Existence Results for Abstract Mixed Type Impulsive Fractional Semilinear Evolution Equations

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***Abstract*—In this paper, we study the existence and uniqueness for PC-mild solutions for abstract Cauchy problms for a class of mixed type semilinear impulsive fractional evolution equations. The results are obtained by using semigroup theory, probability density functions via impulsive conditions, fractional calculus and fixed point theorems.**

***Index Terms*—Impulsive conditions, Caputo fractional derivative, PC-mild solution, Fixed point theorems.**

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I. INTRODUCTION

The theory of impulsive differential equations has become an important area of investigation in the past two decades because of their applications to various problems arising in communications, control technology, impact mechanics, electrical engineering and medical domains. In these models, the investigated simulating processes and phenomena usually have short-time perturbations are considered to take place instantaneously in the form of impulses. For a wide bibliography and exposition on this object see for instance the monographs of [6, 8, 16, 26, 36] and the papers of [1–5, 9, 14, 19, 20, 29, 30].

On the other hand, fractional differential and integrodifferential equations with (or without) impulsive conditions arise from various real processes and phenomena appeared in physics, chemical technology, materials, earthquake analysis, robots, electric fractal network, statistical mechanics, biotechnology, medicine and economics. They have in recent years been an object of investigations with much increasing interest. For more details, one can see the monographs of [17, 21–23, 28] and the papers of [10, 15, 24, 25, 27, 31–35, 37–40].

In particular, X. B. Shu et al. [27] studied the existence of mild solutions for a class of impulsive fractional partial differential equations by using fixed point theorem and assumed that, A is the sectorial operator, and Jaydev Dabas et al. [11] extended the results of [27] into semilinear fractional order differential equations with infinite delay, by assuming that A is the sectorial operator. Very recently, JinRong Wang et al. [35] studied the existence of solutions of impulsive fractional differential equations with nonlocal conditions by using the fixed point techniques and assuming that A is the infinitesimal generator of strongly continuous semigroup $\{T(t),\ t \geq 0\}$ and also they introduced the new concept of solutions of impulsive fractional evolution equations.

Motivated by above mentioned works [4, 35, 37], the purpose of this paper is to study the existence results for the abstract mixed type impulsive fractional semilinear evolution equation of the form:

$$^{C}D_t^{\alpha}x(t) = Ax(t) + f(t, x(t), Tx(t), Sx(t)),\ \alpha \in (0, 1],$$
$$0 \leq t \leq T_0,\ t \neq t_k, \qquad (1.1)$$
$$x(t_k^+) = x(t_k^-) + y_k, \quad k = 1, 2, ..., m, \qquad (1.2)$$
$$x(0) = x_0, \qquad (1.3)$$

where $^{C}D_t^{\alpha}$ is the Caputo fractional derivative of order α, $A : D(A) \subset X \to X$ is the generator of a C_0 semigroup $\{T(t), t \geq 0\}$ on a Banach space X, $f \in C([0, T_0] \times X \times X \times X, X)$ is continuous, x_0, y_k are the elements of X,

$$Tx(t) = \int_0^t K(t, s)x(s)ds, \quad K \in C[D, R^+],$$
$$Sx(t) = \int_0^{T_0} H(t, s)x(s)ds, \quad H \in C[D_0, R^+],$$

$D = \{(t, s) \in R^2 : 0 \leq s \leq t \leq T_0\}$, $D_0 = \{(t, s) \in R^2 : 0 \leq t, s \leq T_0\}$ and $0 = t_0 < t_1 < t_2 < .... < t_m < t_{m+1} = T_0$, $x(t_k^+) = \lim_{h \to 0} x(t_k + h)$ and $x(t_k^-) = x(t_k)$ represent respectively the right and left limits of $x(t)$ at $t = t_k$. And also, we extend the results into the nonlocal Cauchy problem $(1.1) - (1.2)$, with

$$x(0) = x_0 + g(x), \qquad (1.4)$$

where g is a function and constitutes a nonlocal Cauchy problem. The results are obtained by using Banach contraction mapping principle, Schauder's fixed point theorem, Schaefer's fixed point theorem and Krasnoselskii's fixed point theorem. Hence, our results generalize the main results in [35].

The rest of this paper is organized as follows: In section 2, we give some notations and recall some concepts and preparation results. In section 3, we establish the existence results of PC-mild solutions for fractional impulsive mixed Cauchy problems $(1.1) - (1.3)$. In section 4, the existence results of PC-mild solutions for fractional impulsive mixed nonlocal Cauchy problems $(1.1) - (1.2)$ with (1.4).

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II. Preliminaries

In this section, we mention a notations, definitions, lemmas and preliminary facts needed to establish our main results.

Let $L_{T_0}(X)$ be the Banach space of all linear and bounded operators on X. For a C_0 semigroup $\{T(t), t \geq 0\}$ on X, we set $M = \sup_{t\in[0,T_0]} \|T(t)\|_{L_{T_0}(X)}$. Let $C([0,T_0],X)$ be the Banach space of all X-valued continuous functions from $J = [0,T_0]$ into X endowed with the norm $\|x\|_C = \sup_{t\in[0,T_0]} \|x(t)\|$. And the set of functions $PC([0,T_0],X) = \{x : [0,T_0] \to X|\ x$ is continuous at $t \in [0,T_0]\backslash\{t_1,t_2,....,t_m\}$, and x is continuous from left and has right hand limits at $t \in \{t_1,t_2,....,t_m\}\}$. Endowed with the norm

$$\|x\|_{PC} = \max \{\sup_{t\in J} \|x(t+0)\|,\ \sup_{t\in J} \|x(t-0)\|\},$$

it is easy to see $(PC([0,T_0],X), \|\cdot\|_{PC})$ is a Banach space.

