Robustness of CDC Plan using NBIB Mating Designs against Unavailability of One or More Observations

R. Shunmugathai and M.R. Srinivasan

***Abstract*—Mating Designs are the study of progenies developed through various methods like Diallel Cross plans which are subjected to Incomplete Block Designs. The concept of robustness in designs has been studied and available in the literature. The paper dealt with a class of Nested Balanced Incomplete Block Designs for varying parametric values with unavailability of one or more observations and efficiencies are calculated. The effects of missing blocks on Complete Diallel Cross designs are examined in this study. A-efficiencies based on non -zero eigenvalues suggest that these designs are fairly robust. The investigation shows that Nested Balanced Incomplete Block Designs are fairly robust in terms of efficiency. In this paper, the robustness of Nested Balanced Incomplete Block Design when one or more observations are lost has been discussed.**

***Index Terms*—Nested Balanced Incomplete Block Design; Efficiency of residual design; Mating Design; Youden Square Design; Latin Square Design.**

***MSC 2010 Codes* – 45B05, 51E05.**

I. INTRODUCTION

THE Diallel cross is a type of Mating Design used in plant breeding to study the genetic properties of a set of inbred lines. Suppose there are p inbred lines and it is desired to perform a Diallel Cross experiment involving $p(p-1)/2$ crosses of the type (ij), $i < j, i, j = 1, 2, \dots, p$, the lines are to be compared with respect to their General Combining Abilities. Customarily, Diallel cross experiments of the type mentioned above have been conducted using a Completely Randomised Design or a Randomised Complete Block Design. However, with increase in the number of lines p, the number of crosses in the experiment increases rapidly, and in such a situation, adoption of a Complete Block Design is not appropriate. The designs that are considered compare the inbred lines with respect to their General Combining Abilities. For detailed discussion on these designs and further references, Griffing [12], Gilbert [11], Ghosh and Biswas [7], Hanani [16], Dey and Midha [4] have obtained some Optimal Incomplete Block Designs for Diallel Crosses. Discussions on Design of comparative Experiments are studied by Hinkelmann and Kempthorne [13], Raghavarao Damaraju [22], Atkinson, Donev, and Tobias [1], Hinkelmann and Kempthorne [14] and Bailey [3]. Eric W. Weisstein and [6] discussed the Balanced Incomplete Block Design (BIBD) as a well studied experimental design with desirable features from a statistical perspective.Some methods of constructing these designs were also discussed by Gupta and Kageyama [10], Ghosh and Desai [8], Ghosh and Desai [9], Mausami P. Bhatt [19] making use of Nested Balanced Incomplete Block Designs of Preece [21] to arrive at two series of Optimal Block Designs for Diallel crosses.

Preece [21] has introduced a case of two way elimination of heterogeneity, one nested within the other. He also has introduced a Nested Balanced Incomplete Block Design and gave method of construction of Nested Balanced Incomplete Block Design. Further, he listed a table of available Nested Balanced Incomplete Block Designs. Some experimental units are the half -leaves of plants. There may be more treatments than there are suitable half -leaves per plant, where as there may be variation between plants, between leaves within plants and between half -leaves within leaves. Here, both leaves and plants can be thought as system of blocks, one system Nested within the others. Sudesh Srivastav and Arti Shankar [23] on the construction and existence of a certain class of Complete Diallel Crosses designs as Nested Balanced Incomplete Block Designs with sub-block size two are discussed.

A Nested Balanced Incomplete Block Design is defined as a design with two systems of blocks, the second Nested within the first (each block from the first system containing m blocks from the second), such that ignoring either system leaves a Balanced Incomplete Block Design Whose blocks are those of the other systems. A list is given of such designs having each treatment (or variety) replicated not more than fifteen times. The analysis is given for the general Nested Balanced Incomplete Block Design; this analysis is an extension of that developed by Yates [24] for the recovery of inter-block information in Balanced Incomplete Block Designs. A design based on a Nested Balanced Incomplete Block Design was devised by Kleczkowski [17] for laboratory studies on lesions produced by inoculating bean plants with tobacco necrosis virus.

An arrangement of v treatment each replicated r times in two system of block is said to be a Nested Balanced Incomplete Block Design with parameters $(v, b_1, r, k_1, \lambda_1, b_2, k_2, \lambda_2, m)$ if,

1) Second system is nested within the first, with each block from the first system containing m blocks from the second system (sub blocks).
2) Ignoring the second system leaves a Balanced Incomplete Block Design with b_1 blocks each of k_1 units with λ_1 concurrence.

