Wave Propagation in a Homogeneous Isotropic Cylindrical Panel Embedded on Elastic Medium

P. Ponnusamy and R .Selvamani

Abstract—**In this paper the three dimensional wave propagation in a homogenous isotropic cylindrical panel embedded in an elastic medium (Winkler model) is investigated in the context of the three dimensional linear theory of elasticity. The analysis is carried out by introducing three displacement functions so that the equations of motion are uncoupled and simplified. The Bessel function solution with complex arguments is then directly used for the case of complex eigen values. In order to illustrate theoretical development, numerical solutions are obtained and presented graphically for a zinc material with the support of MATLAB.**

Index Terms— **Isotropic cylindrical panel, Winkler foundation, Bessel function.**

MSC 2010 Codes **—74H45, 74J20, 74K25.**

I. Introduction

In view of the increased usage of isotropic materials in the development of the diverse engineering fields and Ultrasonic nondestructive inspection for health monitoring of ailing infrastructure, the vibration of isotropic material plays a vital role. Circular cylindrical panels are often applied as protective tank walls, thick cylindrical covers and roof structures of large-span and open-space buildings etc. The wave propagation in cylindrical panel embedded on elastic medium is common place in the design of intelligent building, aircraft modeling, submarine structure, pressure vessels, chemical pipes and as component in the automotive suspensions. So it is very important for engineers to understand the vibration behavior of such cylindrical panel Structures for more reliable and cost effective designs and scattering of elastic waves in isotropic materials is necessary in order to develop ability to characterize cracks and predicts the reliability.

The theory of elastic vibrations and waves is well established by Love [1]. An excellent collection of works on vibration of shells were published by Leissa [2]. Mirsky [3] analyzed the wave propagation in transversely isotropic circular cylinder of infinite length and presented the numerical results. Gazis [4] has studied the most general form of harmonic waves in a hollow cylinder of infinite length. Sinha et. al. [5] has discussed the axisymmetric wave propagation in circular cylindrical shell immersed in fluid in two parts. In Part I, the theoretical analysis of the propagating modes are discussed and in Part II, the axisymmetric modes excluding torsional modes are obtained theoretically and experimentally and are compared. Vibration of functionally graded multilayered orthotropic cylindrical panel under thermo mechanical load was analyzed by Wang et.al [6]. Three dimensional vibration of a homogenous transversely isotropic thermo elastic cylindrical panel was investigated by Sharma [7]. Free vibration of transversely isotropic piezoelectric circular cylindrical panels was studied by Ding et.al [8]. An iterative approach predicts the frequency of isotropic cylindrical shell and panel was studied by Soldatos and Hadhgeorgian [9]. Free vibration of composite cylindrical panels with random material properties was developed by Sing et.al [10], in this work the effect of variations in the mechanical properties of laminated composite cylindrical panels on its natural frequency has been obtained by modeling these as random variables. Zhang [11] employed a wave propagation method to analysis the frequency of cylindrical panels. Lam and Loy [12] investigated the vibration of thin cylindrical panels of simply supported boundary conditions with Flugge's theory and also studied the vibration of rotating cylindrical panel. Three-dimensional solution for shape control of a simply supported rectangular hybrid plate was obtained by Kaburia et. al [13]. Natural frequency of a cylindrical panel on a kerr foundation was studied by Chen.et.al [14] and he used a Bessel function solution with complex argument directly for the complex eigenvalue case. Free vibrations of thin cylindrical shells having finite lengths with freely supported and clamped edges were discussed by Yu et.al [15]. An interesting problem in engineering is the static and dynamic analysis of plates and shell supported on elastic foundations [16].For isotropic cylindrical shell buried at a depth below the free surface of the ground, Wong et al. [17] gave its dynamic response from the point of view of three-dimensional elastic theory. Paliwal et al. [18] presented a clear investigation on the coupled free vibrations of isotropic circular cylindrical shell on Winkler and Pasternak foundations by employing a membrane theory. Upadhyay and Mishra [19] dealt with the non-axisymmetric dynamic behavior of buried orthotropic cylindrical shells excited by a combination of P-, SV and SH-waves. On natural frequencies of a transversely isotropic Cylindrical panel on a kerr foundation was discussed by Chen et al [20].