Let us recall the following known definitions and lemmas. For more details see [21].

Definition 1. *The fractional integral of order α with the lower limit zero for a function f is defined as*

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(s)}{(t-s)^{1-\alpha}} ds, \quad t > 0, \quad \alpha > 0,$$

provided the right hand-side is point-wise defined on $[0,\infty)$, where $\Gamma(\cdot)$ is the gamma function, which is defined by $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$.

Definition 2. *The Riemann-Liouville fractional derivative of order* $\alpha > 0,\ n-1 < \alpha < n,\ n \in N$, *is defined as*

$$^{(R-L)}D^\alpha_{0+} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_0^t (t-s)^{n-\alpha-1} f(s) ds,$$

where the function $f(t)$ has absolutely continuous derivative up to order $(n-1)$.

Definition 3. *The Caputo derivative of order α for a function* $f : [0,\infty) \to R$ *can be written as*

$$D^\alpha f(t) = D^\alpha \left(f(t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} f^{(k)}(0) \right), \quad t > 0, \quad n-1 < \alpha < n.$$

Remark 1. *(i)If $f(t) \in C^n[0,\infty)$, then*

$$^C D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(s)}{(t-s)^{\alpha+1-n}} ds = I^{n-\alpha} f^{(n)}(t),\ t > 0,\ n-1 < \alpha < n.$$

(ii) The Caputo derivative of a constant is equal to zero.
(iii) If f is an abstract function with values in X, then integrals which appear in Definition 1 and 2 are taken in Bochner's sense.

Definition 4 ([35]). *By a PC-mild solution of the system $(1.1)-(1.3)$, we mean the function $x \in PC([0,T_0],X)$ which satisfies the following integral equation*

$$x(t) = \begin{cases} \mathcal{T}(t)x_0 \\ + \int_0^t (t-s)^{\alpha-1} \mathcal{S}(t-s) f(s, x(s), Tx(s), Sx(s))\, ds, & t \in [0,t_1], \\ \mathcal{T}(t)x_0 + \mathcal{T}(t-t_1)y_1 \\ + \int_0^t (t-s)^{\alpha-1} \mathcal{S}(t-s) f(s, x(s), Tx(s), Sx(s))\, ds, & t \in (t_1,t_2], \\ \vdots \\ \mathcal{T}(t)x_0 + \sum_{i=1}^m \mathcal{T}(t-t_i)y_i \\ + \int_0^t (t-s)^{\alpha-1} \mathcal{S}(t-s) f(s, x(s), Tx(s), Sx(s))\, ds, & t \in (t_m,b], \end{cases}$$

where

$$\mathcal{T} = \int_0^\infty \xi_\alpha(\theta) T(t^\alpha\theta) d\theta, \qquad \mathcal{S} = \alpha \int_0^\infty \theta \xi_\alpha(\theta) T(t^\alpha\theta) d\theta,$$

$$\xi_\alpha(\theta) = \frac{1}{\alpha} \theta^{-1-\frac{1}{\alpha}} \overline{w}_\alpha(\theta^{-\frac{1}{\alpha}}) \geq 0,$$

$$\overline{w}_\alpha(\theta) = \frac{1}{\pi} \sum_{n=1}^\infty (-1)^{n-1} \theta^{-n\alpha-1} \frac{\Gamma(n\alpha+1)}{n!} sin(n\pi\alpha), \theta \in (0,\infty),$$

ξ_α is a probability density function defined on $(0,\infty)$, that is

$$\xi_\alpha(\theta) \geq 0, \quad \theta \in (0,\infty) \quad \text{and} \quad \int_0^\infty \xi_\alpha(\theta) d\theta = 1.$$

Lemma 1. *A measurable function $f : J \to X$ is Bochner integrable if $\|f\|$ is Lebesuge integrable.*

Lemma 2. *For $\sigma \in (0,1]$ and $0 < a \leq b$, we have $|a^\sigma - b^\sigma| \leq (b-a)^\sigma$.*

Lemma 3. *(Holder's Inequality). Assume that $r, p \geq 1$, and $\frac{1}{r} + \frac{1}{p} = 1$. If $l \in L^r(J,R)$, then for $1 \leq p \leq \infty$, $lm \in L^1(J,R)$ and $\|lm\|_{L^1 J} \leq \|l\|_{L^r J} \|m\|_{L^p J}$.*

Lemma 4. *(Lemma $3.2-3.4$, [37]) The operators $\mathcal{T}$ and $\mathcal{S}$ have the following properties:*
(i)For any fixed $t \geq 0$, $\mathcal{T}$ and $\mathcal{S}$ are linear and bounded operators, that is for any $x \in X$,

$$\|\mathcal{T}(t)(x)\| \leq M\|x\| \text{ and } \|\mathcal{S}(t)x\| \leq \frac{\alpha M}{\Gamma(1+\alpha)} \|x\|.$$

(ii) $\{\mathcal{T}(t), t \geq 0\}$ and $\{\mathcal{S}(t), t \geq 0\}$ are strongly continuous.
(iii) For every $t > 0, \mathcal{T}$ and $\mathcal{S}$ are also compact operators if $T(t)$ is compact.

III. Existence results for impulsive Cauchy problems

In this section, we shall present and prove the existence and uniqueness results concering the PC-mild solution for the system $(1.1)-(1.3)$ under the different assumptions of f.

Case 1. f is Lipschitz

To establish our results, let us list the following hypotheses:

(H1) A is the infinitesimal generator of a compact semigroup $\{T(t), t \geq 0\}$ in X.