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3) Ignoring the first system leaves a Balanced Incomplete Block Design with b_2 blocks each of k_2 units with λ_2 concurrences.

The Parametric relationships are given as:

1) $vr = b_1k_1 = b_1k_2m = b_2k_2$.
2) $\lambda_1(v-1) = r(k_1-1) = (v-1)\lambda_2 = r(k_2-1)$.
3) $(\lambda_1 - m_2)(v-1) = r(m-1)$.

Das and Kageyama [5] showed that Balanced Incomplete Block Designs and extended Balanced Incomplete Block Designs are fairly robust against the unavailability of $s(s \leq k)$ observations in any block, while any Youden Design and Latin Square Designs are found to be fairly robust against the loss of any one column. Prescott and Mansson [19] investigated the effect of missing observations on Complete Diallel Cross designs. They examined the robustness of CDC using Balanced Incomplete Block Design and PBIBD. A-efficiencies, based on average variances of the elementary contrasts of the line effects, suggest that Complete Diallel Cross Design is fairly robust against the unavailability of observations. Further the robustness of CDC Design against the loss of a block of observations using BIBD and PBIBD examined by Mansson and Prescott [19].

This paper looks into the robustness of CDC plan when one or more crosses are lost from a block using Nested Balanced Incomplete Block Design. C^*matrix and its non-zero eigenvalues are computed with its corresponding multiplicity and its efficiency. It shows that CDC plans are fairly robust against the loss of one or more crosses from a block. Corresponding C^* matrices and their non-zero eigenvalues with multiplicities are computed for each set of parameters and robustness of CDC designs are examined against the unavailability of one or more crosses from a block.

The robustness criteria against the unavailability of data are:

1) to get the connectedness of the residual design;
2) to have the variance balance of the residual design;
3) to consider the A-efficiency of residual design for the robustness study.

So far, robustness of Incomplete Block Designs and Complete Block Designs are carried out against loss of $s(s \leq k_1)$ observations in one block.

In this investigation, consider a connected CDC plan D. Let D^* be the residual design obtained when one or more observations and two blocks are lost from a design. Assume D^* to be connected. In this case, the criterion of robustness against the unavailability of one or more observations and two blocks is the overall A-efficiency, of the residual design D^*, given by

$$e(s) = \frac{\text{Sum of reciprocals of non-zero eigenvalues of } C}{\text{Sum of reciprocals of non-zero eigenvalues of } C^*}$$

or

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{1}$$

Efficiencies of 43 Nested Balanced Incomplete Block Designs are computed. Thus, it shows that design is fairly robust against loss of one or more observations. C^* matrix and its non-zero eigenvalues are also computed with corresponding multiplicity.

II. C - Matrix of Complete Diallel Crosses Plan

We know that for any block design C matrix can be defined as,

$$C = r\left(I_v - \frac{NN'}{k}\right)$$

Since for Complete Diallel Cross plan C matrix can be given by,

$$C_t = G_{ti} - \frac{NN'}{k}$$

where

$$G_{ti} = \begin{pmatrix} w_{ti} & g'_{ii} \\ - & w'_{ii} \end{pmatrix}$$

and

$$W_{ti} = w_{t'i} = r(k_1 - 1), \quad g_{ii} = 1$$

The NN' matrix of the Complete Diallel Cross Plan can be defined as

$$NN' = \begin{bmatrix} \sum n_{1j}^2 & \sum n_{1j}n_{2j} & \sum n_{1j}n_{vj} \\ \sum n_{2j}n_{1j} & \sum n_{2j}^2 & \sum n_{2j}n_{vj} \\ \sum n_{vj}n_{1j} & \sum n_{vj}n_{2j} & \sum n_{vj}^2 \end{bmatrix}$$

