Robustness of PDC Plan using BIB Mating Designs against Unavailability of One or More Observations

R. Shunmugathai and M.R. Srinivasan

***Abstract*—Mating Designs are the study of progenies developed through various methods like Diallel Cross plans which are subjected to Incomplete Block Designs. The concept of robustness in designs has been studied and available in the literature. The paper dealt with a class of Balanced Incomplete Block Designs for varying parametric values with unavailability of one or more observations and efficiencies are calculated. The effects of missing blocks on Partial Diallel Cross designs are examined in this study. A-efficiencies based on non -zero eigenvalues suggest that these designs are fairly robust. The investigation shows that Balanced Incomplete Block Designs are fairly robust in terms of efficiency. In this paper, the robustness of Balanced Incomplete Block Design when one or more observations are lost has been discussed.**

***Index Terms*—Balanced Incomplete Block Design; Efficiency of residual design; Mating Design; Youden Square Design; Latin Square Design**

***MSC 2010 Codes* – 45B05, 51E05.**

I. Introduction

DIALLEL crossing is a useful method for conducting plant and animal breeding experiments, especially for estimating combining ability effects of lines. Diallel crosses, in which all possible distinct crosses are in pairs among the available lines are called Complete Diallel Crosses (CDC). Diallel crosses in which only a fraction of all possible crosses among the available lines are taken are called Partial Diallel Crosses. When the number of lines is more, it is desirable to adopt Partial Diallel Crosses (PDC) according to Ghosh and Divecha [17]. Partial Diallel Cross designs have been constructed by several authors, Das and Sivaram [11], Arya [2], and Ghosh and Desai [15]. Most of these plans are laid out as Incomplete Block Designs and these Incomplete Block Designs require special analysis.

Partial Diallel Crossing is discussed in detail by Kempthorne and Curnow [26]. These PDC plans based on either circulant designs or PBIB designs according to Fyfe and Gilbert [13], Curnow [7], Hinkelmann and Kempthorne [21] and Hinkelmann [23] are usually carried out by accommodating all the crosses in each block of Randomised Block Designs (RBD). In such experiments, the blocks may be large enough leading to consider able decrease in the efficiency of the design due to larger intrablock variance. In such situations, the device of blocking, if properly employed, is certainly useful to increase the accuracy of combining ability estimates. With this view in end, Incomplete Block Designs (IBD) for PDC are introduced in this chapter, construction of which is proposed through PBIB and BIB designs, and their analyses using the property of partially balanced n-ary designs for evaluating General Combining Ability (GCA) effects of lines have also been indicated. An estimator of one missing observation in these designs is also provided. Moreover, the efficiency of the IBD for PDC is evaluated in comparison to the PDC through Randomised Block Design.

Through PDC, a plant breeder is not only able to estimate the GCA of a large number of parental lines but can also make selection among crosses from a wide range of parents. GCA of each line may be estimated with relatively low precision but larger genetic gains may result from the more intense selection that can be applied to them. For enabling the plant breeders to make use of PDC in their experimentation, it is necessary to develop methods of their construction and analysis. It is equally important to indicate which of the several possible designs for PDC, is the most efficient in the sense that it gives the least average variance of the difference between GCA effects of a pair of lines.

Design of experiments for diallel crosses has received considerable attention in the literature: Curnow [7], Hinkelmann [23] and Gupta, Das and Kageyama [9], for references. Suppose there are v parental lines and let a typical cross be denoted by ($1 \leq i \leq j \leq v$). A diallel cross experiment is complete if it includes all the crosses; otherwise, it is partial. Gupta and Kageyama [18] and Dey and Midha [10] investigated the issue of optimality of Complete Diallel Crosses in Incomplete Blocks when interest lies in comparing the parents with respect to their General Combining Abilities. For large v, a Complete Diallel Cross may, however, become prohibitively large and the use of a Partial Diallel Cross is necessitated. Although efficient designing of Partial Diallel Crosses has been studied by several authors Hinkelmann and Kempthorne [22], Arya [2], no formal optimality result within adequately general classes has yet been reported. The literature on block designs for Partial Diallel Crosses is even scantier and only the recent work of Gupta and Kageyama [18]addresses the issue of orthogonality in this context.

Kempthorne and Curnow [26], Arya and Narain [4], Narain and Arya [29], Agrawal [3], Narain [28] and Kaushik [25] have discussed the advantages of performing only a sample of all possible crosses among a large number of parents rather than making all possible crosses among a smaller number of parents. Prescott and Mansson [33] investigated the effect of missing observations on Complete Diallel Cross designs. They

R. Shunmugathai is a Research Scholar in the Department of Statistics, University of Madras, Chennai-600 005, India (e-mail: shunmuga_77@yahoo.co.in)

Dr. M.R. Srinivasan is a Professor in the Department of Statistics, University of Madras, Chennai-600 005, India (e-mail: mrsvasan8@hotmail.com)

examined the robustness of CDC using BIBD and PBIBD, A-efficiencies, based on average variances of the elementary contrasts of the line effects, and suggest that Complete Diallel Cross Design is fairly robust against the unavailability of observations. Further the robustness of CDC against the loss of a block of observations using BIBD and PBIBD was examined by Mansson and Prescott [32].

Das and Kageyama [7] showed that Balanced Incomplete Block Designs (BIBD) and extended Balanced Incomplete Block Designs are fairly robust against the unavailability of $s(s \leq k)$ observations in any block, while any Youden Design and Latin Square Designs are found to be fairly robust against the loss of any one column. It may be noted that in the construction of Balanced Incomplete Block Design, restriction is imposed in respect of occurrence of pairs of treatments, and on the individual treatment, i.e., each pair of treatments should occur r times. Shunmugathai and Srinivasan [35] obtained the robustness of Partial Diallel Crosses Plan using Doubly Balanced Incomplete Block Mating Design against the unavailability of two blocks.