(H2) $f : [0,T_0] \times X \times X \times X \to X$ is continuous and there exists a constant $q_1 \in (0,\alpha)$ and a real-valued function $L_f \in L^{\frac{1}{q}}([0,T_0], R^+)$ such that $\|f(t,x_1,x_2,x_3) - (t,y_1,y_2,y_3)\| \le L_f(t)\Big[\|x_1 - y_1\| + \|x_2 - y_2\| + \|x_3 - y_3\|\Big]$, $t \in [0,T_0]$, $x_i, y_i \in X$, i=1,2,3.

(H3) Denote $K^\star = \sup_{t\in[0,T_0]} \int_0^t |K(t,s)|dt < \infty$ and $H^* = \sup_{t\in[0,T_0]} \int_0^{T_0} |H(t,s)|dt < \infty.$
For the sake of the shortness, let $T^\star = \Big[\Big(\frac{1-q_1}{\alpha-q_1}\Big)T_0^{\frac{\alpha-q_1}{1-q_1}}\Big]^{1-q_1}\|L_f\|_{L^{\frac{1}{q_1}}(J,R^+)}$ and $Z^\star = [1 + (K^\star + H^\star)T_0\Big].$

Theorem III.1. *Let $(H1)-(H3)$ be satisfied. Then for every $x_0 \in X$, the system $(1.1)-(1.3)$ has a unique PC-mild solution on $[0,T_0]$ provided that*

$$0 < \frac{\alpha M T^\star Z^\star}{\Gamma(1+\alpha)} < 1. \qquad (3.1)$$

Proof: Let $x_0 \in X$ be fixed. Define an operator F on $PC([0,T_0],X)$ by

$$(Fx)(t) = \begin{cases} \mathcal{T}(t)x_0 \\ +\int_0^t (t-s)^{\alpha-1} \\ \mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)ds, \ t\in[0,t_1], \\ \mathcal{T}(t)x_0 + \mathcal{T}(t-t_1)y_1 + \int_0^t (t-s)^{\alpha-1} \\ \mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)ds, \ t\in(t_1,t_2], \\ \vdots \\ \mathcal{T}(t)x_0 + \sum_{i=1}^m \mathcal{T}(t-t_i)y_i + \int_0^t (t-s)^{\alpha-1} \\ \mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)ds, \ t\in(t_m,b]. \end{cases}$$

By our hypotheses and Lemma 1, F is well defined on $PC([0,T_0],X)$.

Step 1: We prove that $Fx \in PC([0,T_0],X)$ for $x \in PC([0,T_0],X)$. For $0 \le \eta \le t \le t_1$, taking into account the hypotheses and applying Lemma 2 and Lemma 4 (i), we obtain

$$\begin{aligned} &\|(Fx)(t) - (Fx)(\eta)\| \\ &\le \|\mathcal{T}(t)x_0 - \mathcal{T}(\eta)x_0\| \\ &\quad + \int_\eta^t (t-s)^{\alpha-1}\|\mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)\|ds \\ &\quad + \int_0^\eta (t-s)^{\alpha-1}\|\mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right) \\ &\quad - \mathcal{S}(\eta-s)f\left(s,x(s),Tx(s),Sx(s)\right)\|ds \\ &\quad + \int_0^\tau |(t-s)^{\alpha-1} - (\tau-s)^{\alpha-1}| \\ &\quad (\times)\ \|\mathcal{S}(\eta-s)f\left(s,x(s),Tx(s),Sx(s)\right)\|ds \\ &\le \|\mathcal{T}(t)-\mathcal{T}(\tau)\|\|x_0\| + \frac{\alpha M}{\Gamma(1+\alpha)}\int_\eta^t (t-s)^{\alpha-1} \\ &\quad (\times)\ \|\mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)\|ds \\ &\quad + \sup_{s\in[0,\eta]}\|\mathcal{S}(t-s)-\mathcal{S}(\eta-s)\|\int_0^\tau (t-s)^{\alpha-1} \\ &\quad (\times)\ \|f\left(s,x(s),Tx(s),Sx(s)\right)\|ds \\ &\quad + \frac{\alpha M\|f\|_{C([0,t_1],X)}}{\Gamma(1+\alpha)}\Big|\int_0^\eta (\eta-s)^{\alpha-1}ds - \int_0^\eta (t-s)^{\alpha-1}ds\Big| \\ &\le \|\mathcal{T}(t)-\mathcal{T}(\tau)\|\|x_0\| \\ &\quad + \frac{t_1^\alpha\|f\|_{PC}}{\alpha}\sup_{s\in[0,\eta]}\|\mathcal{S}(t-s)-\mathcal{S}(\tau-s)\| \\ &\quad + \frac{3M\|f\|_{PC}(t-\eta)^\alpha}{\Gamma(1+\alpha)}, \end{aligned}$$

where we use the inequality $t^\sigma - \eta^\sigma \le (t-\eta)^\sigma$. From Lemma 4 (iii), the first and second terms tend to zero as $t \to \eta$. Moreover, it is obvious that the last terms tends to zero too $t \to \tau$. Thus, we can deduce that $Fx \in C([0,t_1],X)$.

For $t_1 \le \eta < t < t_2$, applying the hypotheses and Lemma 2 and Lemma 4 (i), we have

$$\begin{aligned} &\|(Fx)(t) - (Fx)(\eta)\| \\ &\le \|\mathcal{T}(t)-\mathcal{S}(\eta)\|\|x_0\| + \|\mathcal{T}(t-t_1)-\mathcal{T}(\eta-t_1)\|\|y_1\| \\ &+ \frac{t_2^\alpha\|f\|_{PC}}{\alpha}\sup_{s\in[0,\eta]}\|\mathcal{S}(t-s)-\mathcal{S}(\eta-s)\| + \frac{3M\|f\|_{PC}(t-\eta)^\alpha}{\Gamma(1+\alpha)}. \end{aligned}$$

As $t \to \eta$, the right hand side of the above inequality tends to zero. Thus, we can deduce that $Fx \in C((t_1,t_2],X)$. Similarly, we can also obtain that $Fx \in C((t_2,t_3],X), ..., Fx \in C((t_m,b],X)$. That is $Fx \in PC([0,T_0],X)$.