Thus it is obvious that for this Complete Diallel Cross Plan,

$$\sum n_{vj}^2 = r(k_1-1)^2, \quad \sum n_{ij}n_{mj} = \lambda_1(k_1-1)^2$$

$k = \frac{k_1(k_1-1)}{2}$

Now C matrix is given as,

$$C = \begin{bmatrix} r(k_1-1) & \lambda_1 & \lambda_1 \\ \lambda_1 & r(k_1-1) & \lambda_1 \\ \lambda_1 & \lambda_1 & r(k_1-1) \end{bmatrix} - \frac{\begin{bmatrix} r(k_1-1)^2 & \lambda_1(k_1-1) & \lambda_1(k_1-1) \\ \lambda_1(k_1-1) & r(k_1-1)^2 & \lambda_1(k_1-1) \\ \lambda_1(k_1-1) & \lambda_1(k_1-1) & r(k_1-1)^2 \end{bmatrix}}{\frac{k_1(k_1-1)}{2}}$$

$$C = \theta(I_v - \frac{E_{vv}}{v})$$

The non - zero eigenvalues of C matrix and its corresponding multiplicity of Complete Diallel Cross Plan can be given by,

$$\theta = \frac{\lambda_1 v(k_1-2)}{k_1}$$

with multiplicity $(v-1)$.

III. Robustness of Complete Diallel Cross Plan using NBIB Design against the unavailability of one or more observations

Complete Diallel Crosses Plan was considered Nested Balanced Incomplete Block Design D having parameters $v = p,\ b,\ r,\ k,\ \lambda_1,\ \lambda_2\ m,\ n$. Now consider treatment of the Nested Balanced Incomplete Block Designs as lines and cross them between the lines in each block. This result in Complete Diallel Crosses Plan that involves p line with $k_1(k_1-1)/2$ crosses, each cross repeated λ_1 times. Without loss of generality one or more cross is lost from Complete Diallel Crosses Plan, call this design as a residual design and assume that the residual design D^* is connected. Situation can be treated by separating into three cases:

Case i: $s = 1$
Case ii: $s = 2$
Case iii: $s = k_1$

For all the three cases, the lines between one or more observations are lost in a Nested Balanced Incomplete Block Design. The efficiency factor depends upon the lines between one or more observations. To find the efficiency for all the three cases, the lines between one or more observations are $0, 1, 2, 3, \ldots, (k_1 - 1), k_1$ respectively are studied. Here, the robustness criterion of Nested Balanced Incomplete Block Design is further discussed for the different value of one or more observations.

Case (i): $s = 1$

Consider a Nested Balanced Incomplete Block Design D having parameters $v, b_1, r, k_1, \lambda_1, b_2, k_2, \lambda_2, m$. The Cmatrix of the design is given by

$$C = \theta(I_v - \frac{E_{vv}}{v})$$

where

$$\theta = \lambda_1\nu(k_1 - 2)/k$$

is the eigenvalues of C matrix of design D with multiplicity $(v - 1)$.

Let from this design $s = 1$ line be lost. Call this design as a residual design and assume that the residual design D^* is connected. Let C^* be the information matrix of the residual design be C^* given as,

$$C^* = R^* - N^* K_1^{-1} N'^*$$

The C^* matrix of design D can be written as,

$$k_1(k_1 - 2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where

$\varepsilon_{11} = (k_1 - 2)(\lambda_1\nu - k_1)I_1 - (k_1 - 2)(\lambda_1 - 1)J_{11}$
$\varepsilon_{12} = -(k_1 - 2)\lambda_1 J_{1(\nu - k_1)}$
$\varepsilon_{13} = -(k_1 - 2)(\lambda_1 - 1)J_{1(k_1 - 2)}$
$\varepsilon_{21} = -(k_1 - 2)(\lambda_1)J_(\nu - k_1)(1)$
$\varepsilon_{22} = (k_1 - 2)(\lambda_1\nu)I_(\nu - k_1) - (k_1 - 2)(\lambda_1)J_{(\nu - k_1)(\nu - k_1)}$
$\varepsilon_{23} = (k_1 - 2) - \lambda_1 J_{(\nu - k_1)(k_1 - 2)}$
$\varepsilon_{31} = -(k_1 - 2)(\lambda_1 - 1)J_{(k_1 - 2)(1)}$
$\varepsilon_{32} = -(k_1 - 2)\lambda_1 J_{(k_1 - 2)(\nu - k_1)}$
$\varepsilon_{33} = (k_1 - 2)\lambda_1\nu(I_{(k_1 - 2)} - (k_1 - 2)\lambda_1 + sJ_{(k_1 - 2)(k_1 - 2)})$

The non zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k_1-2)(\lambda_1\nu - k_1)}{k_1}$, with multiplicity 1.
2) $\frac{(k_1-2)\lambda_1\nu}{k_1}$, with multiplicity $(v - 2)$.

