⊆ U\) whenever \(A ⊆ U\) and \(U ∈ (1,2)αO(X)\). The complement of a \((1,2)^*g\) -closed set is called a \((1,2)^*g\) -open set. The family of all \((1,2)^*g\) -closed and the family of all \((1,2)^*g\) -open sets of X are denoted as \((1,2)^*GCL(X)\) and \((1,2)^*GOSO(X)\) respectively.

Definition 3.3 A subset A of a bitopological space \((X, τ_1, τ_2)\) is said to be \((1,2)^*gs\) -closed if \((1,2)gsCl(A) ⊆ U\) whenever \(A ⊆ U\) and \(U ∈ (1,2)^*GOSO(X)\). The complement of a \((1,2)^*gs\) -closed set is called a \((1,2)^*gs\) -open set. The family of all \((1,2)^*gs\) -closed and the family of all \((1,2)^*gs\) -open sets of X are denoted as \((1,2)^*GSCL(X)\) and \((1,2)^*GSO(X)\) respectively.

Example 3.5 Let \(X = \{a, b, c\}, τ_1 = \{∅, X, \{a\}, \{a, b\}\}, τ_2 = \{∅, X, \{b, c\}\}\). \(τ_1 τ_2 O(X) = \{∅, X, \{a\}, \{a, b\}, \{b, c\}\}\), \(τ_1 τ_2 Cl(X) = \{∅, X, \{a\}, \{c\}, \{b\}\}\), \((1,2)\omega Cl(X) = \{∅, X, \{a\}, \{c\}, \{b\}\}\), \((1,2)\omega O(X) = \{∅, X, \{a\}, \{b\}, \{b, c\}\}\), \((1,2)^*GCL(X) = \{∅, X, \{a\}, \{c\}, \{b\}\}\), \((1,2)^*GOSO(X) = \{∅, X, \{a\}, \{b\}, \{b, c\}\}\), \((1,2)^*GSCL(X) = \{∅, X, \{a\}, \{c\}, \{b\}\}\), \((1,2)^*GSO(X) = \{∅, X, \{a\}, \{b\}, \{b, c\}\}\), \((1,2)^*GCL(X) = \{∅, X, \{a\}, \{c\}, \{b\}\}\), \((1,2)^*GOSO(X) = \{∅, X, \{a\}, \{b\}, \{b, c\}\}\)

Proposition 3.6 Every \((1,2)\alpha\) -closed set is \((1,2)\tilde{g}_α\) -closed.

Proof. Let A be a \((1,2)\alpha\) -closed set and U be any \((1,2)^*gs\) -open set containing A. Then \((1,2)αCl(A) = A ⊆ U\). Hence A is \((1,2)\tilde{g}_α\) -closed.

Remark 3.7 The converse of the proposition 3.6 is not true.

Example 3.8 In example 3.5 \(\{a, c\}\) is \((1,2)\tilde{g}_α\) -closed but not \((1,2)\alpha\) -closed.
**Proposition 3.9** Any (1,2)semi-closed set is (1,2)\#gs-closed.

**Proof.** Let A be any (1,2)semi-closed set and U be any (1,2)\#gs-open set containing A, then (1,2)sCl(A) = A ⊆ U. Hence A is (1,2)\#gs-closed.

**Proposition 3.10** Every (1,2)\#gs\_a-closed set is (1,2)gs\_closed.

**Proof.** Let A be a (1,2)\#gs\_a-closed set and U be any (1,2)\#gs-open set containing A. Since any (1,2)\#gs-open set is (1,2)gs-open, we have (1,2)sCl(A) ⊆ (1,2)gsCl(A) ⊆ U. Hence A is (1,2)gs\_closed.

**Proposition 3.11** Every (1,2)\#gs\_a-closed set is (1,2)gs\_closed.

**Proof.** Let A be a (1,2)\#gs\_a-closed set and U be any (1,2)\#gs-open set containing A. Since any (1,2)\#gs-open set is (1,2)gs-open, we have (1,2)sCl(A) ⊆ (1,2)gsCl(A) ⊆ U. Hence A is (1,2)gs\_closed.