Step 2: We show that F is a contraction on $PC([0,T_0],X)$.

For each $t \in [0,t_1]$, using the hypotheses, Lemma 3 and Lemma 4, we have

$$\begin{aligned} \|(Fx)(t) - (Fy)(t)\| &\le \frac{\alpha M}{\Gamma(1+\alpha)}\int_0^t (t-s)^{\alpha-1} \\ &\Big[L_f(s)\{\|x(s)-y(s)\| + \|Tx(s)-Ty(s)\| \\ &\qquad (\times) + \|Sx(s)-Sy(s)\|\}\Big]ds \end{aligned} \qquad (3.2)$$

Now,

$$\begin{aligned} &L_f(t)\int_0^t \|Tx - Ty\|ds \\ &\le L_f(t)\int_0^t\int_0^s \|K(s,\eta)\|\ \|x(\eta)-y(\eta)\|d\eta ds \\ &\le L_f(t)\int_0^t \|x(s)-y(s)\|\ \int_0^s \|K(s,\eta)\|d\eta ds \\ &\le L_f(t)\ \|x(t)-y(t)\|\int_0^t K^\star ds \\ &\le L_f(t)\ \|x-y\|_{PC}K^\star T_0. \end{aligned} \qquad (3.3)$$

Similarly,

$$L_f(t)\int_0^t \|Sx - Sy\|ds \le L_f(t)\|x-y\|_{PC}H^\star T_0. \qquad (3.4)$$

Substitute the equations (3.3) and (3.4) in (3.2), we have

$$\leq \frac{\alpha M}{\Gamma(1+\alpha)}\int_0^t (t-s)^{\alpha-1}L_f(s)\|x(s)-y(s)\| (\times)\Big[1+(K^\star+H^\star)T_0\Big]ds$$

$$\leq \frac{\alpha M\Big[1+(K^\star+H^\star)T_0\Big]\ \|x-y\|_{PC}}{\Gamma(1+\alpha)} (\times)\int_0^t (t-s)^{\alpha-1}L_f(s)ds$$

$$\leq \frac{\alpha M\Big[1+(K^\star+H^\star)T_0\Big]\ \|x-y\|_{PC}}{\Gamma(1+\alpha)} (\times)\Big(\int_0^t (t-s)^{\frac{\alpha-1}{1-q_1}}ds\Big)^{1-q_1}\|L_f\|_{L^{\frac{1}{q_1}}([0,t_1],R^+)}$$

$$\leq \frac{\alpha M\Big[1+(K^\star+H^\star)T_0\Big]\ \|x-y\|_{PC}}{\Gamma(1+\alpha)} (\times)\Big[\Big(\frac{1-q_1}{\alpha-q_1}\Big)t_1^{\frac{\alpha-q_1}{1-q_1}}\Big]^{1-q_1}\|L_f\|_{L^{\frac{1}{q_1}}([0,t_1],R^+)}.$$

In general, for each $t\in(t_k,t_{k+1}]$, using our hypotheses and Lemma 4 again,

$$\|(Fx)(t)-(Fy)(t)\| \leq \frac{\alpha M\Big[1+(K^\star+H^\star)T_0\Big]\ \|x-y\|_{PC}}{\Gamma(1+\alpha)} (\times)\Big[\Big(\frac{1-q_1}{\alpha-q_1}\Big)t_{k+1}^{\frac{\alpha-q_1}{1-q_1}}\Big]^{1-q_1}\|L_f\|_{L^{\frac{1}{q_1}}([t_k,t_{k+1}],R^+)}.$$

Thus,

$$\|Fx-Fy\|_{PC}\leq \frac{\alpha M T^\star Z^\star}{\Gamma(1+\alpha)}\|x-y\|_{PC}.$$

It follows that the operator F is contraction (3.1). By the Banach contraction mapping principle, there exists a unique fixed point $x\in PC([0,T_0],X)$. Therefore the system $(1.1)-(1.3)$ has a unique PC-mild solution on $[0,T_0]$.

Case 2. f is not Lipschitz.
Let us list the following hypotheses.

- **(H4)** $f:[0,T_0]\times X\times X\times X\to X$ is continuous and maps a bounded set into a bounded set.
- **(H5)** For each $x_0\in X$, there exists a constant $r>0$ such that

$$M\Big[\|x_0\|+\sum_{k=1}^m\|y_m\|+\frac{b^\alpha}{\Gamma(1+\alpha)} (\times)\sup_{s\in[0,T_0],\phi\in Y_\Gamma}\|f(s,\phi(s),T\phi(s),S\phi(s))\|\Big] \leq r,$$

where

$$Y_\Gamma=\{\phi\in PC([0,T_0],X)|\ \|\phi\|\leq r,\ \text{for } t\in[0,T_0]\}.$$

Theorem III.2. *Suppose that $(H1),(H4)$ and $(H5)$ are satisfied. Then for every $x_0\in X$, the system $(1.1)-(1.3)$ has at least a PC-mild solution on $[0,T_0]$.*

Proof: Let $x_0\in X$ be fixed. We set the map $F:PC([0,T_0],X)\to PC([0,T_0],X)$ by

$$(Fv)(t)=(F_1v)(t)+(F_2v)(t),$$

where

$$(F_1v)(t)=\mathcal{T}(t)x_0 +\int_0^t (t-s)^{\alpha-1}\mathcal{S}(t-s)f(s,v(s),Tv(s),Sv(s))ds, \quad t\in[0,T_0]\backslash\{t_1,t_2,...,t_m\}, \tag{3.5}$$

and

$$(F_2v)(t)=\begin{cases}0,\ t\in[0,t_1],\\ \sum_{i=1}^k\mathcal{T}(t-t_i)y_i,\ t\in(t_k,t_{k+1}],\ k=1,2,..,m.\end{cases} \tag{3.6}$$