This paper looks into the robustness of PDC plan when one or more crosses are lost from a block using Balanced Incomplete Block Design. C^*matrix and its non-zero eigenvalues are computed with its corresponding multiplicity and its efficiency. It shows that PDC plans are fairly robust against the loss of one or more crosses from a block. Corresponding C^* matrices and their non-zero eigenvalues with multiplicities are computed for each set of parameters and robustness of PDC designs are examined against the unavailability of one or more crosses from a block.

The robustness criteria against the unavailability of data are:

1) to get the connectedness of the residual design;
2) to have the variance balance of the residual design;
3) to consider the A-efficiency of residual design for the robustness study.

So far, robustness of Incomplete Block Designs and Complete Block Designs are carried out against loss of $s(s \leq k_1)$ observations in one block.

In this investigation, consider a connected PDC plan D. Let D^* be the residual design obtained when one or more observations are lost from a design. Assume D^* to be connected. In this case, the criterion of robustness against the unavailability of one or more observations is the overall A-efficiency, of the residual design D^*, given by

$$e(s) = \frac{\text{Sum of reciprocals of non-zero eigenvalues of } C}{\text{Sum of reciprocals of non-zero eigenvalues of } C^*}$$

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \quad (1)$$

Diallel cross experiment involves p lines and $p(p-1)/2$ crosses of these lines in pairs. Several different models have been proposed to describe the effect of these diallel crosses. The most general model includes a contribution from each of the individuals in the pairing, known as the General Combining Ability (GCA) effects of the cross and an interaction term, known as the Specific Combining Ability effect. Hinkelmann [23] discusses the background to these models and considers appropriate methods of analysis under various assumptions Ghosh and Desai [15].

The general form of the underlying model for these Mating Designs, for the cross $i \times j$, contains the General Combining Ability terms, g_i and g_j, and the Specific Combining Ability term s_{ij} however, in this investigation into the robustness of estimating and comparing the General Combining Ability effects, g_i, assume the simple additive model of the form

$$y_{ijl} = \mu + g_i + g_j + \beta_l + \varepsilon_{ijl}, i = 1, \ldots, p; j, l = 1, \ldots, b \quad (1.2)$$

where μ is the overall mean, g_i and g_j are the effects of the i^{th} and j^{th} lines, respectively, β_l is the effect of the l^{th} block, and ε_{ijl} is experimental error assumed to have zero mean and constant variance σ^2. In this model the Specific Combining Ability effect, s_{ij}, has been absorbed into the error term. A least squares analysis used to estimate the parameters in this model, recovering only the intra-block information on the line-effects, produces the normal equations, which, after removal of the block parameters, give the $p \times p$ information matrix, C, for line effects $g_i, i = 1, \ldots, p$, as

$$C = R - NK^{\delta}N',$$

where the diagonal element, r_{ii}, of R is the number of replicates of the i^{th} line, and the off-diagonal element, r_{ij}, is the number of replicates of the cross between the i^{th} and j^{th} lines. The matrix K^{δ} is diagonal and the elements of this matrix are the number of crosses in each block. The $(i,j)^{th}$ element, n_{ij}, of the incidence matrix, N, is the number of times the i^{th} line appears in the j^{th} block of the design.

Efficiencies of 106 Balanced Incomplete Block Designs are computed. Thus, it shows that design is fairly robust against loss of one or more observations. C^* matrix and its non-zero eigenvalues are also computed with corresponding multiplicity.

II. C - Matrix of Partial Diallel Crosses Plan

We know that for any block design C matrix can be defined as,

$$C = r(I_v - \frac{NN'}{k_1})$$

For Partial Diallel Cross plan, C matrix can be given by,

$$C_t = G_{ti} - \frac{NN'}{k_1}$$

where

$$G_{ti} = \begin{pmatrix} w_{ti} & g'_{ii} \\ - & w_{t'i} \end{pmatrix}$$

$$w_{ti} = w_{t'i} = r(k-1) \text{ and } g_{ii} = 1$$

The NN' matrix of the Partial Diallel Cross Plan can be defined as

$$NN' = \begin{bmatrix} \sum n_{1j}^2 & \sum n_{1j}n_{2j} & \sum n_{1j}n_{vj} \\ \sum n_{2j}n_{1j} & \sum n_{2j}^2 & \sum n_{2j}n_{vj} \\ \sum n_{vj}n_{1j} & \sum n_{vj}n_{2j} & \sum n_{vj}^2 \end{bmatrix}$$

Thus it is obvious that for this Partial Diallel Cross Plan,

$$\sum n_{vj}^2 = r(k-1)^2, \sum n_{ij}n_{mj} = \lambda(k-1)^2, k_1 = \frac{k(k-1)}{2}$$

Now C matrix is given as

$$C = \theta(I_v - \frac{E_{vv}}{v})$$

The non-zero eigenvalues of C matrix and its corresponding multiplicity of Partial Diallel Cross Plan can be given by,

$$\theta = \lambda\nu(k-2)/k$$

with multiplicity $(v-1)$.

