Unsteady MHD Laminar Momentum Boundary Layer over a Flat Plate with Leading Edge Accretion (Ablation)

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Abstract— **This paper deals with unsteady, laminar MHD incompressible boundary layer flow over a flat plate with leading edge accretion or ablation. It is found that the application of transverse magnetic field can greatly nullify the instability in the laminar flow, due to these leading edge disturbances.**

Index Terms— **MHD, Laminar boundary layer, Leading edge accretion or ablation, Flat plate.**

MSC 2010 Codes **—76N20, 76R05 and 76M20**

I. INTRODUCTION

IT is well known that the fluid motion pattern and boundary layer separation are severely influenced by unsteady boundary layer owing to extra time-dependent term in the momentum equation[1-3]. The typical examples of unsteady boundary layers in the history of fluid mechanics are the Rayleigh problem and Stokes oscillating plate [2-3]. Two dimensional unsteady boundary layers have been studied by many researchers for different unsteady free stream velocity and flow configuration as discussed in these references and the references therein [4-6]. Similarity transformation technique played a significant role in solving the unsteady boundary layer problem by transforming a partial differential equation (PDE) into an ordinary differential equation (ODE) in most studies. An interesting and new unsteady state boundary layer problem was proposed by Todd [7], where the laminar momentum boundary layer flow over a flat plate entailing the moving leading edge with a certain rate of accretion or ablation, has been analyzed.

In the present work, the laminar momentum boundary layer of this flow pattern, in the presence of an applied magnetic field, is undertaken as magneto-hydrodynamic flows are considered to be important in many geothermal, technological and engineering applications. Indeed, flow characteristics of such an unsteady, MHD laminar boundary layer play a significant role in many engineering and technological problems like start-up process and periodic fluid motion.

II. MATHEMATICAL FORMULATION

Consider two-dimensional laminar incompressible flow over a flat plate with leading edge accretion or ablation. The free stream velocity is U_∞. The x-axis runs along the free stream direction and the y-axis is perpendicular to it. A transverse magnetic field of strength B_0 is applied in $y-$direction normal to the plate with the assumption that the magnetic Reynolds number is small, so that the induced magnetic field can be neglected as compared to the applied magnetic field. Hence, the applied magnetic field contributes only to the Lorentz force which acts in the x- direction. The governing equations for the laminar momentum boundary layer of this problem are given by [2]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

with boundary conditions

$$u(x,0)=0,\ \ v(x,0)=0,\ \ u(x,\infty)=U_\infty \tag{3}$$

where u and v are velocity components in the x and y direction, respectively, t is the time variable, ν, σ and ρ, respectively, denote kinematic viscosity, thermal diffusivity and density.

By choosing a stream function

$$\psi(x,y,t)=U_\infty\sqrt{(\cos\beta)\nu t+(\sin\beta)(\nu x/U_\infty)}\ f(\eta)$$

along with a similarity variable

$\eta = y/\sqrt{(\cos\beta)\nu t+(\sin\beta)(\nu x/U_\infty)}$, which is also called Blasius-Rayleigh-Stokes variable by Todd [7], the continuity equation (1) is automatically satisfied and the Eqn. (2) can be transformed into a similarity equation as

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$$f''' + \frac{1}{2}(\cos\beta)\eta f'' + \frac{1}{2}(\sin\beta) ff'' - Mf' = 0 \qquad (4)$$

along with the associated boundary conditions

$$f(0) = 0,\ f'(0) = 0,\ f'(\infty) = 1 \qquad (5)$$

where β is the leading edge accretion or ablation parameter, and the prime denotes differentiation with respect to η. Based on these definitions, the velocity can be shown as

$u = U_{\infty} f'(\eta)$ and

$v = (\nu/2)(\eta f' - f)(\sin\beta)/\sqrt{(\cos\beta)\nu t + (\sin\beta)(\nu x/U_{\infty})}$.

Equation (4) with the boundary condition (5) will be solved by using an implicit finite difference scheme as described in [8] to determine the velocity field $[F = f'(\eta)]$ and skin-friction parameter $[f''(0)]$ under boundary conditions (5).