For each $t\in(t_k,t_{k+1}],v\in Y_\Gamma$,

$$\|(Fv)(t)\|\leq\|(F_1v)(t)\|+\|(F_2v)(t)\| \leq M\|x_0\|+M\sum_{k=1}^m\|y_k\| +\frac{b^\alpha M}{\Gamma(1+\alpha)}\sup_{s\in[0,T_0],\phi\in Y_\Gamma}\|f(s,\phi(s),T\phi(s),S\phi(s))\|.$$

From $(H5)$, we see that $F:Y_\Gamma\to Y_\Gamma$.

Step 1: We prove that F is a continuous mapping from Y_Γ to Y_Γ. In order to derive the continuity of F, we only need to check that F_1 and F_2 are all continuous.

For this purpose, we assume that $v_n\to v$ in Y_Γ. It comes from the continuity of f that

$$(\cdot-s)^{\alpha-1}f(s,v_n(s),Tv_n(s),Sv_n(s))\to(\cdot-s)^{\alpha-1}f(s,v(s),Tv(s),Sv(s)),$$

$n\to\infty$. Noting that

$$(t-s)^{\alpha-1}\|f(f(s,v_n(s),Tv_n(s),Sv_n(s)))-f(s,v(s),Tv(s),Sv(s))\| \leq (t-s)^{\alpha-1}\sup_{s\in[0,T_0],\phi\in Y_\Gamma}\|f(s,\phi(s),T\phi(s),S\phi(s))\|, \quad \text{for } s\in[0,t],\ t\in[0,T_0],$$

by using Lebesgue dominated convergence theorem, we obtain that

$$\int_0^t(t-s)^{\alpha-1}\|f(s,v_n(s),Tv_n(s),Sv_n(s)) -f(s,v(s),Tv(s),Sv(s))\|ds\to0,\ \text{as } n\to\infty.$$

It is easy to see that for each $t\in[0,T_0]$,

$$\|(F_1v_n)(t)-(F_1v)(t)\| \leq\frac{\alpha M}{\Gamma(1+\alpha)}\int_0^t(t-s)^{\alpha-1}\Big[\|f(s,v_n(s),Tv_n(s),Sv_n(s)) -f(s,v(s),Tv(s),Sv(s))\|\Big]\ ds \tag{3.7}$$

$$\to0,\ \text{as } n\to\infty.$$

Thus, F_1 is continuous. On the other hand, it is obvious that F_2 is continuous. Since F_1 and F_2 are continuous, F is continuous.

Step 2: We prove that F is a compact operator, or F_1 and F_2 are compact operators. The compactness of F_2 is clear since it is a constant map (see(3.5)).

Now we prove the compactness of F_1. For each $t \in [0, T_0]$, the set $\{\mathcal{T}(t)x_0\}$ is precompact in X since $\mathcal{T}(t),\ t > 0$ is compact. Also, for each $t \in [0, T_0]$, arbitrary $\alpha > h > 0,\ \epsilon > 0$, the set

$$\Big\{T(h^\alpha\epsilon)\int_0^{t-h}(t-s)^{\alpha-1}\Big(\alpha\int_\epsilon^\infty \theta\xi_\alpha(\theta)T((t-s)^\alpha\theta - h^\alpha\epsilon)\ d\theta\Big) f(s, v(s), Tv(s), Sv(s))ds|\ v \in Y_\Gamma\Big\}$$
$$= \Big\{\alpha\int_0^{t-h}\int_\epsilon^\infty \theta(t-s)^{\alpha-1}\xi_\alpha(\theta)T((t-s)^\alpha\theta) f(s, v(s), Tv(s), Sv(s))ds \mid v \in Y_\Gamma\Big\}$$

is precompact in X, since $T(h^\alpha\epsilon)$ is compact.

By the proof of Theorem 3.1 in [37], we can obtain

$$\alpha\int_0^{t-h}\int_\epsilon^\infty \theta(t-s)^{\alpha-1}\xi_\alpha(\theta)T((t-s)^\alpha\theta) f(s, v(s), Tv(s), Sv(s))\ d\theta ds \to \alpha\int_0^t\int_0^\infty \theta(t-s)^{\alpha-1}\xi_\alpha(\theta) T((t-s)^\alpha\theta)f(s, v(s), Tv(s), Sv(s))\ d\theta ds,$$

as $h \to 0, \epsilon \to 0$.

Thus, we can conclude that

$$\Big\{\int_0^t(t-s)^{\alpha-1}\mathcal{S}(t-s)f(s, v(s), Tv(s), Sv(s))\ ds \mid v \in Y_\Gamma\Big\}$$
$$= \Big\{\alpha\int_0^t\int_0^\infty \theta(t-s)^{\alpha-1}\xi_\alpha(\theta)T((t-s)^\alpha\theta) f(s, v(s), Tv(s), Sv(s))\ d\theta ds \mid v \in Y_\Gamma\Big\}$$

is precompact in X. Therefore, the set

$$\Big\{\mathcal{T}(t)x_0 + \sum_{i=1}^k \mathcal{T}(t-t_i)y_i + \int_0^t(t-s)^{\alpha-1}\mathcal{S}(t-s)f(s, v(s), Tv(s), Sv(s))\ ds|\ v \in Y_\Gamma\Big\}$$

is precompact in X. Thus, for each $t \in [0, T_0],\ \{(F_1v)(t) \mid v \in Y_\Gamma\}$ is precompact in X.