Theorem 1: Nested Balanced Incomplete Block Designs with parameters $v, b_1,$
$r, k_1, \lambda_1, b_2, k_2, \lambda_2, m$ are fairly robust against the unavailability of $s = 1$ line, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda_1\nu - k_1)(\nu - 1)}{(\lambda_1\nu) + (\nu - 2) + (\lambda_1\nu - k_1)} \quad (2)$$

Proof: Without loss of generality, let $s = 1$ line be lost from the Nested Balanced Incomplete Block Design, C^* matrix of the residual design is given by,

$$k_1(k_1 - 2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where

$$\begin{aligned}
\varepsilon_{11} &= (k_1 - 2)(\lambda_1\nu - k_1)I_1 - (k_1 - 2)(\lambda_1 - 1)J_{11} \\
\varepsilon_{12} &= -(k_1 - 2)\lambda_1 J_{1(\nu - k_1)} \\
\varepsilon_{13} &= -(k_1 - 2)(\lambda_1 - 1)J_{1(k_1 - 2)} \\
\varepsilon_{21} &= -(k_1 - 2)(\lambda_1)J_(\nu - k_1)(1) \\
\varepsilon_{22} &= (k_1 - 2)(\lambda_1\nu)I_(\nu - k_1) \\
&\quad -(k_1 - 2)(\lambda_1)J_{(\nu - k_1)(\nu - k_1)} \\
\varepsilon_{23} &= (k_1 - 2) - \lambda_1 J_{(\nu - k_1)(k_1 - 2)} \\
\varepsilon_{31} &= -(k_1 - 2)(\lambda_1 - 1)J_{(k_1 - 2)(1)} \\
\varepsilon_{32} &= -(k_1 - 2)\lambda_1 J_{(k_1 - 2)(\nu - k_1)} \\
\varepsilon_{33} &= (k_1 - 2)\lambda_1\nu(I_{(k_1 - 2)} \\
&\quad -(k_1 - 2)\lambda_1 + sJ_{(k_1 - 2)(k_1 - 2)})
\end{aligned}$$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k_1-2)(\lambda_1\nu - k_1)}{k_1}$, with multiplicity 1.
2) $\frac{(k_1-2)\lambda_1\nu}{k_1}$, with multiplicity $(v - 2)$.

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \quad (3)$$

Where, $\phi_2(s)$ = sum of reciprocals of non-zero eigenvalues of C matrix of design D and $\phi_1(s)$ = sum of reciprocals of non-zero eigenvalues of C^* matrix of design D*. That is,

$$\phi_2(s) = \frac{k_1(\nu - 1)}{\lambda_1\nu}(k_1 - 2) \quad (4)$$

$$\phi_1(s) = \frac{k_1}{(k_1 - 2)(\lambda_1\nu - k_1)} + \frac{(\nu - 2)(k_1)}{(k_1 - 2)(\lambda_1\nu)}. \quad (5)$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(\lambda_1\nu - k_1)(\nu - 1)}{(\lambda_1\nu) + (\nu - 2) + (\lambda_1\nu - k_1)} \quad (6)$$

Example 1: Let D represent the Nested Balanced Incomplete Block Design with parameters $v = p = 7,\ b_1 = 7,\ b_2 = 14,\ r = 4,\ k_1 = 4,\ k_2 = 2,\ \lambda_1 = 2$. Design D is given by Table I.

Here, when $s = 1$ line is lost from a block 2, C^* matrix of residual design is given by,

$$8C^* = \begin{bmatrix} 27 & -6 & -6 & -6 & -6 & -3 & -3 \\ -6 & 36 & -6 & -6 & -6 & -6 & -6 \\ -6 & -6 & 36 & -6 & -6 & -6 & -6 \\ -6 & -6 & -6 & 36 & -6 & -6 & -6 \\ -3 & -6 & -6 & -6 & 35 & -7 & -7 \\ -3 & -6 & -6 & -6 & -7 & 35 & -7 \\ -3 & -6 & -6 & -6 & -7 & -7 & 35 \end{bmatrix}$$

TABLE I
NESTED BALANCED INCOMPLETE BLOCK DESIGN

Block	NBIB Design	Crosses in the NBIB Design				
1	1	2	3	6	1 × 6	2 × 3
2	2	3	4	7	2 × 7	3 × 4
3	3	4	5	1	1 × 3	4 × 5
4	4	5	6	2	2 × 4	5 × 6
5	5	6	7	3	3 × 5	6 × 7
6	6	7	1	4	4 × 6	1 × 7
7	7	1	2	5	5 × 7	1 × 2

The non-zero eigenvalues with their corresponding multiplicities are,

1) $\frac{30}{4}$, with multiplicities 1.
2) $\frac{42}{4}$, with multiplicities 5.