**Remark 3.12** The converse of the propositions 3.10 and 3.11 are not true.

**Example 3.13** Let X = {a, b, c}, \(\tau_1 = \{\phi, X, \{a\}\}\), \(\tau_2 = \{\phi, X, \{b\}\}\). The set \{b\} is (1,2)gs\_closed and (1,2)gs\_closed but not (1,2)\#gs\_a-closed.

**Proposition 3.14** Every (1,2)\#gs\_a-closed set is (1,2)gs\_sp-closed.

**Proof.** Let A be a (1,2)\#gs\_a-closed set and U be any (1,2)\#gs-open set containing A. Since any (1,2)\#gs-open set is (1,2)gs\_open and any (1,2)\#gs-open set is (1,2)gs\_open, we have (1,2)spCl(A) ⊆ (1,2)gsCl(A) ⊆ U. Hence A is (1,2)gs\_sp-closed.

**Proposition 3.15** Every (1,2)\#gs\_a-closed set is (1,2)gs\_closed.

**Proof.** Let A be a (1,2)\#gs\_a-closed set and U be any (1,2)\#gs-open set containing A. Since any (1,2)\#gs-open set is (1,2)gs\_open and any (1,2)\#gs-open set is (1,2)gs\_open, we have (1,2)pCl(A) ⊆ (1,2)gsCl(A) ⊆ U. Hence A is (1,2)gs\_closed.

**Example 3.16** In Example 3.13 the set \{a, b\} is (1,2)gs\_closed and (1,2)gs\_closed but not (1,2)\#gs\_a-closed.

**Remark 3.17** From the above discussions we have the following figure which gives the relationship between the different generalized closed sets in bitopological spaces.

IV. **Properties of (1,2)\#gs\_a-Closed Sets**

**Theorem 4.1** If a subset A of a bitopological space X is (1,2)\#gs\_a-closed then (1,2)αCl(A) – A contains no nonempty (1,2)\#gs -closed set.

**Proof.** Let A be a (1,2)\#gs\_a-closed set and U be a (1,2)\#gs-open set containing A. Then (1,2)αCl(A) ⊆ U. Let F be a nonempty (1,2)\#gs-open set such that F ⊆ (1,2)αCl(A) – A. Then F' ⊆ [(1,2)αCl(A) – A]' . which implies that A ⊆ F'. Hence (1,2)αCl(A) ⊆ F' and so F ⊆ ((1,2)αCl(A))'. So F ⊆ (1,2)αCl(A) ∩ ((1,2)αCl(A)'). Hence F = \φ.

**Theorem 4.2** If A is (1,2)\#gs-closed and (1,2)\#gs\_a-closed subset of X then A is a (1,2)\#gs-closed subset of X.

**Proof.** Since A is (1,2)\#gs-closed and (1,2)\#gs\_a-closed, (1,2)αCl(A) ⊆ A. Hence A is (1,2)\#gs\_a-closed.

**Theorem 4.3** Let A be a (1,2)\#gs\_a-closed subset of X. If A ⊆ B ⊆ (1,2)αCl(A) then B is also a (1,2)\#gs\_a-closed subset of X.

**Proof.** Let U be a (1,2)\#gs-open set of X such that B ⊆ U. Then A ⊆ U. Since A is an (1,2)\#gs\_a-closed set (1,2)αCl(A) ⊆ U. Also
B \subseteq (1,2)\alpha Cl(A), (1,2)\alpha Cl(B) \subseteq (1,2)\alpha Cl(A) \subseteq U. Hence B is also a (1,2)\tilde{g}_\alpha -closed subset of X.

**Theorem 4.4** A (1,2)\tilde{g}_\alpha -closed subset of X is (1,2)\alpha -closed if and only if (1,2)\alpha Cl(A) – A is (1,2)\alpha -closed.

**Proof.** Let A be (1,2)\alpha -closed then (1,2)\alpha Cl(A) – A = \phi which is (1,2)\tilde{g}_\alpha -closed.

Conversely (1,2)\alpha Cl(A) – A itself is a subset of it. By Theorem 4.1 it is equal to \phi. Hence A is (1,2)\alpha -closed.