Now, we show the equicontinuity of $\Re = \{(F_1v)(\cdot) \mid v \in Y_\Gamma\}$.

The equicontinuity of $\{\mathcal{T}(t)x_0 \mid t \in [0, T_0]\backslash\{t_1, t_2, .., t_m\}\}$, can be shown using the fact of $\mathcal{T}(\cdot)$ is continuous. Now, we only need to check the equicontinuity of the second term in $\Re$. For $t \in [0, T_0]$, let $0 \le t^{'} < t^{''} \le t_1$, set

$$I_1 = \Big\|\int_{t'}^{t''}(t^{''}-s)^{\alpha-1}\mathcal{S}(t^{''}-s)f(s, v(s), Tv(s), Sv(s))ds\Big\|,$$
$$I_2 = \Big\|\int_0^{t'}[(t^{''}-s)^{\alpha-1} - (t^{'}-s)^{\alpha-1}]\mathcal{S}(t^{''}-s) f(s, v(s), Tv(s), Sv(s))ds\Big\|,$$
$$I_3 = \Big\|\int_0^{t'}(t^{'}-s)^{\alpha-1}[\mathcal{S}(t^{''}-s) - \mathcal{S}(t^{'}-s)] f(s, v(s), Tv(s), Sv(s))ds\Big\|.$$

After some calculation, we obtain

$$\Big\|\int_0^{t''}(t^{''}-s)^{\alpha-1}\mathcal{S}(t^{''}-s)f(s, v(s), Tv(s), Sv(s))ds - \int_0^{t'}(t^{'}-s)^{\alpha-1}\mathcal{S}(t^{'}-s)f(s, v(s), Tv(s), Sv(s))ds\Big\| \le I_1 + I_2 + I_3.$$

Now repeating the steps in Theorem 3.1 in [37], we derive that I_1, I_2, I_3 tends to zero as $t^{''} \to t^{'}$. Accordingly, we see that the functions in $\Re$ are equicontinuous. Therefore, F_1 is a compact operator by the Arzela-Ascoli theorem, and hence F is also a compact operator. Now, Schauder's fixed point theorem implies that F has a fixed point, which gives rise to a PC-mild solution.

To end this section, we need the following hypotheses.

(H6) $f : [0, T_0] \times X \times X \times X \to X$ is continuous and there exists a function $m(\cdot) \in L^\infty([0, T_0], R^+)$ such that $\|f(t, x, Tx, Sx)\| \le m(t)$, for all $x \in X$ and $t \in [0, T_0]$.

Theorem III.3. *Assume that* $(H1)$ *and* $(H6)$ *are satisfied. Then the system* $(1.1)-(1.3)$ *has at least a* PC*-mild solution on* $[0, T_0]$.

Proof: We defined that $F : PC([0, T_0], X) \to PC([0, T_0], X)$ as in Theorem 3.2 by
$(Fv)(t) = (F_1v)(t) + (F_2v)(t)$. Then we proceed in the following steps.

Step 1: We prove that F is a continuous mapping from $PC([0, T_0], X)$ to $PC([0, T_0], X)$.

Let $\{v_n\}$ be a sequence in $PC([0, T_0])$ such that $v_n \to v$ in $PC([0, T_0], X)$. From $(H6)$ that

$$(\cdot - s)^{\alpha-1}f(s, v_n(s), T_n(s), S_n(s)) \to (\cdot - s)^{\alpha-1} f(s, v(s), Tv(s), Sv(s)),\ \text{as}\ n \to \infty,$$

and note that

$$(t-s)^{\alpha-1}\|f(s, v_n(s), T_n(s), S_n(s)) - f(s, v(s), Tv(s), Sv(s))\| \le 2m(s)(t-s)^{\alpha-1} \in L^1([0, T_0], R^+),\ \text{for}\ s \in [0, t],\ t \in [0, T_0].$$

Similar to the discussion in Theorem 3.2, one can prove that F is a continuous mapping from $PC([0, T_0], X) \to PC([0, T_0], X)$.

Step 2: F maps bounded sets into bounded sets in $PC([0, T_0], X)$.

So, let us prove that for any $r > 0$ there exists a $M^\star > 0$ such that for each $v \in B_r = \{v \in PC([0, T_0]) \mid \|v\|_{PC} \le r\}$, we have $\|Fv\|_{PC} \le M^\star$.
Indeed, for any $v \in B_r$,

$$\|(Fv)(t)\| \le \|(F_1v)(t)\| + \|(F_2v)(t)\| \le M\|x_0\| + M\sum_{i=1}^m\|y_i\| + \frac{b^\alpha M}{\Gamma(1+\alpha)}\|m\|_{L^\infty([0,T_0],R^+)},$$

which implies

$$\|Fv\|_{PC} \leq M\|x_0\| + M\sum_{i=1}^{m}\|y_i\| + \frac{b^\alpha M}{\Gamma(1+\alpha)}\|m\|_{L^\infty([0,T_0],R^+)} \equiv M^\star.$$

Step 3: F is a compact operator.

In order to verify that F is a compact operator, one can repeat the same process in Step 2 of Theorem 3.2 only need to replace $\sup_{s\in[0,T_0],\phi\in Y_\Gamma} \|f(s,\phi(s),T\phi(s),S\phi(s))\|$ by $\|m\|_{L^\infty([0,T_0],R^+)}$.

Step 4: The set $\Omega = \{x \in PC([0,T_0],X) \mid x = \lambda \in [0,1]\}$ is bounded.

Let $v \in \Omega$. Then $v(t) = \lambda(Fv)(t)$ for some $\lambda \in [0,1]$. Thus, for $t \in [0,T_0]$, directly calculation implies that $\|Fv\|_{PC} \leq M^\star$. Hence, we deduce that Ω is a bounded set.