III. Robustness of PDC Plan using BIB Design against the unavailability of one or more observations

Partial Diallel Crosses Plan was considered Balanced Incomplete Block Design D having parameters

$$v = p,\ b,\ r,\ k,\ \lambda.$$

Now, we consider treatment of the Balanced Incomplete Block Designs as lines and cross them between the lines in each block. This result in Partial Diallel Crosses Plan that involves p line with $k(k-1)/2$ crosses, each cross repeated λ times. Without loss of generality one or more cross is lost from Partial Diallel Crosses Plan, call this design as a residual design and assume that the residual design D^* is connected. Situation can be treated by separating into three cases:

Case i: $s = 1$
Case ii: $s = 2$
Case iii: $s = k$

For all the three cases, the lines between one or more observations are lost in a Balanced Incomplete Block Design. The efficiency factor depends upon the lines between one or more observations. To find the efficiency for all the three cases, the lines between one or more observations are $0, 1, 2, 3, \ldots, (k-1), k$ respectively are studied. Here, the robustness criterion of Balanced Incomplete Block Design is further discussed for the different value of one or more observations.

Case (i): $s = 1$
Consider a Balanced Incomplete Block Design D having parameters v, b, r, k, λ. The Cmatrix of the design is given by,

$$C = \theta(I_v - \frac{E_{vv}}{v})$$

where

$$\theta = \lambda\nu(k-2)/k$$

is the eigenvalues of C matrix of design D with multiplicity $(v-1)$.

Let from this design $s = 1$ line be lost. Call this design as a residual design and assume that the residual design D^* is connected. Let C^* be the information matrix of the residual design be C^* given as,

$$C^* = R^* - N^* K^{-1} N'^*$$

The C^* matrix of the design D can be written as,

$$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where
$\varepsilon_{11} = (k-2)(\lambda\nu - k)I_1 - (k-2)(\lambda-1)J_{11}$
$\varepsilon_{12} = -(k-2)\lambda J_{1(\nu-k)}$
$\varepsilon_{13} = -(k-2)(\lambda-1)J_{1(k-2)}$
$\varepsilon_{21} = -(k-2)(\lambda)J_(\nu-k)(1)$
$\varepsilon_{22} = (k-2)(\lambda\nu)I_(\nu-k) - (k-2)(\lambda)J_{(\nu-k)(\nu-k)}$
$\varepsilon_{23} = (k-2) - \lambda J_{(\nu-k)(k-2)}$
$\varepsilon_{31} = -(k-2)(\lambda-1)J_{(k-2)(1)}$
$\varepsilon_{32} = -(k-2)\lambda J_{(k-2)(\nu-k)}$
$\varepsilon_{33} = (k-2)\lambda\nu(I_{(k-2)} - (k-2)\lambda + sJ_{(k-2)(k-2)})$
The non zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k-2)(\lambda\nu-k)}{k}$, with multiplicity 1.
2) $\frac{(k-2)\lambda\nu}{k}$, with multiplicity $(v-2)$.

Theorem 1: Balanced Incomplete Block Designs with parameters $v, b,$
$r, k, \lambda,$ are fairly robust against the unavailability of $s = 1$ line, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda\nu - k)(\nu - 1)}{(\lambda\nu) + (\nu - 2) + (\lambda\nu - k)} \quad (2)$$

Proof : Without loss of generality, let $s = 1$ line be lost from the Balanced Incomplete Block Design, C^* matrix of the residual design is given by,

$$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where

$\varepsilon_{11} = (k-2)(\lambda\nu - k)I_1 - (k-2)(\lambda-1)J_{11}$
$\varepsilon_{12} = -(k-2)\lambda J_{1(\nu-k)}$
$\varepsilon_{13} = -(k-2)(\lambda-1)J_{1(k-2)}$
$\varepsilon_{21} = -(k-2)(\lambda)J_(\nu-k)(1)$
$\varepsilon_{22} = (k-2)(\lambda\nu)I_(\nu-k) - (k-2)(\lambda)J_{(\nu-k)(\nu-k)}$
$\varepsilon_{23} = (k-2) - \lambda J_{(\nu-k)(k-2)}$
$\varepsilon_{31} = -(k-2)(\lambda-1)J_{(k-2)(1)}$
$\varepsilon_{32} = -(k-2)\lambda J_{(k-2)(\nu-k)}$
$\varepsilon_{33} = (k-2)\lambda\nu(I_{(k-2)} - (k-2)\lambda + sJ_{(k-2)(k-2)})$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k-2)(\lambda\nu-k)}{k}$, with multiplicity 1.
2) $\frac{(k-2)\lambda\nu}{k}$, with multiplicity $(v-2)$.

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \quad (3)$$

Where, $\phi_2(s)$ = sum of reciprocals of non-zero eigenvalues of C matrix of design D and $\phi_1(s)$ = sum of reciprocals of non-zero eigenvalues of C^* matrix of design D*.

That is,

$$\phi_2(s) = \frac{k(\nu-1)}{\lambda\nu(k-2)} \quad (4)$$

$$\phi_1(s) = \frac{k}{(k-2)(\lambda\nu - k)} + \frac{(\nu-2)(k)}{(k-2)(\lambda\nu)}. \quad (5)$$

Finally, A- efficiency is given by

$$e(s) = \frac{(\lambda\nu - k)(\nu - 1)}{(\lambda\nu) + (\nu - 2) + (\lambda\nu - k)} \quad (6)$$

Example 1: Let D represent the Balanced Incomplete Block Design with parameters

$$v = p = 13,\ b = 13,\ r = 4,\ k = 4,\ \lambda = 1,$$

given by Ghosh and Desai[15] is as in Table 1.

Here, when 2 x 5 is lost from second block, C^* matrix of residual design is given below Table 1.

The non-zero eigenvalues with their corresponding multiplicities are,

1) $27/4$, with multiplicities 1.
2) $39/4$, with multiplicities 11.