The overall A- efficiency of the design is, $e(s) = 0.984615$.

Case (ii): $s = 2$

Consider a Nested Balanced Incomplete Block Design D having parameters $v, b_1, r, k_1, \lambda_1, b_2, k_2, \lambda_2, m$.

Let from this design $s = 2$ line be lost. Call this design as a residual design and assume that the residual design D^* is connected. Let C^* be the information matrix of the residual design be C^* given as,

$$C^* = R^* - N^* K_1^- 1 N'^*$$

The C^* matrix of design D can be written as,

$$k_1(k_1-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where

$$\begin{aligned} \varepsilon_{11} &= (k_1-2)(\lambda_1\nu - k_1)I_2 - (k_1-2)(\lambda_1-1)J_{22} \\ \varepsilon_{12} &= -(k_1-2)\lambda_1 J_{(2)(\nu-k_1)} \\ \varepsilon_{13} &= -(k_1-2)(\lambda_1-1)J_{2(k_1-2)} \\ \varepsilon_{21} &= -(k_1-2)(\lambda_1)J_(\nu-k_1)(2) \\ \varepsilon_{22} &= (k_1-2)(\lambda_1\nu)I_(\nu-k_1) \\ &\quad -(k_1-2)(\lambda_1)J_{(\nu-k_1)(\nu-k_1)} \\ \varepsilon_{23} &= (k_1-2)-\lambda_1 J_{(\nu-k_1)(k_1-2)} \\ \varepsilon_{31} &= -(k_1-2)(\lambda_1-1)J_{(k_1-2)(2)} \\ \varepsilon_{32} &= -(k_1-2)\lambda_1 J_{(k_1-2)(\nu-k_1)} \\ \varepsilon_{33} &= (k_1-2)\lambda_1\nu(I_{(k_1-2)} \\ &\quad -(k_1-2)\lambda_1 + sJ_{(k_1-2)(k_1-2)} \end{aligned}$$

The non zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k_1-2)(\lambda_1\nu-k_1)}{k_1}$, with multiplicity 2.
2) $\frac{(k_1-2)\lambda_1\nu}{k_1}$, with multiplicity $(v-3)$.

Theorem 2: Nested Balanced Incomplete Block Designs with parameters $v, b_1, r, k_1, \lambda_1, b_2, k_2, \lambda_2, m$ are fairly robust against the unavailability of $s = 2$ line, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda_1\nu - k_1)(\nu-1)}{(2\lambda_1\nu) + (\nu-3) + (\lambda_1\nu - k_1)} \quad (7)$$

Proof: Without loss of generality, let $s = 2$ line be lost from the Nested Balanced Incomplete Block Design, C^* matrix of the residual design is given by,

$$k_1(k_1-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where

$$\begin{aligned} \varepsilon_{11} &= (k_1-2)(\lambda_1\nu - k_1)I_2 - (k_1-2)(\lambda_1-1)J_{22} \\ \varepsilon_{12} &= -(k_1-2)\lambda_1 J_{(2)(\nu-k_1)} \\ \varepsilon_{13} &= -(k_1-2)(\lambda_1-1)J_{2(k_1-2)} \\ \varepsilon_{21} &= -(k_1-2)(\lambda_1)J_(\nu-k_1)(2) \\ \varepsilon_{22} &= (k_1-2)(\lambda_1\nu)I_(\nu-k_1) \\ &\quad -(k_1-2)(\lambda_1)J_{(\nu-k_1)(\nu-k_1)} \\ \varepsilon_{23} &= (k_1-2)-\lambda_1 J_{(\nu-k_1)(k_1-2)} \\ \varepsilon_{31} &= -(k_1-2)(\lambda_1-1)J_{(k_1-2)(2)} \\ \varepsilon_{32} &= -(k_1-2)\lambda_1 J_{(k_1-2)(\nu-k_1)} \\ \varepsilon_{33} &= (k_1-2)\lambda_1\nu(I_{(k_1-2)} \\ &\quad -(k_1-2)\lambda_1 + sJ_{(k_1-2)(k_1-2)} \end{aligned}$$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k_1-2)(\lambda_1\nu-k_1)}{k_1}$, with multiplicity 2.
2) $\frac{(k_1-2)\lambda_1\nu}{k_1}$, with multiplicity $(v-3)$.