Since we have already proven that F is continuous and compact, by the help of Schaefer's fixed point theorem, F has a fixed point which is a PC-mild solution of system (1.1) – (1.3) on $[0,T_0]$.

Remark 2. *In the hypotheses* $(H6)$*, the condition* $m(\cdot) \in L^\infty([0,T_0],R^+)$ *can be replaced by* $m(\cdot) \in L^{\frac{1}{q_2}}([0,T_0],R^+)$ *where* $\frac{1}{q_2} \in [0,\alpha)$*. The norm of* m *is defined by*

$$\|m\|_{L^{\frac{1}{q_2}}(J,R^+)} = \begin{cases} \left(\int_J \|m(t)\|^{\frac{1}{q_2}} dt\right)^{q_2}, & \text{if} \quad 1 < \frac{1}{q_2} < \infty, \\ \inf_{\mu(\bar{J})=0}\left\{\sup_{t\in J-\bar{J}} \|m(t)\|\right\}, & \text{if} \quad \frac{1}{q_2} = \infty \end{cases}$$

where $\mu(\bar{J})$ is the Lebesgue measure on $\bar{J}$.

IV. Existence results for impulsive nonlocal mixed Cauchy problems

In this section, we study the nonlocal Cauchy problem for impulsive fractional evolution equations. More precisely, we will prove the existence and uniqueness of the PC-mild solutions for the system $(1.1)-(1.2)$ with (1.4). The notion of nonlocal condition has been introduced to extend the study of the classical initial value problems and it is more precise for describing nature phenomena than the classical condition since more information is taken into account, thereby decreasing the negative effects incurred by a possibly erroneous single measurement taken at the initial value. The importance of nonlocal conditions in many applications is discussed in [7, 12, 13, 18, 38].

Definition 5. *By a* PC*-mild solution of the system* (1.1) – (1.2) *with* (1.4)*, we mean that a function* $x \in PC(J,X)$ *which satisfies the following integral equation*

$$x(t) = \begin{cases} \mathcal{T}(t)[x_0 + g(x)] + \int_0^t (t-s)^{\alpha-1}\mathcal{S}(t-s) \\ f\left(s,x(s),Tx(s),Sx(s)\right)ds, \quad t\in[0,t_1], \\ \mathcal{T}(t)[x_0+g(x)] + \mathcal{T}(t-t_1)y_1 + \int_0^t (t-s)^{\alpha-1} \\ \mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)ds, \quad t\in(t_1,t_2], \\ \vdots \\ \mathcal{T}(t)[x_0+g(x)] + \sum_{i=1}^m \mathcal{T}(t-t_i)y_i + \int_0^t (t-s)^{\alpha-1} \\ \mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)ds, \quad t\in(t_m,b]. \end{cases} \tag{4.1}$$

Case 1. g **is Lipschitz**

(H7) $g : PC([0,T_0],X)$ and there exists a constant $L_1 > 0$ such that

$$\|g(x) - g(y)\| \leq L_1\|x-y\|_{PC}, \ x,y \in PC([0,T_0],X).$$

Theorem IV.1. *Let* $(H1)$*,*$(H2)$ *and* $(H7)$ *be satisfied. Then for every* $x_0 \in X$*, the system* (1.1) *and* (1.3) *with* (1.4) *has a unique* PC*- mild solution on* $[0,T_0]$ *provided that*

$$0 < \Theta = ML_1 + \frac{\alpha MT^\star}{\Gamma(1+\alpha)} < 1. \tag{4.2}$$

Proof: Define an operator $P : PC([0,T_0],X) \to PC([0,T_0],X)$ by

$$(Px)(t) = \begin{cases} \mathcal{T}(t)[x_0 + g(x)] + \int_0^t (t-s)^{\alpha-1}\mathcal{S}(t-s) \\ f\left(s,x(s),Tx(s),Sx(s)\right)ds, \ t\in[0,t_1], \\ \mathcal{T}(t)[x_0+g(x)] + \mathcal{T}(t-t_1)y_1 + \int_0^t (t-s)^{\alpha-1} \\ \mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)ds, \ t\in(t_1,t_2], \\ \vdots \\ \mathcal{T}(t)[x_0+g(x)] + \sum_{i=1}^m \mathcal{T}(t-t_i)y_i + \int_0^t (t-s)^{\alpha-1} \\ \mathcal{S}(t-s)f\left(s,x(s),Tx(s),Sx(s)\right)ds, \quad t\in(t_m,b]. \end{cases} \tag{4.3}$$

It is obvious that P is well defined on $PC([0,T_0],X)$.

For $0 \leq \eta < t \leq t_1$, by hypotheses and Lemma 4,

$$\|\mathcal{T}(t)g(x) - \mathcal{T}(\eta)g(x)\| \leq \|\mathcal{T}(t) - \mathcal{T}(\eta)\|(L_1\|x\|_{PC} + \|g(0)\|).$$

As $t \to \eta$, the right hand side of the above inequality tend to zero due to Lemma $4(iii)$ again. Recall the Step 1 in Theorem 3.1, we know that $Px \in PC([0,T_0],X)$.

Step 2: P is contraction.

We only take $t \in (t_k, t_{k+1}]$, then we have

$$\|(Px)(t) - (Py)(t)\| \leq \left[ML_1 + \frac{\alpha MT^\star Z^\star}{\Gamma(1+\alpha)}\right]\|x-y\|_{PC}.$$

So we get

$$\|Px - Py\|_{PC} \leq \Theta\|x-y\|_{PC},$$

where

$$\Theta = ML_1 + \frac{\alpha MT^\star Z^\star}{\Gamma(1+\alpha)}.$$

Hence, the condition (4.2) allows us to conclude, in view of the Banach contraction mapping principle again, that P has a unique fixed point $x \in PC([0,T_0],X)$ which is the PC-mild solution of the system (1.1) and (1.3) with (1.4).