The overall A- efficiency of the design is, $e(s) = 0.964286$.

Case (ii): $s = 2$

Consider a Balanced Incomplete Block Design D having parameters v, b, r, k, λ. Let from this design $s = 2$ line be lost. Call this design as a residual design and assume that the residual design D^* is connected. Let C^* be the information matrix of the residual design be C^* given as,

$$C^* = R^* - N^* K^{-1} N'^*$$

The C^* matrix of the design D can be written as,

$$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where

$\varepsilon_{11} = (k-2)(\lambda\nu - k)I_2 - (k-2)(\lambda - 1)J_{22}$
$\varepsilon_{12} = -(k-2)\lambda J_{(2)(\nu-k)}$
$\varepsilon_{13} = -(k-2)(\lambda - 1)J_{2(k-2)}$
$\varepsilon_{21} = -(k-2)(\lambda)J_(\nu - k)(2)$
$\varepsilon_{22} = (k-2)(\lambda\nu)I_(\nu - k) - (k-2)(\lambda)J_{(\nu-k)(\nu-k)}$
$\varepsilon_{23} = (k-2) - \lambda J_{(\nu-k)(k-2)}$
$\varepsilon_{31} = -(k-2)(\lambda - 1)J_{(k-2)(2)}$
$\varepsilon_{32} = -(k-2)\lambda J_{(k-2)(\nu-k)}$
$\varepsilon_{33} = (k-2)\lambda\nu(I_{(k-2)} - (k-2)\lambda + sJ_{(k-2)(k-2)})$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k-2)(\lambda\nu-k)}{k}$, with multiplicity 2.
2) $\frac{(k-2)\lambda\nu}{k}$, with multiplicity $(v-3)$.

Theorem 2: Balanced Incomplete Block Designs with parameters v, b, r, k, λ are fairly robust against the unavailability of $s = 2$ line, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda\nu - k)(\nu - 1)}{(2\lambda\nu) + (\nu - 3) + (\lambda\nu - k)} \quad (7)$$

Proof : Without loss of generality, let $s = 2$ line be lost from the Balanced Incomplete Block Design, C^* matrix of the residual design is given by,

$$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where

$\varepsilon_{11} = (k-2)(\lambda\nu - k)I_2 - (k-2)(\lambda - 1)J_{22}$
$\varepsilon_{12} = -(k-2)\lambda J_{(2)(\nu-k)}$
$\varepsilon_{13} = -(k-2)(\lambda - 1)J_{2(k-2)}$
$\varepsilon_{21} = -(k-2)(\lambda)J_(\nu - k)(2)$
$\varepsilon_{22} = (k-2)(\lambda\nu)I_(\nu - k) - (k-2)(\lambda)J_{(\nu-k)(\nu-k)}$
$\varepsilon_{23} = (k-2) - \lambda J_{(\nu-k)(k-2)}$
$\varepsilon_{31} = -(k-2)(\lambda - 1)J_{(k-2)(2)}$
$\varepsilon_{32} = -(k-2)\lambda J_{(k-2)(\nu-k)}$
$\varepsilon_{33} = (k-2)\lambda\nu(I_{(k-2)} - (k-2)\lambda + sJ_{(k-2)(k-2)})$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k-2)(\lambda\nu-k)}{k}$, with multiplicity 2.
2) $\frac{(k-2)\lambda\nu}{k}$, with multiplicity $(v-3)$.

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \quad (8)$$

That is,

$$\phi_2(s) = \frac{k(\nu - 1)}{\lambda\nu(k-2)} \quad (9)$$

and

$$\phi_1(s) = \frac{2k}{(k-2)(\lambda\nu - k)} + \frac{(\nu - 3)(k)}{(k-2)(\lambda\nu)}. \quad (10)$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(\lambda\nu - k)(\nu - 1)}{(2\lambda\nu) + (\nu - 3) + (\lambda\nu - k)} \quad (11)$$

Example 2: Let D represent the Balanced Incomplete Block Design with parameters $v = p = 13,\ b = 13,\ r = 4,\ k = 4,\ \lambda = 1$, given by Ghosh and Desai[15] is as in Table 2.

Here, when $s = 2$ lines be lost from a block 3 and block 5, C^* matrix of residual design is given below Table 2.

The non-zero eigenvalues with their corresponding multiplicities are,

1) $18/4$, with multiplicities 2.
2) $26/4$, with multiplicities 10.

The overall A- efficiency of the design is, $e(s) = 0.931034$.

Case (iii): $s = k$

Consider a Balanced Incomplete Block Design D having parameters v, b, r, k, λ. Let from this design $s = k$ line be lost. Call this design as a residual design and assume that the residual design D^* is connected. Let C^* be the information matrix of the residual design be C^* given as

$$C^* = R^* - N^* K^{-1} N'^*$$

The C^* matrix of the design D can be written as

$$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix}$$

where

$\varepsilon_{11} = (k-2)(\lambda_1\nu - k)I_k - (k-2)(\lambda - 1)J_{kk}$
$\varepsilon_{12} = -(k-2)\lambda J_{(k)(\nu-k)}$

