

Robust Guaranteed Cost Control for Uncertain T-S Fuzzy Time-Delay Sampled-Data Systems with Nonlinear and Linear Fractional Perturbations

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Abstract—The guaranteed cost control for uncertain Takagi-Sugeno (T-S) fuzzy time-delay systems with sampled-data input is considered in this paper. In order to treat the sampled-data input problem, the time-varying delay transformation technique and LMI optimization approach are provided to find the upper bound of sampled time and solve the minimization of guaranteed cost. A numerical example is illustrated to show the use of the main results.

Index Terms—T-S fuzzy time-delay system, sampled-data input, guaranteed cost control, LMI optimization approach, time-varying delay, linear fractional perturbation.

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I. INTRODUCTION

Time-delay phenomena are often encountered in various practical systems; such as aircraft stabilization, manual control, models of lasers, neural networks, nuclear reactors, ship stabilization, and systems with lossless transmission lines. Hence there are many results proposed to deal the stability for time-delay systems. In complex and practical systems, Takagi-Sugeno (T-S) fuzzy system model [1-2] had been provided to represent the systems by some fuzzy sets and reasoning. On the other hand, time delay often generates instability and oscillations in many systems. Hence stability analysis and stabilization of T-S fuzzy time-delay systems have been investigated in recent years [3-6]. Parallel distributed control (PDC) is a useful technique to stabilize the stability and performance of T-S fuzzy system, but PDC is difficult to implement by analog elements and devices. Hence sampled-data input will be an available consideration for implementing this PDC. Suppose the PDC is calculated by computer, then the input value will be held until next sample instant to reflect the input [7]. This important issue to guarantee the stabilization is arisen from the sampled time $T > 0$. In [7]-[12], a time-varying delay input approach had been used to represent the sampled-data input. This technique provides a useful analytic tool to estimate the upper bound of sampled time $T > 0$ and attain the design performance.

H_∞ control and guaranteed cost control are two stabilization schemes with some respective performance indices

[7], [12]. H_∞ control concept was proposed to reduce the effect of the disturbance input on the regulated output within a prescribed level and guarantee that the closed-loop system is stable. In our past results in [7], H_∞ control problem of fuzzy time-delay system with sampled-data input had been considered. Guaranteed cost control which not only makes the closed-loop system asymptotically stable but also guarantees an adequate level of performance. In [12], Lyapunov-Krasovskii functional with Leibniz-Newton formula had been used to find guaranteed control for T-S fuzzy systems by using time-varying delay input approach. In [13], some generalized parametric (linear fractional) perturbations are introduced and studied for T-S fuzzy time-delay systems. In this paper, the guaranteed cost control for T-S fuzzy time-delay systems with nonlinear and linear fractional perturbations are considered. Some additional nonnegative inequalities are introduced to improve the conservativeness of the proposed results in this paper.

Notations. For a matrix A , we denote symmetric negative definite by $A < 0$. I which means identity matrix. $A \leq B$ means that matrix $B - A$ is symmetric positive semi-definite.

II. GUARANTEED COST CONTROL FOR T-S FUZZY SYSTEMS WITH NONLINEAR PERTURBATION AND SAMPLED-DATA CONTROL INPUT

Consider a continuous-time fuzzy time-delay system with sampled input and nonlinear perturbations, which is represented by a Takagi-Sugeno (T-S) fuzzy model [1-2] composed of a set of fuzzy implications. Each implication is expressed by a system with nonlinear perturbation and the i th rule of the T-S model is written in the following form:

Rule i :

If $z_1(t)$ is M_{i1} and $\dots z_r(t)$ is M_{ir} , then

$$\dot{x}(t) = A_{i1}x(t) + A_{i2}x(t-\tau) + \Delta f_i(x(t)) + B_i u(t), \quad t \geq 0, \quad (1a)$$

$$x(t) = \varphi_i(t), \quad t \in [-\tau, 0], \quad i = 1, 2, \dots, m, \quad (1b)$$

where $z_1(t), z_2(t), \dots, z_r(t)$ are premise variables, M_{ij} , $i \in \{1, 2, \dots, m\}$, $j \in \{1, \dots, r\}$ are fuzzy sets, and m is the rules number. $x(t) \in \mathcal{R}^n$ is the system state, $u(t) \in \mathcal{R}^p$ is the sampled-data control input. Matrices A_{i1} , A_{i2} , and B_i , are given, and the vectors φ_i , $i \in \{1, 2, \dots, m\}$, belong to the set of continuous functions. $\Delta f_i(x(t))$ is a perturbed nonlinear function satisfying

$$\|\Delta f_i(x(t))\| \leq \|F_i x(t)\|, \quad i \in \{1, 2, \dots, m\}, \quad (2)$$

where F_i is a given constant matrix. The sampled-data control input is described as follows:

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$$u(t) = p(nT, x(nT)), \quad \forall nT \leq t < (n+1)T, \quad (3)$$

where the function $p(t, x(t))$ is designed by any control theory and may depend on t and state $x(t)$, $T > 0$ is the sample time.

The final state output of fuzzy system is inferred as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^m v_i(z(t)) \cdot [A_{1i}x(t) + A_{2i}x(t-\tau) + \Delta f_i(x(t)) + B_i u(t)] / \sum_{i=1}^m v_i(z(t)) \\ &= \sum_{i=1}^m \eta_i(z(t)) \cdot [A_{1i}x(t) + A_{2i}x(t-\tau) + \Delta f_i(x(t)) + B_i u(t)], \end{aligned} \quad (4a)$$

$$t \geq 0, \quad (4a)$$

$$x(t) = \sum_{i=1}^m \eta_i(z(t)) \cdot \varphi_i(t), \quad t \in [-\tau, 0], \quad i = 1, 2, \dots, m, \quad (4b)$$

where $v_i(z(t)) = \prod_{j=1}^r \Omega_{ij}(z_j(t))$, $\eta_i(z(t)) = \frac{v_i(z(t))}{\sum_{i=1}^m v_i(z(t))}$, $\Omega_{ij}(z_j(t))$ is the

grade of the membership of $z_j(t)$ in fuzzy set M_{ij} . In this

paper, $v_i(z(t)) \geq 0$, $i \in \{1, \dots, m\}$, and $\sum_{i=1}^m v_i(z(t)) > 0$ are

assumed. Hence $\eta_i(z(t)) \geq 0$ and $\sum_{i=1}^m \eta_i(z(t)) = 1$.