Theorem IV.2. *Assume that* $(H1)$, $(H6)$ *and* $(H7)$ *are satisfied. If* $ML_1 < \frac{1}{2}$ *then system* $(1.4)-(1.6)$ *has at least a* PC*-mild solution on* $[0, T_0]$.

Proof: Choose

$$\kappa \geq 2M\Big[(\|x_0\| + \|g(0)\|) + \sum_{k=1}^{m}\|y_k\| + \frac{b^\alpha}{\Gamma(1+\alpha)}\|m\|_{L^\infty(J,R^+)}\Big].$$

Consider $B_\kappa = \{x \in PC([0,T_0],X) \mid \|x\|_{PC} \leq \kappa\}$. Define the operators Υ on B_κ by

$$(\Upsilon x)(t) = (\Upsilon_1 x)(t) + (\Upsilon_2 x)(t) + (\Upsilon_3 x)(t)$$

where

$$(\Upsilon_1 x)(t) = \mathcal{T}(t)[x_0 + g(x)],\ t \in [0,T_0],$$
$$(\Upsilon_2 x)(t) = \int_0^t (t-s)^{\alpha-1}\mathcal{S}(t-s)f(s,x(s),Tx(s),Sx(s))ds,\quad t \in [0,T_0] \quad (4.4)$$

and Υ_3 is the same as the operator F_2 defined in Theorem 3.2.

It suffices to proceed exactly steps of the proof in Theorem 3.2 while replacing B_r by B_κ to obtain that $\Upsilon_2 + \Upsilon_3$ are continuous and compact. We want to use the Krasnoselkii's fixed point theorem. Thus, to complete the rest of the proof of this theorem, it suffices to show that Υ_1 is a contraction mapping and that if $x, y \in B_\kappa$, then $\Upsilon_1 x + (\Upsilon_2 + \Upsilon_3)y \in B_\kappa$. Indeed, for any $x \in B_\kappa$, we have

$$\|\Upsilon_1 x\|_{PC} + \|\Upsilon_2 y\|_{PC} + \|\Upsilon_3 y\|_{PC} \leq M(\|x_0\| + \|g(0)\| + L_1\kappa) + M\sum_{k=1}^{m}\|y_k\| + \frac{b^\alpha M}{\Gamma(1+\alpha)}\|m\|_{L^\infty(J,R^+)}.$$

Since $ML_1 < \frac{1}{2}$, we obtain that

$$\|\Upsilon_1 x + (\Upsilon_2 + \Upsilon_3)y\|_{PC} \leq \kappa.$$

Next, for any $t \in (t_k, t_{k+1}],\ x, y \in C((t_k, t_{k+1}], X)$,

$$\|\Upsilon_1 x - \Upsilon_1 y\|_{C((t_k,t_{k+1}],X)} \leq ML_1\|x-y\|_{C((t_k,t_{k+1}],X)}.$$

Therefore, we can deduce that Υ_1 is contraction from $ML_1 < 1$. Moreover, $\Upsilon_2 + \Upsilon_3$ is compact and continuous. Hence, by the well known Krasnoselskii's fixed point theorem, we can conclude that system $(1.1)-(1.2)$ with (1.4) has at least one PC-mild solution on $[0, T_0]$.

Case 2. g **is not Lipschitz**

(H8) $PC([0,T_0],X) \to X$ is continuous and maps a bounded set into a bounded set.

(H9) For each $x_0 \in X$, there exists a constant $r' > 0$ such that

$$M\Big[\|x_0\| + \sup_{\phi \in Y_{\Gamma'}}\|g(\phi)\|\Big] + M\sum_{k=1}^{m}\|y_m\| + \frac{b^\alpha M}{\Gamma(1+\alpha)}\sup_{s\in[0,T_0],\phi\in y_{\Gamma'}}\|f(s,\phi(s),T\phi(s),S\phi(s))\| \leq r',$$

where

$$Y_{\Gamma'} = \Big\{\phi \in PC([0,T_0],X) \mid \|\phi\| \leq r' \text{ for } t \in [0,T_0]\Big\}.$$

Theorem IV.3. *Assume that* $(H1)$, $(H3)$, $(H7)$ *and* $(H8)$ *are satisfied. Then, for every* $x_0 \in X$, *the system* $(1.1)-(1.2)$ *with* (1.4) *has at least a* PC*-mild solution on* $[0, T_0]$.

Proof: Define the operator $\mathcal{A}$ on $PC([0,T_0],X)$ by

$$(\mathcal{A}v)(t) = (\mathcal{A}_1 v)(t) + (\mathcal{A}_2 v)(t)$$

where

$$(\mathcal{A}_1 v)(t) = \mathcal{T}(t)[x_0 + g(x)] + \int_0^t (t-s)^{\alpha-1}\mathcal{S}(t-s)f(s,v(s),Tv(s),Sv(s))ds,\ t \in J$$

and $\mathcal{A}_2$ is the same as F_2 defined in Theorem $III.2$. Thus, we need to check that, for each $t \in [0,T_0]$, the set $\{\mathcal{T}[x_0 + g(v)] \mid v \in Y_{\Gamma'}\}$ is precompact in X since $\Big\{\mathcal{T}[x_0 + g(v)] \mid t \in J,\ v \in Y_{\Gamma'}\Big\}$ can be shown using the same idea.

Therefore, $\mathcal{A}$ is also a compact operator. By Schauder's fixed point theorem again, $\mathcal{A}$ has a fixed point, which gives rise to a PC-mild solution. This completes the proof.

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