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \quad (8)$$

That is,

$$\begin{aligned} \phi_2(s) &= \frac{k_1(\nu-1)}{\lambda_1\nu}(k_1-2) \\ \phi_1(s) &= \frac{2k_1}{(k_1-2)(\lambda_1\nu-k_1)} + \frac{(\nu-3)(k_1)}{(k_1-2)(\lambda_1\nu)} \end{aligned} \quad (9)$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(\lambda_1\nu - k_1)(\nu-1)}{(2\lambda_1\nu) + (\nu-3) + (\lambda_1\nu - k_1)} \quad (10)$$

Example 2: Let D represent the Nested Balanced Incomplete Block Design with parameters $v = p = 7,\ b_1 = 7,\ b_2 = 14,\ r = 4,\ k_1 = 4,\ k_2 = 2,\ \lambda_1 = 2$. Design D is given by Table II.

TABLE II
NESTED BALANCED INCOMPLETE BLOCK DESIGN

Block	NBIB Design	Crosses in the NBIB Design				
1	1	2	3	6	1 × 6	2 × 3
2	2	3	4	7	**2** × **7**	3 × 4
3	3	4	5	1	1 × 3	4 × 5
4	4	5	6	2	2 × 4	5 × 6
5	5	6	7	3	**3** × **5**	6 × 7
6	6	7	1	4	4 × 6	1 × 7
7	7	1	2	5	5 × 7	1 × 2

Here, when $s = 2$ line is lost from a block 2 and block 5 C^* matrix of residual design is given by,

$$8C^* = \begin{bmatrix} 18 & -2 & -4 & -4 & -4 & -2 & -2 \\ -2 & 18 & -4 & -4 & -4 & -2 & -2 \\ -4 & -4 & 24 & -4 & -4 & -4 & -4 \\ -4 & -4 & -4 & 24 & -6 & -6 & -6 \\ -4 & -4 & -4 & -4 & 24 & -4 & -4 \\ -2 & -2 & -4 & -4 & -4 & 22 & -6 \\ -2 & -2 & -4 & -4 & -4 & -6 & 22 \end{bmatrix}$$

The non-zero eigenvalues with their corresponding multiplicities are,

1) $\frac{20}{4}$, with multiplicities 2.
2) $\frac{28}{4}$, with multiplicities 4.

The overall A- efficiency of the design is, $e(s) = 0.969697$.

Case (iii): $s = k_1$

Consider a Nested Balanced Incomplete Block Design D having parameters $v, b_1, r, k_1, \lambda_1, b_2, k_2, \lambda_2, m$.

Let from this design $s = k_1$ line be lost. Call this design as a residual design and assume that the residual design D^* is connected.

Let C^* be the information matrix of the residual design be C^* given as,

$$C^* = R^* - N^* K_1^- 1 N'^*$$

The C^* matrix of design D can be written as,

$$k_1(k_1-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix}$$

where

$$\begin{aligned} \varepsilon_{11} &= (k_1-2)(\lambda_1\nu - k_1)I_{k_1} - (k_1-2)(\lambda_1-1)J_{k_1k_1} \\ \varepsilon_{12} &= -(k_1-2)\lambda_1 J_{(k_1)(\nu-k_1)} \\ \varepsilon_{21} &= -(k_1-2)(\lambda_1)J_(\nu-k_1)(k_1) \\ \varepsilon_{22} &= (k_1-2)(\lambda_1\nu)I_(\nu-k_1) \\ &\quad -(\lambda_1)\nu^{-1}(k_1-2)J_{(\nu-k_1)(\nu-k_1)} \end{aligned}$$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k_1-2)(\lambda_1\nu-k_1)}{k_1}$, with multiplicity $k_1 - 1$.
2) $\frac{(k_1-2)\lambda_1\nu}{k_1}$, with multiplicity $(v - k_1)$.