We define the cost function of system (1) with (2) as follows:

$$J = \int_0^{\infty} [x^T(s) \cdot S_1 \cdot x(s) + u^T(s) \cdot S_2 \cdot u(s)] ds, \quad (5)$$

where $S_1 \in \mathfrak{R}^{n \times n}$ and $S_2 \in \mathfrak{R}^{p \times p}$ are two given positive definite symmetric matrices. We wish to design a sampled-data state feedback control in (3) and find a positive constant J^* , such that the system (1) with (2) is asymptotically stable and $J \leq J^*$, where J^* is the guaranteed cost for this sampled-data state feedback control in (3) of system (1) with (2).

The following fuzzy rule for fuzzy-model-based control is used in this paper:

Rule i :

If $z_1(t)$ is about M_{i1} and \dots $z_r(t)$ is about M_{ir} , then

$$p(t, x(t)) = -K_i x(t), \quad t \geq 0,$$

where $K_i \in \mathfrak{R}^{p \times n}$ will be designed in this paper. The final feedback control is inferred as

$$p(t, x(t)) = -\sum_{i=1}^m \eta_i(z(t)) \cdot K_i x(t), \quad t \geq 0. \quad (6)$$

In this paper, we will provide a concept to treat the original system with sampled-data input by time-varying delay. With (6), the sampled-data control input can be described as follows:

$$\begin{aligned} u(t) &= p(nT, x(nT)) = -\sum_{i=1}^m \eta_i(z(nT)) \cdot K_i x(nT) \\ &= -\sum_{i=1}^m \eta_i(z(t-h(t))) \cdot K_i x(t-h(t)), \quad \forall nT \leq t < (n+1)T, \end{aligned} \quad (7a)$$

where $h(t)$ is specified by

$$h(t) = t - nT, \quad nT \leq t < (n+1)T. \quad (7b)$$

The system (4) can be rewritten as follows:

$$\dot{x}(t) = \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t)) \cdot \eta_j(z(t-h(t))).$$

$$[A_{1i}x(t) + A_{2i}x(t-\tau) + \Delta f_i(x(t)) - B_i K_j x(t-h(t))], \quad t \geq 0, \quad (8a)$$

$$x(t) = \sum_{i=1}^m \eta_i(z(t)) \cdot \varphi_i(t), \quad t \in [-\tau, 0], \quad i = 1, 2, \dots, m. \quad (8b)$$

From (6b), we have $0 \leq h(t) < T$, $t \geq 0$. The following lemma will be used to design the state feedback control.

Lemma 1. (Schur complement of [14]). For a given matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} \text{ with } S_{11} = S_{11}^T, \quad S_{22} = S_{22}^T, \text{ then the following}$$

conditions are equivalent:

$$(1) \quad S < 0,$$

$$(2) \quad S_{22} < 0, \quad S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0.$$

Now we present a result to design the sampled-data guaranteed cost control (7) for system (8) with (2).

Theorem 1. Suppose for a given constant $\eta > 0$, the following LMIs :

$$\begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} \\ * & \hat{Q}_{22} \end{bmatrix} > 0, \quad \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} \\ * & \hat{R}_{22} \end{bmatrix} > 0, \quad \hat{R}_{11} > \hat{Q}_{11},$$

$$\hat{P}_2 > \hat{Q}_{22}, \quad \hat{P}_2 > \hat{R}_{22}, \quad (9a)$$

$$T \cdot \hat{P}_2 < \eta^{-1} \cdot \hat{P}_0, \quad (9b)$$

$$\hat{\Pi}_{ij} = \begin{bmatrix} \hat{\Pi}_{11ij} & \hat{\Pi}_{12ij} & 0 & \hat{\Pi}_{14ij} & \hat{\Pi}_{15ij} & 0 & \hat{\Pi}_{17ij} & \hat{\Pi}_{18ij} & \hat{\Pi}_{19ij} \\ * & 0 & 0 & 0 & 0 & \hat{\Pi}_{26ij} & \hat{\Pi}_{27ij} & 0 & 0 \\ * & * & \hat{\Pi}_{33ij} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Pi}_{44ij} & 0 & 0 & \hat{\Pi}_{47ij} & 0 & 0 \\ * & * & * & * & \hat{\Pi}_{55ij} & 0 & \hat{\Pi}_{57ij} & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66ij} & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{77ij} & 0 & 0 \\ * & * & * & * & * & * & * & \hat{\Pi}_{88ij} & 0 \\ * & * & * & * & * & * & * & * & \hat{\Pi}_{99ij} \end{bmatrix}$$

$$+ \begin{bmatrix} \hat{Q} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \text{ for all } i, j \in \{1, \dots, m\}, \quad (9c)$$

have a solution with positive constants ε_i , positive definite symmetric matrices $\hat{P}_0, \hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{Q}_{22}, \hat{R}_{22} \in \mathfrak{R}^{n \times n}$, $\hat{Q}_{11} \in \mathfrak{R}^{3n \times 3n}$, $\hat{R}_{11} \in \mathfrak{R}^{3n \times 3n}$, matrices $\hat{Q}_{12} \in \mathfrak{R}^{3n \times n}$, $\hat{R}_{12} \in \mathfrak{R}^{3n \times n}$, $\hat{K}_i \in \mathfrak{R}^{p \times n}$ where * represents the symmetric form in the matrix and