III. RESULTS AND DISCUSSION

In the absence of magnetic field ($M = 0$), the laminar momentum boundary layers have been discussed extensively by Todd [7] and the present results of $f''(0)$, in the absence of magnetic field ($M = 0$), are in good agreement with those of [7], where Eqn. (4) has solution only when $-\pi/4 \le \beta < 3\pi/4$ [See Fig.1]. In fact, in the absence of magnetic field $(M = 0)$, when $0 < \beta < \pi/2$, there is a leading edge accretion with a rate of $|U_{\infty} \cot\beta|$, namely the leading edge disturbances progressing upstream; when $\pi/2 < \beta \le \beta_0$, where $\beta_0 \approx 105^0$, there is a leading edge ablation with a rate of $|U_{\infty} \cot\beta|$; when $-\pi/4 \le \beta < 0$, there is a "backward" boundary layer with trailing edge accretion. At the critical points $\beta = 0$ and $\pi/2$, the results are corresponding to the well known Rayleigh-Stokes problem and Blasius flat plate boundary layers, respectively.

It is also seen in Fig. 1 that there are two solutions for Eqn. (4) in small range viz., when $\pi/2 < \beta \le \beta_0$, revealing the chaotic behavior in the flow. However, flow becomes stable with the application of the transverse magnetic field. Indeed, skin friction parameter $[f''(0)]$ decreases smoothly passing over the critical points viz., $\beta = 0$ and $\pi/2$, and finally vanishes, authenticating the stability of the flow. This can be attributed to the fact that in the presence of the transverse magnetic field $(M \neq 0)$, the Lorentz force produces more resistance to the transport phenomena and thereby nullifies the effect of the leading edge accretion/ablation. The laminar momentum boundary layer thus becomes stable.

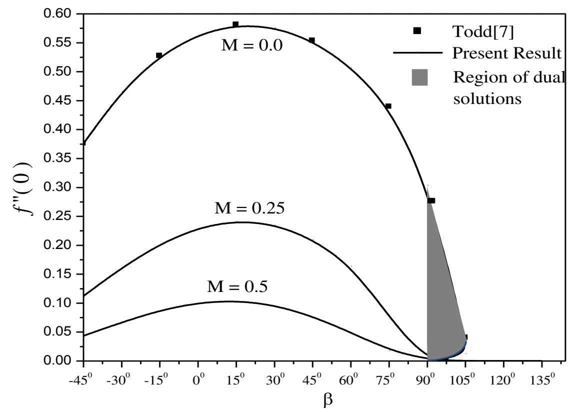

Fig 1: The solution domain of the skin friction parameter $[f''(0)]$

The unsteady velocity field [F] shown in Fig.2 reveals that varying leading edge accretion/ablation parameter can greatly change the velocity distribution and momentum transfer near the wall. And at the same time, the effect of transverse magnetic field $(M \neq 0)$, is clearly visible in establishing the velocity profiles to smoothly sail over to far away asymptotic boundary conditions.

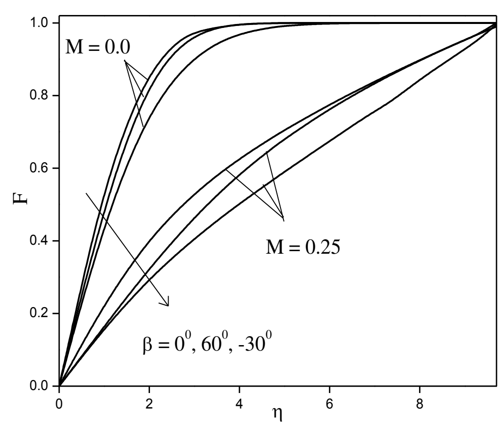

Fig 2: Velocity profile for various of β when $M = 0.0$ and $M = 0.25$

IV. CONCLUSION

In this paper, the unsteady laminar boundary layer over a semi-infinite flat plate under the influence of transverse magnetic field is analyzed. It is observed that the leading edge accretion or ablation parameter is found to affect the laminar momentum boundary layer extensively, in absence of the magnetic field.

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