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In this paper the three dimensional wave propagation in a homogenous isotropic cylindrical panel embedded in an elastic medium is investigated in the context of the three dimensional linear theory of elasticity. The analysis is carried out by introducing three displacement functions so that the equations of motion are uncoupled and simplified**.** A modified Bessel function solution with complex arguments is then directly used for the case of complex eigen values.

II. FORMULATION OF THE PROBLEM

Consider a cylindrical panel embedded on elastic medium as shown in Fig.1 of length L having inner and outer radius a and b with thickness h. The angle subtended by the cylindrical panel, which is known as center angle, is denoted by α. The deformation of the cylindrical panel in the direction r, θ, and z are defined by u, v and w. The cylindrical panel is assumed to be homogenous, isotropic and linearly elastic with Young's modulus E, poisson ratio ν and density ρ in an undisturbed state.

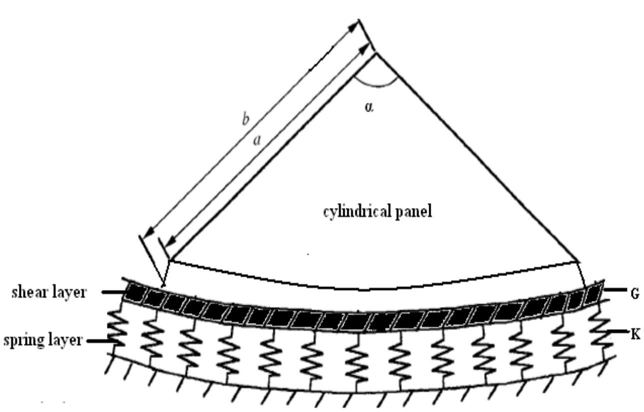

Figure 1. Geometry of the problem

The governing three dimensional equation of motion in cylindrical coordinate (r,θ,z) system in the absence of body force are:

$$\sigma_{rr,r}+\frac{1}{r}\sigma_{r\theta,r}+\sigma_{rz,r}+\frac{1}{r}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)=\rho\frac{\partial^2 u}{\partial t^2}$$
$$\sigma_{r\theta,r}+\frac{1}{r}\sigma_{\theta\theta,\theta}+\sigma_{\theta z,z}+\frac{2}{r}\sigma_{r\theta}=\rho\frac{\partial^2 v}{\partial t^2} \qquad (1)$$
$$\sigma_{rz,r}+\frac{1}{r}\sigma_{\theta z,\theta}+\sigma_{zz,z}+\frac{1}{r}\sigma_{rz}=\rho\frac{\partial^2 w}{\partial t^2}$$

where $\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz}$ are the normal stress components and $\sigma_{r\theta},\sigma_{\theta z},\sigma_{zr}$ are the shear stress components and ρ is the mass density of the cylindrical panel.

The stress-strain relation for an isotropic material by generalized Hook's law is given by

$$\sigma_{rr}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{rr}$$
$$\sigma_{\theta\theta}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{\theta\theta}$$
$$\sigma_{zz}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{zz}$$
$$\sigma_{r\theta}=\mu\gamma_{r\theta},\sigma_{rz}=\mu\gamma_{rz},\sigma_{\theta z}=\mu\gamma_{\theta z} \qquad (2)$$

The strain-displacement relation is given as

$$e_{rr}=\frac{\partial u}{\partial r},e_{\theta\theta}=\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta},e_{zz}=\frac{\partial w}{\partial z}$$
$$\gamma_{r\theta}=\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right),\gamma_{rz}=\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right) \qquad (3)$$
$$\gamma_{z\theta}=\left(\frac{\partial v}{\partial z}+\frac{1}{r}\frac{\partial w}{\partial\theta}\right)$$

where u,v,w are displacements components along radial, circumferential and axial directions respectively. $e_{rr},e_{\theta\theta},e_{zz}$ are normal strain components and $e_{r\theta},e_{\theta z},e_{zr}$ are shear strain components, λ and μ are the Lame's constants.