$$\begin{aligned} \hat{\Pi}_{11ij} &= A_{i1} \hat{P}_0 + \hat{P}_0 A_{i1}^T + \hat{P}_1 + \hat{P}_3, \quad \hat{\Pi}_{12ij} = -B_i \hat{K}_j, \quad \hat{\Pi}_{14ij} = A_{2i} \hat{P}_0, \\ \hat{\Pi}_{15ij} &= \varepsilon_i \cdot I, \quad \hat{\Pi}_{17ij} = \hat{P}_0 A_{i1}^T, \quad \hat{\Pi}_{18ij} = \hat{P}_0 F_i^T, \quad \hat{\Sigma}_{19ij} = \hat{P}_0, \\ \hat{\Pi}_{26ij} &= -\hat{K}_j^T, \quad \hat{\Pi}_{27ij} = -\hat{K}_j^T B_i^T, \quad \hat{\Pi}_{33ij} = -\hat{P}_1, \quad \hat{\Pi}_{44ij} = -\hat{P}_3, \\ \hat{\Pi}_{47ij} &= \hat{P}_0 A_{2i}^T, \quad \hat{\Pi}_{55ij} = -\varepsilon_i \cdot I, \quad \Pi_{57ij} = \varepsilon_i \cdot I, \quad \hat{\Pi}_{66ij} = -S_2^{-1}, \\ \hat{\Pi}_{77ij} &= -\eta \cdot \hat{P}_0, \quad \hat{\Pi}_{88ij} = -\varepsilon_i \cdot I, \quad \hat{\Pi}_{99ij} = -S_1^{-1}, \\ \hat{\Omega} &= T \cdot \hat{R}_{11} + \hat{Q}_{12} [I \quad -I \quad 0] + [I \quad -I \quad 0]^T \hat{Q}_{12}^T \\ &\quad + \hat{R}_{12} [0 \quad I \quad -I] + [0 \quad I \quad -I]^T \hat{R}_{12}^T. \end{aligned} \quad (9d)$$

Then the system (4) with (2) is stabilizable by sampled-data input (6) and (7) with $K_j = \hat{K}_j \hat{P}_0^{-1}$ and the guaranteed cost is given by

$$\begin{aligned} J^* &= x^T(0) P_0 x(0) + \int_{-T}^0 x^T(s) P_1 x(s) ds \\ &\quad + \int_{-T}^0 (s+T) \dot{x}^T(s) P_2 \dot{x}(s) ds + \int_{-T}^0 x^T(s) P_3 x(s) ds, \end{aligned} \quad (9e)$$

where $P_0 = \hat{P}_0^{-1}$, $P_i = \hat{P}_0^{-1} \hat{P}_i \hat{P}_0^{-1}$, $i \in \{1, 2, 3\}$.

Proof. Define the Lyapunov functional

$$\begin{aligned} V(x_t) &= x^T(t) P_0 x(t) + \int_{t-T}^t x^T(s) P_1 x(s) ds \\ &\quad + \int_{t-T}^t (s-(t-T)) \dot{x}^T(s) P_2 \dot{x}(s) ds + \int_{t-T}^t x^T(s) P_3 x(s) ds, \end{aligned} \quad (10a)$$

where $P_0 = \hat{P}_0^{-1}$, $P_i = \hat{P}_0^{-1} \hat{P}_i \hat{P}_0^{-1}$, $i \in \{1, 2, 3\}$, are positive definite symmetric matrices. The time derivatives of $V(x_t)$, along the trajectories of system (8) with (2) satisfy

$$\begin{aligned} \dot{V}(x_t) &= \sum_{i=1}^m \eta_i(z(t)) \cdot \left\{ x^T(t) (P_0 A_{i1} + A_{i1}^T P_0) x(t) \right. \\ &\quad \left. + 2x^T(t) P_0 [A_{2i} x(t-T) + \Delta f_i(x(t))] \right\} \\ &\quad - \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t)) \cdot \eta_j(z(t-h(t))) \cdot \left\{ 2x^T(t) P_0 B_i K_j x(t-h(t)) \right\} \\ &\quad + x^T(t) P_1 x(t) - x^T(t-T) P_1 x(t-T) + T \cdot \dot{x}^T(t) P_2 \dot{x}(t) \\ &\quad - \left[\int_{t-T}^{t-h(t)} \dot{x}^T(s) P_2 \dot{x}(s) ds + \int_{t-h(t)}^t \dot{x}^T(s) P_2 \dot{x}(s) ds \right] \\ &\quad + x^T(t) P_3 x(t) - x^T(t-T) P_3 x(t-T). \end{aligned} \quad (10b)$$

Now we define a vector by

$$X^T(t) = [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-T)].$$

By Leibniz-Newton formula and LMIs (9a), the following additional nonnegative inequalities can be introduced:

$$\begin{aligned} &\int_{t-h(t)}^t \begin{bmatrix} X(t) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} X(t) \\ \dot{x}(s) \end{bmatrix} ds \\ &= h(t) \cdot X^T(t) Q_{11} X(t) + 2X^T(t) Q_{12} [x(t) - x(t-h(t))] \\ &\quad + \int_{t-h(t)}^t \dot{x}^T(s) Q_{22} \dot{x}(s) ds \geq 0, \end{aligned} \quad (10c)$$

$$\begin{aligned} &\int_{t-T}^{t-h(t)} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix} ds \\ &= [T-h(t)] \cdot X^T(t) R_{11} X(t) \\ &\quad + 2X^T(t) R_{12} [x(t-h(t)) - x(t-T)] \end{aligned}$$

$$+ \int_{t-T}^{t-h(t)} \dot{x}^T(s) R_{22} \dot{x}(s) ds \geq 0, \quad (10d)$$

where

$$\begin{aligned} \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} &= \begin{bmatrix} \hat{P}_0^{-1} & 0 & 0 & 0 \\ 0 & \hat{P}_0^{-1} & 0 & 0 \\ 0 & 0 & \hat{P}_0^{-1} & 0 \\ 0 & 0 & 0 & \hat{P}_0^{-1} \end{bmatrix} \begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} \\ * & \hat{Q}_{22} \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \hat{P}_0^{-1} & 0 & 0 & 0 \\ 0 & \hat{P}_0^{-1} & 0 & 0 \\ 0 & 0 & \hat{P}_0^{-1} & 0 \\ 0 & 0 & 0 & \hat{P}_0^{-1} \end{bmatrix} > 0, \\ \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} &= \begin{bmatrix} \hat{P}_0^{-1} & 0 & 0 & 0 \\ 0 & \hat{P}_0^{-1} & 0 & 0 \\ 0 & 0 & \hat{P}_0^{-1} & 0 \\ 0 & 0 & 0 & \hat{P}_0^{-1} \end{bmatrix} \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} \\ * & \hat{R}_{22} \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \hat{P}_0^{-1} & 0 & 0 & 0 \\ 0 & \hat{P}_0^{-1} & 0 & 0 \\ 0 & 0 & \hat{P}_0^{-1} & 0 \\ 0 & 0 & 0 & \hat{P}_0^{-1} \end{bmatrix} > 0. \end{aligned}$$