TABLE I
BALANCED INCOMPLETE BLOCK DESIGN

Block	BIB Design				Crosses the PDC Design					
1	1	2	4	10	1 × 2	1 × 4	1 × 10	2 × 4	2 × 10	4 × 10
2	2	3	5	11	2 × 3	**2 × 5**	2 × 11	3 × 5	3 × 11	5 × 11
3	3	4	6	12	3 × 4	3 × 6	3 × 12	4 × 6	4 × 12	6 × 12
4	4	5	7	13	4 × 5	4 × 7	4 × 13	5 × 7	5 × 13	7 × 13
5	5	6	8	1	5 × 6	5 × 8	1 × 5	6 × 8	1 × 6	1 × 8
6	6	7	9	2	6 × 7	6 × 9	2 × 6	7 × 9	2 × 7	2 × 9
7	7	8	10	3	7 × 8	7 × 10	3 × 7	8 × 10	3 × 8	3 × 10
8	8	9	11	4	8 × 9	8 × 11	4 × 8	9 × 11	4 × 9	4 × 11
9	9	10	12	5	9 × 10	9 × 12	5 × 9	10 × 12	5 × 10	5 × 12
10	10	11	13	6	10 × 11	10 × 13	6 × 10	11 × 13	6 × 11	6 × 13
11	11	12	1	7	11 × 12	1 × 11	7 × 11	1 × 12	7 × 12	1 × 7
12	12	13	2	8	12 × 13	2 × 12	8 × 12	2 × 13	8 × 13	2 × 8
13	13	1	3	9	1 × 13	3 × 13	9 × 13	1 × 3	1 × 9	3 × 9

$$8C^* = \begin{bmatrix} 27 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & 0 & 0 & 0 \\ -3 & 36 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & 36 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & 36 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & 36 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & -3 & 36 & -3 & -3 & -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & -3 & -3 & 36 & -3 & -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & -3 & -3 & -3 & 36 & -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & 36 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & 36 & -3 & -3 & -3 \\ 0 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & 35 & -4 & -4 \\ 0 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -4 & 35 & -4 \\ 0 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -4 & -4 & 35 \end{bmatrix}$$

TABLE II
BALANCED INCOMPLETE BLOCK DESIGN

Block	BIB Design				Crosses the PDC Design					
1	1	2	4	10	1 × 2	1 × 4	1 × 10	2 × 4	2 × 10	4 × 10
2	2	3	5	11	2 × 3	2 × 5	2 × 11	3 × 5	3 × 11	5 × 11
3	3	4	6	12	**3 × 4**	3 × 6	3 × 12	4 × 6	4 × 12	6 × 12
4	4	5	7	13	4 × 5	4 × 7	4 × 13	5 × 7	5 × 13	7 × 13
5	5	6	8	1	**5 × 6**	5 × 8	1 × 5	6 × 8	1 × 6	1 × 8
6	6	7	9	2	6 × 7	6 × 9	2 × 6	7 × 9	2 × 7	2 × 9
7	7	8	10	3	7 × 8	7 × 10	3 × 7	8 × 10	3 × 8	3 × 10
8	8	9	11	4	8 × 9	8 × 11	4 × 8	9 × 11	4 × 9	4 × 11
9	9	10	12	5	9 × 10	9 × 12	5 × 9	10 × 12	5 × 10	5 × 12
10	10	11	13	6	10 × 11	10 × 13	6 × 10	11 × 13	6 × 11	6 × 13
11	11	12	1	7	11 × 12	1 × 11	7 × 11	1 × 12	7 × 12	1 × 7
12	12	13	2	8	12 × 13	2 × 12	8 × 12	2 × 13	8 × 13	2 × 8
13	13	1	3	9	1 × 13	3 × 13	9 × 13	1 × 3	1 × 9	3 × 9

$\varepsilon_{21} = -(k-2)(\lambda)J_(\nu-k)(k)$
$\varepsilon_{22} = (k-2)(\lambda\nu)I_(\nu-k) - (\lambda)\nu^{-1}(k-2)J_{(\nu-k)(\nu-k)}$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k-2)(\lambda\nu-k)}{k}$, with multiplicity $k-1$.
2) $\frac{(k-2)\lambda\nu}{k}$, with multiplicity $(v-k)$.

Theorem 3: Balanced Incomplete Block Designs with parameters v, b, r, k, λ are fairly robust against the unavailability of $s = k$ line, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda\nu - k)(\nu-1)}{(\lambda\nu)(k-1) + (\nu-k)(\lambda\nu-k)} \qquad (12)$$

Proof : Without loss of generality, when $s = k$ lines is lost from the Balanced Incomplete Block Design, C^* matrix of the residual design is given by,

$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix}$ where

$\varepsilon_{11} = (k-2)(\lambda_1\nu - k)I_k - (k-2)(\lambda-1)J_{kk}$
$\varepsilon_{12} = -(k-2)\lambda J_{(k)(\nu-k)}$
$\varepsilon_{21} = -(k-2)(\lambda)J_(\nu-k)(k)$
$\varepsilon_{22} = (k-2)(\lambda\nu)I_(\nu-k) - (\lambda)\nu^{-1}(k-2)J_{(\nu-k)(\nu-k)}$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k-2)(\lambda\nu-k)}{k}$, with multiplicity $k-1$.
2) $\frac{(k-2)\lambda\nu}{k}$, with multiplicity $(v-k)$.

$$8C^* = \begin{bmatrix} 18 & 0 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 0 & 0 \\ 0 & 18 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 0 & 0 \\ -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 \\ 0 & 0 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 22 & -4 \\ 0 & 0 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -4 & 22 \end{bmatrix}$$

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{13}$$

That is,

$$\phi_2(s) = \frac{k(\nu-1)}{\lambda\nu(k-2)} \tag{14}$$

$$\phi_1(s) = \frac{k}{(k-2)(k-1)(\lambda\nu-k)} + \frac{(\nu-1)(k)}{(k-2)(\lambda\nu)}. \tag{15}$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(\lambda\nu-k)(\nu-1)}{(\lambda\nu)(k-1)+(\nu-k)(\lambda\nu-k)} \tag{16}$$

Example 3: Let D represent the Balanced Incomplete Block Design with parameters

$$v = p = 13,\ b = 13,\ r = 4,\ k = 4,\ \lambda = 1,$$

given by Ghosh and Desai[15] is as in Table 3.