Substituting the eqs. (3) and (2) in Eq. (1), gives the following three displacement equations of motions

$$\mu\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+r^{-2}\left(\lambda+2\mu\right)v_{,\theta\theta}+\mu v_{,zz}$$
$$+r^{-2}\left(\lambda+3\mu\right)u_{,\theta}+r^{-1}\left(\lambda+\mu\right)u_{,r\theta}+r^{-1}\left(\lambda+\mu\right)w_{,\theta z}=\rho v_{,tt}$$
$$\left(\lambda+2\mu\right)\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+\mu r^{-2}u_{,\theta\theta}+\mu u_{,zz} \qquad (4)$$
$$+r^{-1}\left(\lambda+\mu\right)v_{,r\theta}-r^{-2}\left(\lambda+3\mu\right)v_{,\theta}+\left(\lambda+\mu\right)w_{,rz}=\rho u_{,tt}$$
$$\left(\lambda+2\mu\right)w_{,zz}+\mu\left(w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta}\right)$$
$$+\left(\lambda+\mu\right)u_{,rz}+r^{-1}\left(\lambda+\mu\right)v_{,\theta z}+r^{-1}\left(\lambda+\mu\right)u_{,z}=\rho w_{,tt}$$

The comma in the subscripts denotes the partial differentiation with respect to the variables. The Eq. (4) is a coupled partial differential equations of three displacement components. To uncouple the Eq. (4), we follow Sharma [7] and assuming the solution of Eq. (4) as follows.

$$u=\frac{1}{r}\psi,_{\theta}-\phi,_{r};v=-\frac{1}{r}\phi,_{\theta}-\psi,_{\varpi};w=-\chi,_{z} \qquad (5)$$

Substituting Eq. (5) in Eq. (4) yields the following second order partial differential equation with constant coefficients.

$$((\lambda+2\mu)\nabla^2{}_1+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2})\phi-(\lambda+\mu)\frac{\partial^2\chi}{\partial z^2}=0 \quad (6a)$$
$$(\mu\nabla_1{}^2+(\lambda+2\mu)\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2})\chi-(\lambda+\mu)\nabla_1{}^2\phi=0 \quad (6b)$$
$$(\nabla_1{}^2+\frac{\partial^2}{\partial z^2}-\frac{\rho}{\mu}\frac{\partial^2}{\partial t^2})\psi=0 \quad (6c)$$

where, $\nabla_1{}^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$.

Eq. (6c) in ψ gives a purely transverse wave. This wave is polarized in planes perpendicular to the z-axis is a SH wave. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$ and hence the system of equations in (6a) - (6c) becomes

$$((\lambda+2\mu)\nabla^2{}_1+\mu\frac{\partial^2}{\partial z^2}+\rho\omega^2)\phi-(\lambda+\mu)\frac{\partial^2\chi}{\partial z^2}=0 \quad (7a)$$

$$(\mu\nabla_1{}^2+(\lambda+2\mu)\frac{\partial^2}{\partial z^2}-\rho\omega^2)\chi-(\lambda+\mu)\nabla_1{}^2\phi=0 \quad (7b)$$

$$(\nabla_1{}^2+\frac{\partial^2}{\partial z^2}-\frac{\rho}{\mu}\omega^2)\psi=0 \quad (7c)$$

III. Solution to the Problem

The equation (7) is coupled partial differential equations of the three displacement components. To uncouple equation (7), we can write three displacement functions which satisfies the simply supported boundary conditions followed by Sharma [9]

$$\begin{aligned}\psi(r,\theta,z,t)&=\overline{\psi}(r)\sin(m\pi z)\cos(n\pi\theta/\alpha)e^{i\omega t}\\ \phi(r,\theta,z,t)&=\overline{\phi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}\\ \chi(r,\theta,z,t)&=\overline{\chi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}\end{aligned} \quad (8)$$

where m is the circumferential mode and n is the axial mode, ω is the angular frequency of the cylindrical panel, c_1 wave velocity of the cylindrical panel, and the dimensionless quantities are

$$r^{'}=\frac{r}{R},z^{'}=\frac{z}{L},\delta=\frac{n\pi}{\alpha},t_L=\frac{m\pi R}{L}$$
$$\overline{\lambda}=\frac{\lambda}{\mu},\in_4=\frac{1}{2+\overline{\lambda}},c_1{}^2=\frac{\lambda+2\mu}{\rho}$$

Substituting the Eq. (8) in the Eq. (7), we obtain the following second order partial differential equation:

$$(\nabla_2^2+k_1{}^2)\overline{\psi}=0 \quad (9a)$$

$$(\nabla_2^2+g_2)\overline{\phi}+g_2\overline{\chi}=0 \quad (9b)$$

$$(\nabla_2^2+g_3)\overline{\chi}-(1+\overline{\lambda})\nabla_2^2\overline{\phi}=0 \quad (9c)$$

in which $\nabla_2^2=\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)-\frac{\delta^2}{r^2}$ and

$$g_1=(2+\overline{\lambda})(t^2{}_L-\Omega^2)$$
$$g_2=\in_4(1+\overline{\lambda})t_L{}^2$$
$$g_3=(\Omega^2-\in_4 t_L{}^2)$$