From condition (2), we have

$$x^T(t) F_i^T F_i x(t) - \Delta f_i^T(x(t)) \Delta f_i(x(t)) \geq 0, \quad i \in \{1, 2, \dots, m\}. \quad (11)$$

From the input in (7), we have

$$\begin{aligned} u^T(t) S_2 u(t) &= \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t-h(t))) \cdot \eta_j(z(t-h(t))) \\ &\quad \cdot [-K_j x(t-h(t))]^T S_2 [-K_j x(t-h(t))] \\ &\leq \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t-h(t))) \cdot \eta_j(z(t-h(t))) \\ &\quad \cdot [-K_j x(t-h(t))]^T S_2 [-K_j x(t-h(t))] \\ &\quad + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t-h(t))) \cdot \eta_j(z(t-h(t))) \\ &\quad \cdot [-K_j x(t-h(t))]^T S_2 [-K_j x(t-h(t))] \\ &= \sum_{j=1}^m \eta_j(z(t-h(t))) \cdot [-K_j x(t-h(t))]^T S_2 [-K_j x(t-h(t))] \\ &= \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t)) \cdot \eta_j(z(t-h(t))) \\ &\quad \cdot [-K_j x(t-h(t))]^T S_2 [-K_j x(t-h(t))]. \end{aligned} \quad (12a)$$

By the similar derivation of (12), condition (9b), and system (8)

with $P_2 = \hat{P}_0^{-1} \hat{P}_2 \hat{P}_0^{-1}$, we have

$$T \cdot \dot{x}^T(t) P_2 \dot{x}(t) \leq \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t)) \cdot \eta_j(z(t-h(t))).$$

$$\begin{aligned} &[A_{i1} x(t) + A_{2i} x(t-T) + \Delta f_i(x(t)) - B_i K_j x(t-h(t))]^T \cdot (\eta \cdot \hat{P}_0)^{-1} \cdot \\ &[A_{i1} x(t) + A_{2i} x(t-T) + \Delta f_i(x(t)) - B_i K_j x(t-h(t))]. \end{aligned} \quad (12b)$$

From (9a) and (10)-(12), we have

$$\dot{V}(x_t) + x^T(t) S_1 x(t) + u^T(t) S_2 u(t)$$

$$\begin{aligned}
& + \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t)) \cdot \eta_j(z(t-h(t))) \\
& \cdot \varepsilon_i^{-1} \cdot [x^T(t) F_i^T F_i x(t) - \Delta f_i^T(x(t)) \Delta f_i(x(t))] \\
\leq & \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t)) \cdot \eta_j(z(t-h(t))) \\
& \cdot [Y_i^T(t) \cdot \Pi_{ij} \cdot Y_i(t) - h(t) \cdot X^T(t) (R_{11} - Q_{11}) X(t)] \\
& - \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t)) \cdot \eta_j(z(t-h(t))) \\
& \cdot \left[\int_{t-T}^{t-h(t)} \dot{x}^T(s) (P_2 - R_{22}) \dot{x}(s) ds + \int_{t-h(t)}^t \dot{x}^T(s) (P_2 - Q_{22}) \dot{x}(s) ds \right] \\
\leq & \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t)) \cdot \eta_j(z(t-h(t))) \cdot [Y_i^T(t) \cdot \Pi_{ij} \cdot Y_i(t)], \quad (13a)
\end{aligned}$$

where

$$\begin{aligned}
Y_i^T(t) &= [X^T(t) \quad x^T(t-\tau) \quad \Delta f_i^T(x(t))], \\
\Pi_{ij} &= \begin{bmatrix} \Pi_{11ij} & \Pi_{12ij} & 0 & \Pi_{14ij} & \Pi_{15ij} \\ * & 0 & 0 & 0 & 0 \\ * & * & \Pi_{33ij} & 0 & 0 \\ * & * & * & \Pi_{44ij} & 0 \\ * & * & * & * & \Pi_{55ij} \end{bmatrix} \\
& + \begin{bmatrix} 0 \\ \Pi_{26ij} \\ 0 \\ 0 \\ 0 \end{bmatrix} (S_2^{-1})^{-1} \begin{bmatrix} 0 \\ \Pi_{26ij} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \Pi_{17ij} \\ \Pi_{27ij} \\ 0 \\ \Pi_{47ij} \\ \Pi_{57ij} \end{bmatrix} (\eta \cdot \hat{P}_0)^{-1} \begin{bmatrix} \Pi_{17ij} \\ \Pi_{27ij} \\ 0 \\ \Pi_{47ij} \\ \Pi_{57ij} \end{bmatrix}^T \\
& + \begin{bmatrix} \Omega & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (13b)
\end{aligned}$$

$$\begin{aligned}
\Pi_{11ij} &= P_0 A_i + A_i^T P_0 + P_1 + P_3 + \varepsilon_i^{-1} \cdot F_i^T F_i + S_1, \\
\Pi_{12ij} &= -P_0 B_i K_j, \quad \Pi_{14ij} = P_0 A_{2i}, \quad \Pi_{15ij} = P_0, \\
\Pi_{17ij} &= A_i^T, \quad \Pi_{26ij} = -K_j^T, \quad \Pi_{27ij} = -K_j^T B_i^T, \quad \Pi_{33ij} = -P_1, \\
\Pi_{44ij} &= -P_3, \quad \Pi_{47ij} = A_{2i}^T, \quad \Pi_{55ij} = -\varepsilon_i^{-1} \cdot I, \quad \Pi_{57ij} = I, \\
\Omega &= T \cdot R_{11} + Q_{12} [I \quad -I \quad 0] + [I \quad -I \quad 0]^T Q_{12}^T \\
& + R_{12} [0 \quad I \quad -I] + [0 \quad I \quad -I]^T R_{12}^T.
\end{aligned}$$