Here, when block 12 is lost, C^* matrix of residual design is given below Table 3.

The non-zero eigenvalues with their corresponding multiplicities are,

1) $40/4$, with multiplicities 3.
2) $48/4$, with multiplicities 4.

The overall A- efficiency of the design is, $e(s) = 0.921053$.

IV. Conclusion

Mating Designs are the study of progenies developed through various methods like Diallel Cross Plans which are subjected to Incomplete Block Designs. The analysis of such plans, namely the estimation of variance components, design and genetic, is available in the literature. However, the primary interest in this study is to examine the robustness of various Mating Designs as it depends on the underlying experimental design. Robustness of Partial Diallel Cross Plan is examined using Balanced Incomplete Block Design.There are varying Balanced Incomplete Block Designs for different parametric values with unavailability of one or more observations. Results have shown that efficiency from a Balanced Incomplete Block Designs for three case, $s = 1$ will be more than $s = 2$, and $s = k$ with the same parameters. The theoretical results show that for one or more missing crosses from a Partial Diallel Cross, Designs are reasonably robust. Balanced Incomplete Block Designs, with one or more crosses missing the efficiency occurs between 90 - 99 percentage.

The efficiency is obtained and the cases are compared to determine the robustness of the Balanced Incomplete Block Designs. It may be observed that the Balanced Incomplete Block Mating Design is fairly robust for different set of parameters with one or more observations are lost in a block. Efficiency of designs depends on the parameters v, b, r, k, λ in the case of Balanced Incomplete Block Design. It has been observed that efficiency increases with the increase of number of treatment v, but not on b, r, k, λ and δ. However for the same set of parameter efficiency gets reduced for case (i) to case (iii) in all situations that as s gets increased. But in all cases it has been found to be highly robust.

Acknowledgment

The author thanks the referee for his valuable comments and suggestions on an earlier version of the paper.

References

[1] A.C. Atkinson, A.N. Donev and R.D Tobias, *Optimum Experimental Designs with SAS*, Oxford University Press, 2007.

[2] A.S. Arya, "Circulant plans for partial diallel crosses," *Biometrics,* vol. 39, pp. 43-52, 1988.

[3] H.C. Agrawal, "A four class cyclic association scheme and related partial diallel crosses," *Sankhya B,* vol. 47, pp. 78-90, 1985.

[4] A.S. Arya, and P. Narain, "Partial diallel crosses based on some association schemes with three and four associate classes," *Sankhya B,* vol.39, pp. 394-399, 1997.

[5] C.B. Ogbunugafor, J.B. Pease and P.E. Turner, "On the possible role of robustness in the evolution of infectious diseases," *Chaos,* vol. 20, 026108, 2010.

[6] R.A. Bailey, *Design of Comparative Experiments*, Cambridge University Press, 2008.

[7] R.N. Curnow, "Sampling the diallel cross," *Biometrics*, vol. 19, pp. 287-336, 1963.

[8] R.E. Comstock and H.F. Robinson, "Genetic parameters, their Estimation and significance," *Proc. Sixth Int. Grosslands Conf.*, pp. 284-291, 1952.

[9] A. Das and S. Kageyama, "Robustness of BIB and extended BIB designs against the unavailability of any number of observations in a block," *Comput. Statist. Data Anal.,* vol. 14, no. 3, pp. 343-358, 1992.

[10] A. Dey and C.K. Midha, "Optimal block designs for diallel cross," *Biometrika,* vol. 83, no. 2, pp. 484-489, 1996.

[11] M.N. Das and K. Sivaram, "Partial diallel crosses and incomplete block design," Contribution on statistics and agricultural science, Industrial Agriculture Stat, New Delhi, 1968.

[12] E.W. Weisstein, "Block Design from math world a wolfram Web Resource," Math world, Wolfram. Com, 2011.

[13] J.S. Fyfe and N. Gilbert, "Partial diallel crosses," *Biometrics,* vol. 19, pp. 278-286, 1963.

[14] D.K. Ghosh, and P.C. Biswas, "Robust designs for diallel crosses against the missing of one block," *J. Applied Statistics,* vol. 27, no.6, pp. 715-723, 2000.