The system of equations given in Eq. (9) has trivial solution. To obtain the non-trivial solutions, the coefficient of the determinant is equal to zero, that is

$$\begin{vmatrix}(\nabla_2^2+g_3) & -g_2\\ (1+\overline{\lambda})\nabla_2^2 & (\nabla_2^2-g_1)\end{vmatrix}(\overline{\phi},\overline{\chi})=0 \quad (10)$$

The equation (10), on simplification reduces to the following differential equation. In addition, α_1 and α_2 ($\mathrm{Re}(\alpha_1\geq 0)$ and $\mathrm{Re}(\alpha_2\geq 0)$ are the two roots of the following equation

$$(\nabla_2^4+B\nabla_2^2+C)\overline{\phi}=0 \quad (11)$$

where $B=-g_1+g_2(1+\overline{\lambda})+g_3, C=-g_1g_3$.

The solution of equation (11) is

$$\begin{aligned}\overline{\phi}(r)&=\sum_{i=1}^{2}\left[A_iJ_\delta(\alpha_ir)+B_iY_\delta(\alpha_ir)\right]\\ \overline{\chi}(r)&=\sum_{i=1}^{2}d_i\left[A_iJ_\delta(\alpha_ir)+B_iY_\delta(\alpha_ir)\right]\end{aligned} \quad (12)$$

Here, $(\alpha_ir)^2$ are the non-zero roots of the algebraic equation

$$(\alpha_ir)^4+B(\alpha_ir)^2-C=0 \quad (13)$$

The arbitrary constant d_i is obtained from

$$d_i=\frac{(1+\overline{\lambda})(\alpha_ir)^2}{(\alpha_ir)^2+g_3} \quad (14)$$

Equation (9a) is a Bessel equation with its possible solutions is

$$\overline{\psi}=\begin{cases}A_3J_\delta(k_1r)+B_3Y_\delta(k_1r) & k_1^2>0\\ A_3r^\delta+B_3r^{-\delta} & k_1^2=0\\ A_3I_\delta(k_1^{'}r)+B_3K_\delta(k_1^{'}r) & k_1^2<0\end{cases} \quad (15)$$

where $k_1^{'2}=-k_1^2$,and , J_δ and Y_δ are Bessel functions of the first and second kinds respectively while, I_δ and k_δ are modified Bessel functions of first and second kinds respectively. A_3 and B_3 are two arbitrary constants. Generally $k_1^2\neq 0$,so that the situation $k_1^2\neq 0$ is will not be discussed in the following. For convenience, we consider the case of $k_1^2>0$,and the derivation for the case of $k_1^2<0$ is similar.

$$\overline{\psi}(r)=A_3J_\delta(k_1r)+B_3Y_\delta(k_1r) \quad (16)$$

where $k_1{}^2=(2+\overline{\lambda})\Omega^2-t_L{}^2$.

IV. Boundary Conditions and Frequency Equation

In this section, we shall derive the secular equation for the three dimensional vibrations cylindrical panel subjected to traction free boundary conditions at the upper and lower surfaces at $r=a,b$