Pre- and post-multiplying the matrix Π_{ij} in (13b) by

$$\begin{aligned}
& \text{diag}[P_0^{-1} \quad P_0^{-1} \quad P_0^{-1} \quad P_0^{-1} \quad \varepsilon_i \cdot I] \\
& = \text{diag}[\hat{P}_0 \quad \hat{P}_0 \quad \hat{P}_0 \quad \hat{P}_0 \quad \varepsilon_i \cdot I] > 0
\end{aligned}$$

with

$$\hat{P}_0 = P_0^{-1}, \quad \hat{P}_i = P_0^{-1} P_i P_0^{-1}, \quad i \in \{1, 2, 3\}, \quad \hat{K}_j = K_j P_0^{-1},$$

we have

$$\begin{aligned}
\hat{\Pi}_{ij} &= \begin{bmatrix} \hat{\Pi}_{11ij} & \hat{\Pi}_{12ij} & 0 & \hat{\Pi}_{14ij} & \hat{\Pi}_{15ij} \\ * & 0 & 0 & 0 & 0 \\ * & * & \hat{\Pi}_{33ij} & 0 & 0 \\ * & * & * & \hat{\Pi}_{44ij} & 0 \\ * & * & * & * & \hat{\Pi}_{55ij} \end{bmatrix} \\
& + \begin{bmatrix} 0 \\ \hat{\Pi}_{26ij} \\ 0 \\ 0 \\ 0 \end{bmatrix} (S_2^{-1})^{-1} \begin{bmatrix} 0 \\ \hat{\Pi}_{26ij} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \hat{\Pi}_{17ij} \\ \hat{\Pi}_{27ij} \\ 0 \\ \hat{\Pi}_{47ij} \\ 0 \end{bmatrix} (\eta \cdot \hat{P}_0)^{-1} \begin{bmatrix} \hat{\Pi}_{17ij} \\ \hat{\Pi}_{27ij} \\ 0 \\ \hat{\Pi}_{47ij} \\ 0 \end{bmatrix}^T \\
& + \begin{bmatrix} \hat{\Omega} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (14)
\end{aligned}$$

where $\hat{\Pi}_{11ij} = \hat{\Pi}_{11ij} + \varepsilon_i^{-1} \cdot \hat{P}_0 F_i^T F_i \hat{P}_0 + \hat{P}_0 S_1 \hat{P}_0$, $\hat{\Pi}_{klj}$, $k, l \in \{1, 2, \dots, 7\}$ are defined in (9d). By Lemma 1, LMI $\hat{\Pi}_{ij} < 0$ in (9c) is equivalent to $\hat{\Pi}_{ij} < 0$ in (14). The condition $\hat{\Pi}_{ij} < 0$ in (14) is also equivalent to $\Pi_{ij} < 0$ in (13b) for all $i, j \in \{1, \dots, m\}$. From (11) and (13a) with the condition $\Pi_{ij} < 0$, there exists a $\rho > 0$ such that

$$\dot{V}(x_i) \leq -\rho \cdot \|x(t)\|^2.$$

We conclude that the fuzzy system (4) with (2) is asymptotically stabilizable by sampled-data input in (6) with (7). Integrating the equation in (13a) from 0 to ∞ with $\Pi_{ij} < 0$, we have

$$V(x_\infty) - V(\phi) + \int_0^\infty [x^T(t) S_1 x(t) + u^T(t) S_2 u(t)] dt \leq 0.$$

With $V(x_\infty) \geq 0$, we have

$$\int_0^\infty [x^T(t) S_1 x(t) + u^T(t) S_2 u(t)] dt \leq V(\phi) = J^*,$$

where J^* is the guaranteed cost and given in (9e). The system (4) with (2) is stabilizable by sampled-data input (6) with (7) and $K_j = \hat{K}_j \hat{P}_0^{-1}$. \square

In the next results, the optimal guaranteed cost control for system (4) with (2) is provided. The minimization for the cost function in (9e) is given in the following result.

Corollary 1.

Suppose for a given constant $\eta > 0$, the following optimization problem:

$$\text{Minimize} \quad \alpha + \text{trace}(W_1^T \Phi_1 W_1 + W_2^T \Phi_2 W_2 + W_3^T \Phi_3 W_3), \quad (15a)$$

subject to

$$\text{(i)} \quad (9a)-(9c),$$

$$(ii) \begin{bmatrix} -\alpha & x(0)^T \\ x(0) & -\hat{P}_0 \end{bmatrix} < 0, \begin{bmatrix} -2\hat{P}_0 + \hat{P}_1 & I \\ I & -\Phi_1 \end{bmatrix} < 0, \\ \begin{bmatrix} -2\hat{P}_0 + \hat{P}_2 & I \\ I & -\Phi_2 \end{bmatrix} < 0, \begin{bmatrix} -2\hat{P}_0 + \hat{P}_3 & I \\ I & -\Phi_3 \end{bmatrix} < 0, \quad (15b)$$

has a solution with constants $\alpha > 0$, $\varepsilon_i > 0$, positive definite matrices $\hat{P}_0, \hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{Q}_{22}, \hat{R}_{22}, \Phi_1, \Phi_2, \Phi_3 \in \mathfrak{R}^{n \times n}$, $\hat{Q}_{11} \in \mathfrak{R}^{3n \times 3n}$, $\hat{R}_{11} \in \mathfrak{R}^{3n \times 3n}$, matrices $\hat{Q}_{12} \in \mathfrak{R}^{3n \times n}$, $\hat{R}_{12} \in \mathfrak{R}^{3n \times n}$, $\hat{K}_i \in \mathfrak{R}^{p \times n}$, where

$$\int_{-T}^0 x(s)x^T(s)ds = W_1 W_1^T, \int_{-T}^0 (s+T)\dot{x}(s)x^T(s)ds = W_2 W_2^T, \\ \int_{-T}^0 x(s)x^T(s)ds = W_3 W_3^T. \quad (15c)$$