[15] D.K. Ghosh, and N.R. Desai, "Robustness of a complete diallel crosses plan with an unequal number of crosses to the unavailability of one block," *J. Applied Statistics,* vol. 26, no.5, pp.563-577, 1999.
[16] D.K. Ghosh, and N.R. Desai, "Robustness of complete diallel crosses plans to the unavailability of one block," *J. Applied Statistics,* vol. 25, no. 6, pp. 827-837, 1998.
[17] D.K. Ghosh, J. Divecha and M.N. Das, "PBIB (2) designs and partial diallel crosses," *Biometrika,* vol.84, pp. 245- 248, 1997.
[18] S. Gupta and S. Kageyama, "Optimal Complete diallel crosses," *Biometrika,* vol. 81, pp. 420-424, 1994.
[19] B. Gilbert, "Diallel cross in plant breeding," *Heredity,* vol. 12, pp. 477-492, 1958.
[20] B. Griffing, "Concepts of general and specific combining ability in relation to Diallel crossing systems," *J. Biol. Sci.,* vol. 9, pp. 463-493, 1956.
[21] K. Hinkelmann and O. Kempthorne, *Design Analysis of Experiments*, I and II (Second Ed.) Wiley, 2008.
[22] K. Hinkelmann and O. Kempthorne, *Design and Analysis of Experiments*, Vol. 2: Advanced Experimental Design," (First Ed.) Wiley,2005.
[23] K. Hinkelmann, "Design of Genetical experiments." In: Srivastava, J.N. (Ed.), A Survey of Statistical Design and Linear Models, vol. 42, North-Holland, Amsterdam, pp. 243-269 ,1975.
[24] H. Hanani, "BIBDs and related designs," *Discrete Mathematics,* vol. 11, 255-369, 1975.
[25] L.S. Kaushik, P.D. Puri and B.D. Mehta, "Augmented PDC based on Two associate class association scheme", *Statistica* vol. 47, pp. 389-401, 1987,
[26] O. Kempthorne and R.N. Curnow, "The partial diallel cross," *Biometrics,* vol.17, pp. 229-250, 1961.
[27] S.N. Mathur and P. Narain, "Some optimal plans for partial diallel cross," *Indian J. Genet.,* vol.36, no. 3, pp. 301-308, 1976.
[28] P. Narain, *Statistical Genetics*, Wiley Eastern Ltd., New Delhi, 1993.
[29] P. Narain and A.S. Arya, "Truncated triangular association scheme and related partial diallel cross," *Sankhya B,* vol. 43, no. 1, pp. 93-103, 1981.
[30] P. Narain, C. Subbarao and A.K. Nigam, "Partial Diallel crosses based on extended triangular association scheme," *Indian J. Genet.,* vol. 34, pp. 309-317, 1974.
[31] B.P. Mausami, *Robustness of balanced incomplete block designs with repeated blocks and other IBD against the unavailability of observations and blocks*, Ph. D. Thesis, Saurashtra University, Rajkot, 2008.
[32] R.A. Mansson, and P. Prescott,"Robustness of a class of partial diallel crosses designs to the unavailability of a complete block of observations," *J. Applied Statistics,* vol.31, no.2, pp. 145-160, 2004.
[33] P. Prescott, and R.A. Mansson,"Robustness of a class of partial diallel crosses designs to the unavailability of a complete block of observations," *J. Applied Statistics,* vol. 31, no. 2, pp. 145-160, 2004.
[34] L.V. Raghavarao Damaraju and Padgett, "Block Designs: Analysis Combinatorics and Applications," 2005.
[35] R. Shunmugathai and M.R. Srinivasan, "Robustness of DBIB Mating Designs against Unavailability of Two Blocks," *J. ProbStatForum*, vol. 4, pp.63-77, 2011.
[36] R. Shunmugathai and M.R. Srinivasan, "Robustness of CDC Plan using NBIB Mating Designs against Unavailability of One or More Observations," *International J. Math. Sci. Computing*, vol. 2, no. 1, pp. 44-50, 2011.
[37] R. Shunmugathai and M.R. Srinivasan, "Robustness of CDC Plan using NBIB Mating Designs against Unavailability of Two Blocks," *Journal of Arts and Science and Commerce*, vol. II, no. 3, ISSN: 2229 - 4686, 2011.
[38] S.K. Sudesh, and A. Shankar, "On the construction and existence of a certain class of complete diallel cross designs," *Statistics and Probability Letters,* vol. 77, pp. 111-115, 2007.
[39] J. Schmidt, "La Valeur de l' medividu a titre de generateur Apprecice suivant la method du croisement Diallel," *Compt. Rend. Lab. Carlsberg,* 14, No.633,1919.
[40] F. Yates, "The recovering of inter-block information in balanced incomplete block designs," *Annals of Human Genetics,* vol. 10, pp. 317 - 325, 1940.

TABLE III
BALANCED INCOMPLETE BLOCK DESIGN

Block	BIB Design				Crosses the PDC Design					
1	1	2	4	10	1 × 2	1 × 4	1 × 10	2 × 4	2 × 10	4 × 10
2	2	3	5	11	2 × 3	2 × 5	2 × 11	3 × 5	3 × 11	5 × 11
3	3	4	6	12	3 × 4	3 × 6	3 × 12	4 × 6	4 × 12	6 × 12
4	4	5	7	13	4 × 5	4 × 7	4 × 13	5 × 7	5 × 13	7 × 13
5	5	6	8	1	5 × 6	5 × 8	1 × 5	6 × 8	1 × 6	1 × 8
6	6	7	9	2	6 × 7	6 × 9	2 × 6	7 × 9	2 × 7	2 × 9
7	7	8	10	3	7 × 8	7 × 10	3 × 7	8 × 10	3 × 8	3 × 10
8	8	9	11	4	8 × 9	8 × 11	4 × 8	9 × 11	4 × 9	4 × 11
9	9	10	12	5	9 × 10	9 × 12	5 × 9	10 × 12	5 × 10	5 × 12
10	10	11	13	6	10 × 11	10 × 13	6 × 10	11 × 13	6 × 11	6 × 13
11	11	12	1	7	11 × 12	1 × 11	7 × 11	1 × 12	7 × 12	1 × 7
12	**12**	**13**	**2**	**8**	**12 × 13**	**2 × 12**	**8 × 12**	**2 × 13**	**8 × 13**	**2 × 8**
13	13	1	3	9	1 × 13	3 × 13	9 × 13	1 × 3	1 × 9	3 × 9

$$8C^* = \begin{bmatrix} 18 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ 0 & 18 & 0 & 0 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ 0 & 0 & 18 & 0 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ 0 & 0 & 0 & 18 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 24 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 24 & -4 & -4 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -4 & 24 & -4 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -4 & -4 & 24 \end{bmatrix}$$