**Remarks 4.5** Union of two (1,2)\tilde{g}_\alpha -closed sets need not be a (1,2)\tilde{g}_\alpha -closed set.

**Example 4.6** Let X=\{a,b,c,d\}
\tau_1=\{ \phi, X,\{a\},\{b\},\{a,b\},\{b,c,d\}\}, \tau_2=\{ \phi, X,\{a,d\},\{a,c,d\}\}
(1,2)\tilde{g}_\alpha Cl(X)=\{ \phi, X,\{a\},\{c\},\{d\},\{c,d\},\{a,c,d\}\}
The sets \{a\} and \{c\} are (1,2)\tilde{g}_\alpha -closed sets. But their union \{a,c\} is not (1,2)\tilde{g}_\alpha -closed.

**Definition 4.7** The intersection of all (1,2)\tilde{g}_\alpha -open subsets of X containing A is called the (1,2)\tilde{g}_\alpha -kernel of A and is denoted by (1,2)\tilde{g}_\alpha ker(A).

**Lemma 4.8** A subset A of X is (1,2)\tilde{g}_\alpha -closed if and only if (1,2)\alpha Cl(A) \subseteq (1,2)\alpha gs – ker(A).

**Proof.** Suppose that A is (1,2)\tilde{g}_\alpha -closed in X. Then (1,2)\alpha Cl(A) \subseteq U whenever A \subseteq U and U is (1,2)\alpha gs-open in X. Let x \in (1,2)\alpha Cl(A). If x \not\in (1,2)\tilde{g}_\alpha ker(A) then there is a (1,2)\tilde{g}_\alpha -open set U such that x \not\in U. Since U is a (1,2)\tilde{g}_\alpha -open set containing A, we have x \not\in (1,2)\alpha Cl(A), a contradiction.

Conversely let (1,2)\alpha Cl(A) \subseteq (1,2)\tilde{g}_\alpha ker(A). If U is any (1,2)\tilde{g}_\alpha -open set containing A, then (1,2)\alpha Cl(A) \subseteq (1,2)\tilde{g}_\alpha ker(A) \subseteq U. Therefore A is (1,2)\tilde{g}_\alpha -closed.

**Theorem 4.9 (5)** For a subset A of X
(1,2)\alpha Cl(A^c) = ((1,2)\alpha dInt(A))^c

**Theorem 4.10** A subset A of X is (1,2)\tilde{g}_\alpha -open if and only if F \subseteq (1,2)\alpha dInt(A) whenever F is (1,2)\tilde{g}_\alpha -gs - closed and F \subseteq A.

**Proof.** Necessity Let A be a (1,2)\tilde{g}_\alpha -open set in X. Let F be a (1,2)\tilde{g}_\alpha -gs-closed such that F \subseteq A. Then A^c \subseteq F^c where F^c is (1,2)\tilde{g}_\alpha -gs-open. A^c is (1,2)\tilde{g}_\alpha -closed implies that (1,2)\alpha Cl(A^c) \subseteq F^c i.e.((1,2)\alpha dInt(A))^c \subseteq F^c. That is F \subseteq (1,2)\alpha dInt(A)

Sufficiency. Suppose F is (1,2)\tilde{g}_\alpha -gs-closed and F \subseteq A. Also F \subseteq (1,2)\alpha dInt(A). Let A^c \subseteq U where U is (1,2)\tilde{g}_\alpha -open.Then U^c \subseteq A where U^c is (1,2)\tilde{g}_\alpha -gs-closed. By hypothesis \forall U^c \subseteq (1,2)\alpha dInt(A) that is \forall U^c \subseteq ((1,2)\alpha dInt(A))^c \subseteq U . This implies that A^c is (1,2)\tilde{g}_\alpha -closed. Hence A is (1,2)\tilde{g}_\alpha -open.

**Theorem 4.11** If (1,2)\alpha dInt(A) \subseteq B \subseteq A and A is (1,2)\tilde{g}_\alpha -open then B is (1,2)\tilde{g}_\alpha -open.