Then the control (6) with (7) and the gain $K_j = \hat{K}_j \hat{P}_0^{-1}$ is the guaranteed cost control of system (4) with (2), and the guaranteed cost is given in (9e) with $P_0 = \hat{P}_0^{-1}$, $P_i = \hat{P}_0^{-1} \hat{P}_i \hat{P}_0^{-1}$.

Proof. By lemma 1, LMIs (15b) are equivalent to

$$x^T(0)P_0 x(0) < \alpha, -2\hat{P}_0 + \hat{P}_k + \Phi_k^{-1} < 0, k \in \{1, 2, 3\}. \quad (16)$$

Note that

$$\begin{bmatrix} \hat{P}_0 & -\Phi_k^{-1} \\ -\Phi_k^{-1} & \hat{P}_0 \end{bmatrix} \begin{bmatrix} \hat{P}_0 & -\Phi_k^{-1} \\ -\Phi_k^{-1} & \hat{P}_0 \end{bmatrix} = \hat{P}_0 \Phi_k \hat{P}_0 - 2\hat{P}_0 + \Phi_k^{-1} \geq 0, k \in \{1, 2, 3\}.$$

The following results are obtained from condition (16):

$$-\hat{P}_0 \Phi_k \hat{P}_0 + \hat{P}_k < 0, k \in \{1, 2, 3\}. \quad (17)$$

Conditions in (17) are equivalent to

$$P_k = \hat{P}_0^{-1} \hat{P}_k \hat{P}_0^{-1} < \Phi_k, k \in \{1, 2, 3\}.$$

Hence we have

$$\int_{-T}^0 x^T(s)P_1 x(s)ds = \text{trace}\left(\int_{-T}^0 x^T(s)P_1 x(s)ds\right) = \text{trace}\left(P_1 \int_{-T}^0 x(s)x^T(s)ds\right) \\ = \text{trace}(P_1 W_1 W_1^T) = \text{trace}(W_1^T P_1 W_1) \leq \text{trace}(W_1^T \Phi_1 W_1), \\ \int_{-T}^0 (s+T)x^T(s)P_2 \dot{x}(s)ds = \text{trace}(W_2^T Q_2 W_2) \leq \text{trace}(W_2^T \Phi_2 W_2), \\ \int_{-T}^0 x^T(s)P_3 x(s)ds = \text{trace}(W_3^T P_3 W_3) \leq \text{trace}(W_3^T \Phi_3 W_3).$$

By the similar formulation of [15-16], we can complete this proof. \square

Remark 1. For a given constant $\eta > 0$, the LMI optimization problem in (15) can be solved by the LMI Toolbox of Matlab. Simple “for loop” can be used to find the minimization of guaranteed cost.

III. GUARANTEED COST CONTROL FOR T-S FUZZY SYSTEMS WITH NONLINEAR AND LINEAR FRACTIONAL PERTURBATIONS

Consider the system (1) with nonlinear and parametric perturbations in the following form:

Rule i :

If $z_1(t)$ is M_{i1} and $\dots z_r(t)$ is M_{ir} , then

$$\dot{x}(t) = \bar{A}_{1i}(t)x(t) + \bar{A}_{2i}(t)x(t-\tau) + \bar{B}_i(t)u(t) + \Delta f_i(x(t)), t \geq 0, \quad (18a)$$

$$x(t) = \varphi_i(t), t \in [-\tau, 0], i = 1, 2, \dots, m, \quad (18b)$$

where $\bar{A}_{1i}(t)$, $\bar{A}_{2i}(t)$, and $\bar{B}_i(t)$ are some system matrices with perturbations and satisfying

$$\begin{bmatrix} \bar{A}_{1i}(t) & \bar{A}_{2i}(t) & \bar{B}_i(t) \end{bmatrix} \\ = \begin{bmatrix} A_{1i} + \Delta A_{1i}(t) & A_{2i} + \Delta A_{2i}(t) & B_i + \Delta B_i(t) \end{bmatrix}, \\ i \in \{1, 2, \dots, m\}, \quad (19a)$$

where constant matrices A_{1i} , A_{2i} , and B_i are given, $\Delta A_{1i}(t)$, $\Delta A_{2i}(t)$, and $\Delta B_i(t)$ are some perturbed matrices and satisfying

$$\begin{bmatrix} \Delta A_{1i}(t) & \Delta A_{2i}(t) & \Delta B_i(t) \end{bmatrix} = M_i \cdot \Delta_i(t) \cdot \begin{bmatrix} N_{1i} & N_{2i} & N_{3i} \end{bmatrix}, \\ i \in \{1, 2, \dots, m\}, \quad (19b)$$

$$\Delta_i(t) = [I - F_i(t)\Theta_i]^{-1} F_i(t), \Theta_i \Theta_i^T < I, \quad (19c)$$

where M_i , N_{ki} , $k \in \{1, 2, 3\}$, and Θ_i are some given constant matrices with appropriate dimensions. $F_i(t)$ is an unknown matrix representing the parameter perturbations which satisfies

$$F_i^T(t)F_i(t) \leq I. \quad (19d)$$

Remark 2. The linear fractional perturbations in (19) are the generalization form of the parametric perturbations in [12].