TABLE IV
EFFICIENCY TABLE WHEN ONE OR MORE CROSSES IS LOST FROM A BALANCED INCOMPLETE BLOCK DESIGN

D.N	$v = p$	b	r	k	λ	Case i	Case ii	Case iii
1	9	24	8	3	2	0.97561	0.952381	0.952381
2	9	12	4	3	1	0.941176	0.888889	0.888889
3	6	10	5	3	2	0.9375	0.882353	0.882353
4	16	20	5	4	1	0.978261	0.957447	0.9375
5	21	21	5	5	1	0.984615	0.969697	0.941176
6	11	11	5	5	2	0.971429	0.944444	0.894737
7	13	26	6	3	1	0.97561	0.952381	0.952381
8	13	13	4	4	1	0.964286	0.931034	0.9
9	7	14	6	3	2	0.956522	0.916667	0.916667
10	10	15	6	4	2	0.972973	0.947368	0.923077
11	25	30	6	5	11	0.999229	0.998459	0.996923
12	31	31	6	6	1	0.992063	0.984252	0.961538
13	16	16	6	6	2	0.984848	0.970149	0.928571
14	16	35	7	3	1	0.984848	0.970149	0.970149
15	8	14	7	4	3	0.972222	0.945946	0.921053
16	15	21	7	5	2	0.985915	0.972222	0.945946
17	30	42	7	6	1	0.991453	0.983051	0.958678
18	43	43	7	7	1	0.995392	0.990826	0.972973
19	22	22	7	7	3	0.994382	0.988827	0.967213
20	15	15	7	7	3	0.987013	0.974359	0.926829
21	9	24	8	3	2	0.97561	0.952381	0.952381
22	25	50	8	4	1	0.992126	0.984375	0.976744
24	13	26	8	4	2	0.985075	0.970588	0.956522
25	9	18	8	4	3	0.978723	0.958333	0.938776
26	21	28	8	6	2	0.991736	0.983607	0.96
27	49	56	8	7	1	0.99654	0.993103	0.979592
28	57	57	8	8	1	0.997093	0.994203	0.98
29	19	57	9	3	1	0.989691	0.979592	0.979592
30	10	30	9	3	2	0.980769	0.962264	0.962264
31	7	21	9	3	3	0.972973	0.947368	0.947368
32	28	63	9	4	1	0.993865	0.987805	0.981818
33	10	18	9	5	4	0.984375	0.962931	0.940299
34	16	40	10	4	2	0.990566	0.981308	0.972222
35	46	69	9	6	1	0.9966678	0.993377	0.983607
36	21	42	10	5	2	0.993289	0.9986667	0.973684
37	11	22	10	5	4	0.987342	0.975	0.95122
38	51	85	10	6	1	0.99734	0.994695	0.986842
39	21	30	10	7	3	0.993789	0.987654	0.963855
40	36	45	10	8	2	0.996441	0.992908	0.97561
41	81	90	10	9	1	0.99844	0.996885	0.987654
42	91	91	10	10	1	0.99863	0.997264	0.987805
43	46	46	10	10	2	0.997297	0.994609	0.97619
44	31	31	10	10	3	0.996	0.992032	0.965116
45	45	99	11	5	1	0.997167	0.99435	0.988764
46	12	22	11	6	5	0.99	0.980198	0.951923
47	45	55	11	9	2	0.997481	0.994975	0.980198
48	100	110	11	10	1	0.998879	0.99776	0.99
49	111	111	11	11	1	0.999001	0.998004	0.990099
50	56	56	11	11	2	0.998024	0.996055	0.980583
51	23	32	11	11	5	0.995215	0.990476	0.954128
52	25	100	12	3	1	0.99435	0.988764	0.988764
53	9	36	12	3	3	0.984615	0.969697	0.969697
54	7	28	12	3	4	0.980392	0.961538	0.961538
55	37	111	12	4	1	0.996644	0.993311	0.99
56	19	57	12	4	2	0.993506	0.987097	0.980769
57	25	60	12	5	2	0.995392	0.990826	0.981818
58	61	122	12	6	1	0.998185	0.996377	0.990991
59	31	62	12	6	2	0.996441	0.992908	0.982456
60	21	42	12	6	3	0.994764	0.989583	0.974359
61	16	32	12	6	4	0.993151	0.986395	0.966667
62	13	26	12	6	5	0.991597	0.983333	0.95935
63	22	33	12	8	4	0.995261	0.990566	0.967742
64	33	44	12	9	3	0.996885	0.993789	0.97561
65	55	66	12	10	2	0.998152	0.99631	0.983607
66	121	132	12	11	1	0.999167	0.998336	0.991736
67	133	133	12	12	1	0.999249	0.9985	0.991803
68	67	67	12	12	2	0.998512	0.997028	0.983871
69	45	45	12	12	3	0.997788	0.995585	0.97619
70	34	34	12	12	4	0.997076	0.994169	0.96875
71	27	117	13	3	1	0.995215	0.990476	0.990476
72	40	130	13	4	1	0.997159	0.994334	0.991525
73	66	143	13	6	1	0.998464	0.996933	0.992366
74	14	26	13	7	6	0.993056	0.986207	0.959732
75	27	39	13	9	4	0.996516	0.993056	0.972789
77	40	52	13	10	3	0.997674	0.99536	0.979452
78	66	78	13	11	2	0.998603	0.997211	0.986207
79	144	156	13	12	1	0.999365	0.99873	0.993056
80	157	157	13	13	1	0.999422	0.998844	0.993103
81	79	79	13	13	2	0.998852	0.997706	0.986395