$$u=\left(-\overline{\phi}'-\frac{\delta\overline{\psi}'}{r}\right)\sin(m\pi z)\sin(\delta\theta)e^{i\omega t}$$
$$v=\left(-\overline{\psi}'-\frac{\delta\overline{\phi}'}{r}\right)\sin(m\pi z)\cos(\delta\theta)e^{i\omega t}$$
$$w=\overline{\chi}t_L\cos(m\pi z)\sin(\delta\theta)e^{i\omega t}$$
$$\overline{\sigma}_{rr}=[\left(2+\overline{\lambda}\right)\delta\left(\frac{\overline{\psi}'}{r}-\frac{\overline{\psi}}{r^2}\right)+\left(2+\overline{\lambda}\right)\left(\frac{1}{r}\overline{\varphi}'+(\alpha_i^2-\frac{\delta^2}{r^2}\overline{\varphi})\right)$$
$$+\overline{\lambda}\left(\frac{\delta}{r^2}\overline{\psi}-\frac{1}{r}\overline{\phi}'-\frac{\delta^2}{r^2}\overline{\varphi}-\frac{\delta}{r}\overline{\psi}'-t_L^2\overline{\chi}\right)]\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$
$$\overline{\sigma}_{r\theta}=2[\frac{1}{r}\overline{\psi}+(\alpha_i^2-\frac{\delta^2}{r^2})\overline{\psi}-\frac{2\delta}{r}\overline{\varphi}'$$
$$+\frac{2\delta}{r^2}\overline{\varphi}+\frac{\overline{\psi}'}{r}-\frac{\delta^2}{r^2}\overline{\psi}]\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$
$$\overline{\sigma}_{rz}=2t_L\left(-\overline{\varphi}'-\frac{\delta}{r}\overline{\psi}+\overline{\chi}'\right)\cos(m\pi)z\sin(\delta\theta)e^{i\omega t} \tag{17}$$

Where prime denotes the differention with respect to r, $\overline{u}_i=\frac{u_i}{R},(i=r,\theta,z)$ are three non –dimensional displacements and $\overline{\sigma}_{rr}=\sigma_{rr}/\mu,\overline{\sigma}_{r\theta}=\sigma_{r\theta}/\mu,\overline{\sigma}_{rz}=\sigma_{rz}/\mu$ are three non-dimensional stresses. The mechanical boundary conditions of a cylindrical panel is

$$\sigma_{rr}=\sigma_{r\theta}=\sigma_{rz}=0,(r=a,b) \tag{18}$$

Using the result obtained in the equations (1)-(3) in (18) we can get the frequency equation of uncoupled free vibration as follows:

$$\left|E_{ij}\right|=0,\qquad (\text{i, j = 1, 2, 3...,6}) \tag{19}$$

$$E_{11}=(2+\overline{\lambda})\begin{pmatrix}(\delta J_\delta(\alpha_1t_1)/t_1^{\;2}-\frac{\alpha_1}{t_1}J_{\delta+1}(\alpha_1t_1))\\-((\alpha_1t_1)^2R^2-\delta^2)J_\delta(\alpha_1t_1)/t_1^{\;2}\end{pmatrix}$$
$$+\overline{\lambda}\left(\delta(\delta-1)J_\delta(\alpha_1t_1)/t_1^{\;2}-\frac{\alpha_1}{t_1}J_{\delta+1}(\alpha_1t_1)\right)$$
$$+\overline{\lambda}d_1t_L^{\;2}J_\delta(\alpha_1t_1)$$

$$E_{15}=(2+\overline{\lambda})\left((\frac{k_1\delta}{t_1}J_{\delta+1}(k_1t_1)-\delta(\delta-1)J_\delta(k_1t_1)/t_1^{\;2}\right)$$
$$+\overline{\lambda}\left(\delta(\delta-1)J_\delta(k_1t_1)/t_1^{\;2}-\frac{k_1\delta}{t_1}J_{\delta+1}(k_1t_1)\right)$$
$$E_{21}=2\delta\left((\alpha_1/t_1)J_{\delta+1}(\alpha_1t_1)-\delta(\delta-1)J_\delta(\alpha_1t_1)\right)$$
$$E_{23}=2\delta\left((\alpha_2/t_1)J_{\delta+1}(\alpha_2t_1)-\delta(\delta-1)J_\delta(\alpha_2t_1)\right)$$
$$E_{25}=(k_1t_1)^2R^2J_\delta(k_1t_1)-2\delta(\delta-1)J_\delta(k_1t_1)/t_1^{\;2}+k_1/t_1J_{\delta+1}(k_1t_1)$$
$$E_{31}=-t_L(1+d_1)\left(\delta/t_1J_\delta(\alpha_1t_1)-\alpha_1J_{\delta+1}(\alpha_1t_1)\right)$$
$$E_{33}=-t_L(1+d_2)\left(\delta/t_1J_\delta(\alpha_2t_1)-\alpha_2J_{\delta+1}(\alpha_2t_1)\right)$$
$$E_{35}=-t_L(\delta/t_1)J_\delta(k_1t_1)$$

In which $t_1=a/R=1-t^*/2$ $,t_2=b/R=1+t^*/2$ and $t^*=b-a/R$ is the thickness -to-mean radius ratio of the panel. Obviously $E_{ij}\left(j=2,4,6\right)$ can obtained by just replacing modified Bessel function of the first kind in $E_{ij}\left(i=1,3,5\right)$ with the ones of the second kind, respectively, while $E_{ij}\left(i=4,5,6\right)$ can be obtained by just replacing t_1 in $E_{ij}\left(i=1,2,3\right)$ with t_2.