**Proof.** (1,2)\alpha dInt(A) \subseteq B \subseteq A implies A^c \subseteq B^c \subseteq ((1,2)\alpha dInt(A))^c i.e. A^c \subseteq B^c \subseteq ((1,2)\alpha Cl(A^c)) and A^c is (1,2)\tilde{g}_\alpha -closed. By theorem 4.3 B^c is (1,2)\tilde{g}_\alpha -closed. Hence B is (1,2)\tilde{g}_\alpha -open.

V. **Applications**

**Definition 5.1** A space (X, \tau_1, \tau_2) is said to be a (1,2)T_{\tilde{g}_\alpha} -space if every (1,2)\tilde{g}_\alpha -closed set in X is (1,2)\alpha -closed.

**Theorem 5.2 (5)** For any subset A of X, x \in (1,2)\alpha Cl(A) if and only if (1,2)\alpha -open set U containing x intersects A.

**Theorem 5.3** For a space X the following conditions are equivalent.
(i) X is a (1,2)T_{\tilde{g}_\alpha} -space.
(ii) Every singleton of X is either (1,2)\tilde{g}_\alpha s-closed or (1,2)\alpha -open.

**Proof.** (i) \Rightarrow (ii) Let x \in X suppose that \{x\} is not a (1,2)\tilde{g}_\alpha gs-closed set of X. Then X – \{x\} is not a (1,2)\tilde{g}_\alpha gs-open set. So X is the only (1,2)\tilde{g}_\alpha gs-open set containing X – \{x\}. Then X – \{x\} is a (1,2)\tilde{g}_\alpha -closed set of X. Since X is a (1,2)T_{\tilde{g}_\alpha} -space X – \{x\} is a (1,2)\alpha -closed set of X and hence \{x\} is a (1,2)\alpha -open set of X.

(ii) \Rightarrow (i) Let A be a (1,2)\tilde{g}_\alpha -closed set of X. A \subseteq (1,2)\alpha Cl(A) Let x \in (1,2)\alpha Cl(A) by ii) \{x\} is either (1,2)\tilde{g}_\alpha gs-closed or (1,2)\alpha -open.

Case i) suppose that \{x\} is (1,2)\tilde{g}_\alpha gs-closed. If x \not\in A, (1,2)\alpha Cl(A) – A contains a non empty (1,2)\tilde{g}_\alpha -gs-closed set \{x\}. By Theorem 4.1 we arrive at a contradiction. Thus x \in A.

Case ii) suppose that \{x\} is (1,2)\alpha -open. Since x \in (1,2)\alpha Cl(A) \cap A \neq \phi . This implies x \in A. Thus in any case x \in A . So (1,2)\alpha Cl(A) \subseteq A. Therefore (1,2)\alpha Cl(A) = A or equivalently A is (1,2)\alpha -closed. Hence X is a (1,2)T_{\tilde{g}_\alpha} -space.

**Definition 5.4 (6)** A space (X, \tau_1, \tau_2) is called an Ultra-T_{1/2} -space if every (1,2)\alpha g-closed set in it is (1,2)\alpha -closed.

**Definition 5.5 (6)** A space (X, \tau_1, \tau_2) is called an Ultra semi T_{1/2} -space if every (1,2)\tilde{g}_\alpha g-closed set in it is (1,2)semi-closed.
Definition 5.6 A space \((X, \tau_1, \tau_2)\) is called a \((1, 2)^{\#} \widetilde{T}_{\alpha^\prime}^\alpha\)-space if every \((1, 2) \widetilde{g}_\alpha\)-set in it is \((1, 2)\)semi-closed.

Proposition 5.7 Every Ultra-\((1, 2)^{T}_{1/2}^\alpha\)-space is a \((1, 2)^{T}_{\alpha^\prime}^\alpha\)-space.

Proof. Let \(X\) be an Ultra-\((1, 2)^{T}_{1/2}^\alpha\)-space and \(U\) be a \((1, 2) \widetilde{g}_\alpha\)-closed set in it. Since every \((1, 2) \widetilde{g}_\alpha\)-closed set is \((1, 2)g\)-closed, \(U\) is \((1, 2)\alpha\)g-closed. Since \(X\) is an Ultra-\((1, 2)^{T}_{1/2}^\alpha\)-space \(U\) is a \((1, 2)\alpha\)-closed set. Hence \(X\) is a \((1, 2)^{T}_{\alpha^\prime}^\alpha\)-space.