Theorem 3: Nested Balanced Incomplete Block Designs with parameters $v, b_1, r, k_1, \lambda_1, b_2, k_2, \lambda_2, m$ are fairly robust against the unavailability of $s = k_1$ line, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda_1\nu - k_1)(\nu-1)}{(\lambda_1\nu)(k_1-1) + (\nu-k_1)(\lambda_1\nu - k_1)} \quad (11)$$

Proof: Without loss of generality, when $s = k_1$ lines is lost from the Partially Balanced Incomplete Block Design, C^* matrix of the residual design is given by,

$$k_1(k_1-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix}$$

where

$$\begin{aligned} \varepsilon_{11} &= (k_1-2)(\lambda_1\nu - k_1)I_{k_1} - (k_1-2)(\lambda_1-1)J_{k_1k_1} \\ \varepsilon_{12} &= -(k_1-2)\lambda_1 J_{(k_1)(\nu-k_1)} \\ \varepsilon_{21} &= -(k_1-2)(\lambda_1)J_(\nu-k_1)(k_1) \\ \varepsilon_{22} &= (k_1-2)(\lambda_1\nu)I_(\nu-k_1) \\ &\quad -(\lambda_1)\nu^{-1}(k_1-2)J_{(\nu-k_1)(\nu-k_1)} \end{aligned}$$

The nonzero eigenvalues of C^* matrix with their corresponding multiplicities are,

1) $\frac{(k_1-2)(\lambda_1\nu-k_1)}{k_1}$, with multiplicity $k_1 - 1$.
2) $\frac{(k_1-2)\lambda_1\nu}{k_1}$, with multiplicity $(v - k_1)$.

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \quad (12)$$

That is,

$$\begin{aligned} \phi_2(s) &= \tfrac{k_1(\nu-1)}{\lambda_1\nu}(k_1-2) \\ \phi_1(s) &= \tfrac{k_1}{k_1-1}(k_1-2)(\lambda_1\nu - k_1) + \tfrac{(\nu-1)(k_1)}{(k_1-2)(\lambda_1\nu)} \end{aligned} \quad (13)$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(\lambda_1\nu - k_1)(\nu-1)}{(\lambda_1\nu)(k_1-1) + (\nu-k_1)(\lambda_1\nu - k_1)} \quad (14)$$

Example 3: Let D represent the Nested Balanced Incomplete Block Design with parameters $v = p = 7,\ b_1 = 7,\ b_2 = 14,\ r = 4,\ k_1 = 4,\ k_2 = 2,\ \lambda_1 = 2$. Design D is given by Table III.

Here, when block 7 is lost, C^* matrix of residual design is given by,

$$8C^* = \begin{bmatrix} 18 & -2 & -2 & -2 & -4 & -4 & -4 \\ -2 & 18 & -2 & -2 & -4 & -4 & -4 \\ -2 & -2 & 18 & -2 & -4 & -4 & -4 \\ -2 & -2 & -2 & 18 & -4 & -4 & -4 \\ -4 & -4 & -4 & -4 & 24 & -4 & -4 \\ -4 & -4 & -4 & -4 & -4 & 24 & -4 \\ -4 & -4 & -4 & -4 & -4 & -4 & 24 \end{bmatrix}$$

TABLE III
NESTED BALANCED INCOMPLETE BLOCK DESIGN

Block	NBIB Design	Crosses in the NBIB Design				
1	1	2	3	6	1×6	2×3
2	2	3	4	7	2×7	3×4
3	3	4	5	1	1×3	4×5
4	4	5	6	2	2×4	5×6
5	5	6	7	3	3×5	6×7
6	6	7	1	4	4×6	1×7
7	**7**	**1**	**2**	**5**	$\mathbf{5 \times 7}$	$\mathbf{1 \times 2}$

The non-zero eigenvalues with their corresponding multiplicities are,

1) $\frac{20}{4}$, with multiplicities 3.
2) $\frac{28}{4}$, with multiplicities 3.

The overall A- efficiency of the design is, $e(s) = 0.969697$.