Lemma 2. [13] Suppose that $\Delta_i(t)$ is defined in (19c) and satisfying (19d), then for real matrices U_i , W_i and X_i with

$X_i = X_i^T$, the following statements are equivalent:

(I) The inequality is satisfied

$$X_i + U_i \Delta_i(t) W_i + W_i^T \Delta_i^T(t) U_i^T < 0,$$

(II) There exists a scalar $\mu_i > 0$, such that

$$\begin{bmatrix} X_i & \mu_i \cdot U_i & W_i^T \\ * & -\mu_i \cdot I & \mu_i \cdot \Theta_i^T \\ * & * & -\mu_i \cdot I \end{bmatrix} < 0, \quad (20)$$

where the matrix Θ_i is defined in (19c).

From Corollary 1 with the fuzzy system in (18) with (2), (19), and Lemma 2, we can obtain the following results.,

Theorem 2.

Suppose for a given constant $\eta > 0$, the following optimization problem:

$$\text{Minimize } \alpha + \text{trace}(W_1^T \Phi_1 W_1 + W_2^T \Phi_2 W_2 + W_3^T \Phi_3 W_3), \quad (21a)$$

subject to

(i) (9a), (9c), (15b),

$$(ii) \tilde{\Pi}_{ij} = \begin{bmatrix} \tilde{\Pi}_{ij11} & \tilde{\Pi}_{ij12} \\ * & \tilde{\Pi}_{ij22} \end{bmatrix}$$

$$+ \begin{bmatrix} \hat{\Omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0,$$

for all $i, j \in \{1, \dots, m\}$,

(22b)

where

$$\tilde{\tilde{\Pi}}_{ij11} =$$

$$\begin{bmatrix} \hat{\Pi}_{11ij} & \hat{\Pi}_{12ij} & 0 & \hat{\Pi}_{14ij} & \hat{\Pi}_{15ij} & 0 & \hat{\Pi}_{17ij} & \hat{\Pi}_{18ij} & \hat{\Pi}_{19ij} \\ * & 0 & 0 & 0 & 0 & \hat{\Pi}_{26ij} & \hat{\Pi}_{27ij} & 0 & 0 \\ * & * & \hat{\Pi}_{33ij} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Pi}_{44ij} & 0 & 0 & \hat{\Pi}_{47ij} & 0 & 0 \\ * & * & * & * & \hat{\Pi}_{55ij} & 0 & \hat{\Pi}_{57ij} & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66ij} & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{77ij} & 0 & 0 \\ * & * & * & * & * & * & * & \hat{\Pi}_{88ij} & 0 \\ * & * & * & * & * & * & * & * & \hat{\Pi}_{99ij} \end{bmatrix},$$

$$\tilde{\tilde{\Pi}}_{ij12} = \begin{bmatrix} \hat{\Pi}_{110ij} & \hat{\Pi}_{111ij} \\ 0 & \hat{\Pi}_{211ij} \\ 0 & 0 \\ 0 & \hat{\Pi}_{411ij} \\ 0 & 0 \\ 0 & 0 \\ \hat{\Pi}_{710ij} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{\tilde{\Pi}}_{ij22} = \begin{bmatrix} -\mu_i \cdot I & \mu_i \cdot \Theta_i^T \\ * & -\mu_i \cdot I \end{bmatrix},$$

has a solution with constants $\alpha > 0$, $\varepsilon_i > 0$, $\mu_i > 0$, positive definite matrices \hat{P}_0 , \hat{P}_1 , \hat{P}_2 , \hat{P}_3 , \hat{Q}_{22} , \hat{R}_{22} , Φ_1 , Φ_2 , $\Phi_3 \in \mathfrak{R}^{n \times n}$, $\hat{Q}_{11} \in \mathfrak{R}^{3n \times 3n}$, $\hat{R}_{11} \in \mathfrak{R}^{3m \times 3m}$, matrices $\hat{Q}_{12} \in \mathfrak{R}^{3n \times n}$, $\hat{R}_{12} \in \mathfrak{R}^{3n \times n}$, $\hat{K}_i \in \mathfrak{R}^{p \times n}$, where matrices W_1 , W_2 , and W_3 are given in (15c), $\hat{\Pi}_{klj}$, $k, l \in \{1, 2, \dots, 9\}$, and $\hat{\Omega}$ are defined in (9d), and $\hat{\Pi}_{110ij} = \hat{\Pi}_{710ij} = \mu_i \cdot M_i$, $\hat{\Pi}_{111ij} = \hat{P}_0 N_{li}^T$, $\hat{\Pi}_{211ij} = -\hat{K}_j^T N_{3i}^T$, $\hat{\Pi}_{411ij} = \hat{P}_0 N_{2i}^T$. Then the control (6) with (7) and the gain $K_j = \hat{K}_j \hat{P}_0^{-1}$ is the guaranteed cost control of the fuzzy system in (18) with (2) and (19), and the guaranteed cost is given in (9e) with $P_0 = \hat{P}_0^{-1}$, $P_i = \hat{P}_0^{-1} \hat{P}_i \hat{P}_0^{-1}$.

Proof. Consider the T-S fuzzy system (18) with (2) and (19),

the stability LMI condition in (9b) should be rewritten as follows:

$$\tilde{\tilde{\Pi}}_{ij} = \begin{bmatrix} \tilde{\tilde{\Pi}}_{11ij} & \tilde{\tilde{\Pi}}_{12ij} & 0 & \tilde{\tilde{\Pi}}_{14ij} & \hat{\Pi}_{15ij} & 0 & \tilde{\tilde{\Pi}}_{17ij} & \hat{\Pi}_{18ij} & \hat{\Pi}_{19ij} \\ * & 0 & 0 & 0 & 0 & \hat{\Pi}_{26ij} & \tilde{\tilde{\Pi}}_{27ij} & 0 & 0 \\ * & * & \hat{\Pi}_{33ij} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Pi}_{44ij} & 0 & 0 & \tilde{\tilde{\Pi}}_{47ij} & 0 & 0 \\ * & * & * & * & \hat{\Pi}_{55ij} & 0 & \hat{\Pi}_{57ij} & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66ij} & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{77ij} & 0 & 0 \\ * & * & * & * & * & * & * & \hat{\Pi}_{88ij} & 0 \\ * & * & * & * & * & * & * & * & \hat{\Pi}_{99ij} \end{bmatrix}$$