Proposition 5.8 Every Ultra semi-\((1, 2)^{T}_{1/2}^\alpha\)-space is a \((1, 2)^{T}_{\alpha^\prime}^\alpha\)-space.

Proof. Let \(X\) be an Ultra semi-\((1, 2)^{T}_{1/2}^\alpha\)-space and \(U\) be a \((1, 2) \widetilde{g}_\alpha\)-closed set in it. Since every \((1, 2) \widetilde{g}_\alpha\)-closed set is \((1, 2)sg\)-closed \(U\) is \((1, 2)\alpha\)-closed. Since \(X\) is an Ultra semi-\((1, 2)^{T}_{1/2}^\alpha\)-space \(U\) is a \((1, 2)\alpha\)semi-closed set. Hence \(X\) is a \((1, 2)^{T}_{\alpha^\prime}^\alpha\)-space.

Remark 5.3 The converse of the propositions 5.7 and 5.8 need not be true.

Example 5.10 Let \(X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{b\}\}\). The space \(X\) is a \((1, 2)^{T}_{\alpha^\prime}^\alpha\)-space but not an Ultra semi-\((1, 2)^{T}_{1/2}^\alpha\)-space.

Example 5.11 Let \(X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{b, c\}\}\). The space \(X\) is a \((1, 2)^{T}_{\alpha^\prime}^\alpha\)-space but not an Ultra-\((1, 2)^{T}_{1/2}^\alpha\)-space.

Definition 5.12 (7) A function \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is said to be
(i) \((1, 2)\alpha\)continuous if the inverse image of every \((1, 2)\alpha\)-closed set in \(Y\) is \((1, 2)\alpha\)-closed in \(X\).
(ii) \((1, 2)sg\)-continuous if the inverse image of every \((1, 2)\alpha\)-closed set in \(Y\) is \((1, 2)sg\)-closed in \(X\).
(iii) \((1, 2)gsp\)-continuous if the inverse image of every \((1, 2)\alpha\)-closed set in \(Y\) is \((1, 2)gsp\)-closed in \(X\).
(iv) \((1, 2)\alpha\)-irresolute if the inverse image of every \((1, 2)\alpha\)-closed set in \(Y\) is \((1, 2)\alpha\)-closed in \(X\).
(v) \((1, 2)\alpha\)g-continuous if the inverse image of every \((1, 2)\alpha\)-closed set in \(Y\) is \((1, 2)\alpha\)-g-closed in \(X\).

Definition 5.13 A function \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is said to be \((1, 2) \widetilde{g}_\alpha\)-continuous if the inverse image of every \((1, 2)\alpha\)-closed set in \(Y\) is \((1, 2) \widetilde{g}_\alpha\)-closed in \(X\).

Proposition 5.14 Every \((1, 2)\alpha\)-continuous function is \((1, 2) \widetilde{g}_\alpha\)-continuous.

Proof. Let \(U\) be a \((1, 2)\alpha\)-closed set in \(Y\). Then \(f^{-1}(U)\) is a \((1, 2)g\)-closed set in \(X\). Since every \((1, 2)\alpha\)-closed set is \((1, 2)g\)-closed, \(f(U)\) is \((1, 2) \widetilde{g}_\alpha\)-continuous.

Remark 5.15 The converse of the proposition 5.14 need not be true.

Example 5.16 Let \(X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a, b\}\}, \tau_2 = \{\phi, X, \{b, c\}\}\).
f^{-1}(g^{-1}(U)) = (gof)^{-1}(U) \text{ is } (1,2) \g_a - closed in X. Hence gof \text{ is } (1,2) \g_a - continuous.

Proposition 5.26 For every (1,2) \g_a - irresolute function f \text{ is } (1,2) \g_a - continuous.

Proposition 5.29 Composition of two (1,2) \g_a - irresolute functions is (1,2) \g_a - irresolute.

REFERENCES