CONCLUSION

Various methods like Diallel Cross plans which are subjected to Incomplete Block Designs are developed for Mating Designs. Robustness of Complete Diallel Cross Plan is examined using Nested Balanced Incomplete Block Design. There are varying Nested Balanced Incomplete Block Designs for different parametric values with unavailability of one or more observations. Results show that from a Nested Balanced Incomplete Block Designs, efficiency for $s = 1$ will be more than $s = 2$ and $s = k$ with the same parameters. The theoretical results show that for one or more missing crosses from either a Complete Diallel Cross, numerical results for these designs are reasonably robust. In the examples with one or more crosses missing, the efficiency occurs between 90 - 99 percentage. The efficiency is obtained and the cases are compared to determine the robustness of the Nested Balanced Incomplete Block Designs. It appears that Nested Balanced Incomplete Block Designs are fairly robust against the unavailability of one or more observations corresponding to the same test treatment. Preece [15], Sudesh, Srivastav and Arti Shankar [16] and Mausami P. Bhatt [12] have listed a series of Nested Balanced Incomplete Block Designs potential for construction of Diallel Mating Crosses. This paper the robustness of the designs are examined and listed. It may be observed that the Nested Balanced Incomplete Block Mating Design is fairly robust for different set of parameters with one or more observations are lost in a block.

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TABLE IV
EFFICIENCY TABLE WHEN TWO BLOCKS IS LOST FROM A PARTIALLY BALANCED INCOMPLETE BLOCK DESIGN

D.N	$v = p$	b_1	b_2	r	k_1	k_2	λ	Case i	Case ii	Case iii
1	5	5	10	4	4	2	3	0.963	0.929	0.963
2	7	7	21	6	6	2	5	0.990	0.980	0.990
3	7	7	14	6	6	3	5	0.985	0.970	0.970
4	8	14	28	7	4	2	3	0.987	0.975	0.987
5	9	18	36	8	4	2	3	0.990	0.980	0.990
6	9	12	24	8	6	3	5	0.991	0.982	0.982
7	9	9	36	8	8	2	7	0.996	0.992	0.996
8	9	9	13	8	8	4	7	0.992	0.983	0.975
9	10	15	45	9	6	2	5	0.995	0.991	0.995
10	10	15	45	9	6	3	5	0.993	0.986	0.986
11	10	10	30	9	9	3	8	0.996	0.991	0.991
12	6	15	30	10	4	2	6	0.988	0.977	0.988
13	11	11	55	10	10	2	9	0.998	0.996	0.998
14	11	11	22	10	10	2	9	0.998	0.996	0.998
15	12	33	66	11	4	2	3	0.995	0.989	0.995
16	12	22	66	11	6	2	5	0.997	0.994	0.997
17	12	22	44	11	6	3	5	0.995	0.991	0.991
18	7	21	42	12	4	2	6	0.992	0.984	0.992
19	13	39	78	12	4	2	3	0.996	0.991	0.996
20	13	26	78	12	6	2	3	0.996	0.991	0.996
21	13	26	52	12	6	3	3	0.993	0.986	0.986
22	13	13	78	12	12	4	11	0.998	0.995	0.993
23	13	13	52	12	12	3	11	0.998	0.996	0.996
24	13	13	49	12	12	4	11	0.998	0.995	0.993
25	13	13	26	12	12	4	11	0.998	0.995	0.993
26	15	35	105	14	6	2	3	0.997	0.993	0.997
27	15	35	70	14	6	3	3	0.995	0.990	0.990
28	15	21	105	14	10	2	9	0.999	0.998	0.999
29	15	21	42	14	10	2	9	0.999	0.998	0.999
30	15	15	105	14	14	2	13	0.999	0.999	0.999
31	15	15	30	14	14	2	13	0.999	0.999	0.999
32	16	60	120	15	4	2	3	0.997	0.994	0.997
33	16	40	120	15	6	2	5	0.998	0.997	0.998
34	16	40	80	15	6	3	5	0.997	0.995	0.995
35	16	30	60	15	8	4	7	0.998	0.995	0.993
36	16	24	120	15	10	2	9	0.999	0.998	0.999
37	16	24	48	15	10	5	9	0.998	0.995	0.990
38	16	20	120	15	12	2	11	0.999	0.998	0.999
39	16	20	80	15	12	3	11	0.999	0.998	0.998
40	16	20	60	15	12	4	11	0.998	0.997	0.995
41	16	20	40	15	12	6	11	0.998	0.995	0.988
42	16	16	80	15	15	3	14	0.999	0.998	0.998
43	16	16	48	15	15	5	14	0.998	0.997	0.994