$$+ \begin{bmatrix} \hat{\Omega} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\Pi}_{11ij} & \hat{\Pi}_{12ij} & 0 & \hat{\Pi}_{14ij} & \hat{\Pi}_{15ij} & 0 & \hat{\Pi}_{17ij} & \hat{\Pi}_{18ij} & \hat{\Pi}_{19ij} \\ * & 0 & 0 & 0 & 0 & \hat{\Pi}_{26ij} & \hat{\Pi}_{27ij} & 0 & 0 \\ * & * & \hat{\Pi}_{33ij} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Pi}_{44ij} & 0 & 0 & \hat{\Pi}_{47ij} & 0 & 0 \\ * & * & * & * & \hat{\Pi}_{55ij} & 0 & \hat{\Pi}_{57ij} & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66ij} & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{77ij} & 0 & 0 \\ * & * & * & * & * & * & * & \hat{\Pi}_{88ij} & 0 \\ * & * & * & * & * & * & * & * & \hat{\Pi}_{99ij} \end{bmatrix}$$

$$+ \begin{bmatrix} M_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M_i \\ 0 \\ 0 \end{bmatrix} \Delta_i(t) + \begin{bmatrix} \hat{P}_0 N_{li}^T \\ -\hat{K}_j^T N_{3i}^T \\ 0 \\ \hat{P}_0 N_{2i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \hat{P}_0 N_{li}^T \\ -\hat{K}_j^T N_{3i}^T \\ 0 \\ \hat{P}_0 N_{2i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta_j^T(t) + \begin{bmatrix} M_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M_i \\ 0 \\ 0 \end{bmatrix}^T$$

$$+ \begin{bmatrix} \hat{\Omega} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \quad (22)$$

where $\hat{\Pi}_{klj}$, $k, l \in \{1, 2, \dots, 9\}$, and $\hat{\Omega}$ are defined in (9d), and

$$\tilde{\Pi}_{11ij} = \bar{A}_{1i}(t)\hat{P}_0 + \hat{P}_0\bar{A}_{1i}^T(t) + \hat{P}_1 + \hat{P}_3, \quad \tilde{\Pi}_{12ij} = -\bar{B}_i(t)\hat{K}_j,$$

$$\tilde{\Pi}_{14ij} = \bar{A}_{2i}(t)\hat{P}_0, \quad \tilde{\Pi}_{17ij} = \hat{P}_0\bar{A}_{1i}^T(t), \quad \tilde{\Pi}_{27ij} = -\hat{K}_j^T\bar{B}_i^T(t),$$

$$\hat{\Pi}_{47ij} = \hat{P}_0\bar{A}_{2i}^T(t).$$

This proof can be completed in the similar formulation of Theorem 1 and Corollary 1. \square

IV. NUMERICAL EXAMPLE

Consider the T-S fuzzy system in (18) with (2), (19), and the following parameters:

$$\begin{aligned} m = 2, \quad A_{11} &= \begin{bmatrix} -2 & 0.1 \\ 1 & -1.1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \\ A_{21} &= \begin{bmatrix} -1 & 0.2 \\ -0.8 & -0.8 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -0.8 & 0 \\ -1 & -1.1 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0.8 \\ -0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ -0.4 \end{bmatrix}, \quad F_1 = F_2 = 0.1 \cdot I, \\ \Theta_1 = \Theta_2 &= 0.1, \quad \tau = 2, \quad M_1 = M_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \\ N_{11} = N_{12} &= [0.2 \quad 0.1], \quad N_{21} = N_{22} = [0.1 \quad 0.05], \\ N_{31} = N_{32} &= 0.05, \quad S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S_2 = 1, \\ x(t) &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad t \in [-2, 0]. \end{aligned} \quad (23)$$

(A) If the upper bound of sampled time is given by $T = 0.1$, the optimization problem in Theorem 2 with $\eta = 10.4$ has a feasible solution

$$\hat{K}_1 = \hat{K}_2 = [0.4187 \quad -0.1290], \quad \hat{P}_0 = \begin{bmatrix} 0.0894 & -0.0813 \\ -0.0813 & 0.1625 \end{bmatrix}.$$

The fuzzy system in (18) with (2), (19), and (23) is stabilizable by sampled-data input in (6) and (7) with

$$K_1 = \hat{K}_1\hat{P}_0^{-1} = [7.27 \quad 2.8441], \quad K_2 = \hat{K}_2\hat{P}_0^{-1} = [7.27 \quad 2.8441].$$

The guaranteed cost is given by $J^* = 36.35$. In this case, the guaranteed cost performance can be provided via the stabilizing robust control in (6) and (7) with the sampled time less than 0.1 second.

(B) If the upper bound of sampled time is given by $T = 0.2$, the optimization problem in Theorem 2 with $\eta = 5.7$ has a feasible solution

$$\hat{K}_1 = \hat{K}_2 = [0.1388 \quad -0.0122], \quad \hat{P}_0 = \begin{bmatrix} 0.0651 & -0.0578 \\ -0.0578 & 0.1330 \end{bmatrix}.$$

The fuzzy system in (18) with (2), (19), and (23) is stabilizable by sampled-data input in (6) and (7) with

$$K_1 = \hat{K}_1\hat{P}_0^{-1} = [3.3365 \quad 1.3575], \quad K_2 = \hat{K}_2\hat{P}_0^{-1} = [3.3365 \quad 1.3575].$$

The guaranteed cost is given by $J^* = 56.40$. In this case, the guaranteed cost performance can be provided via the stabilizing

robust control in (6) and (7) with the sampled time less than 0.2 second.

V. CONCLUSION

In this paper, the guaranteed cost control problem for a class of uncertain T-S fuzzy time-delay system with sampled-data input has been studied. Based on the LMI optimization approach and time-varying delay transformation technique, some delay-dependent criteria have been proposed to minimize the upper bound of the guaranteed cost for the system with nonlinear and linear fractional perturbations.

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