Now we consider the coupled free vibration problem. Allowing for the effect of the surrounded elastic medium, which is treated as the Pasternak model [14], the boundary conditions at the inner and outer surfaces *r* = *a*,*b* can consider as follows

$$\sigma_{rr}=\sigma_{r\theta}=\sigma_{rz}=0,\ T_{,r}=0 \qquad (r=a)$$
$$\sigma_{rr}=-Ku+G\Delta u;\ \sigma_{r\theta}=\sigma_{rz}=0 \qquad (r=b)$$

where $\Delta=\partial^2/\partial z^2+\left(1/r^2\right)\partial^2/\partial\theta^2$, K is the foundation modulus and G is the shear modulus of the foundation. It is mentioned here that the elastic medium can be modeled as Winkler type by setting G=0 in the above and the results obtained in the preceding section, we get the coupled free vibration frequency equation as follows:

$$\left|E_{ij}^2\right|=0 \qquad (\text{i, j = 1, 2,....,8}) \tag{20}$$
$$E_{ij}^2=E_{ij}^1\left(i=1,\ 2,\ ...,8;\ \text{j}=1,\ 2,\ ...,8\ \right)$$
$$E_{51}^2=E_{51}^1-p\delta J_\delta\left(\alpha_1t_2\right)/t_2$$
$$E_{52}^2=E_{52}^1-p\delta Y_\delta\left(\alpha_1t_2\right)/t_2$$
$$E_{53}^2=E_{53}^1-p\left(\delta/t_2J_\delta\left(\alpha_2t_2\right)-t_2J_{\delta+1}\left(\alpha_2t_2\right)\right)$$
$$E_{54}^2=E_{54}^1-p\left(\delta/t_2Y_\delta\left(\alpha_2t_2\right)-t_2Y_{\delta+1}\left(\alpha_2t_2\right)\right)$$

$$E_{55}^2 = E_{55}^1 - p\left(\delta/t_2 J_\delta(\alpha_3 t_2) - t_2 J_{\delta+1}(\alpha_3 t_2)\right)$$
$$E_{56}^2 = E_{56}^1 - p\left(\delta/t_2 Y_\delta(\alpha_3 t_2) - t_2 Y_{\delta+1}(\alpha_3 t_2)\right)$$
$$E_{57}^2 = E_{57}^1 - p\left(\delta/t_2 J_\delta(k_1 t_2) - t_2 J_{\delta+1}(k_1 t_2)\right)$$
$$E_{58}^2 = E_{58}^1 - p\left(\delta/t_2 Y_\delta(k_1 t_2) - t_2 Y_{\delta+1}(k_1 t_2)\right)$$
$$p = p_1 + p_2\left(t_L^2 + n^2/t_2^2\right)$$

where $p_1 = KR/\mu$ and $p_2 = G/R\mu$.

Numerical Results and Discussion

The coupled free wave propagation in a simply supported homogenous isotropic cylindrical panel embedded in a Winkler type of elastic medium is numerically solved for Zinc material by setting $p_2 = 0$ and Winkler elastic modulus $K = 1.5\times10^7$.For the purpose of numerical computation we consider the closed circular cylindrical shell with the center angle $\alpha = 2\pi$ and the integer n must be even since the shell vibrates in circumferential full wave. The frequency equation for a closed cylindrical shell can be obtained by setting $\delta = l\,(l = 1,2,3.....)$ where l the circumferential wave number in equations (18) is. The material properties of a Zinc is

$$\rho = 7.14\times10^3\,\text{kgm}^{-3}$$
$$\mu = 0.508\times10^{11}\,\text{Nm}^{-2}$$
$$\nu = 0.3,\ \lambda = 0.385\times10^{11}\,\text{Nm}^{-2}$$

The roots of the algebraic equation (11) were calculated using a combination of Birge-Vita method and Newton-Raphson method. In the present case simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. This combination has overcome the difficulties in finding the roots of the algebraic equations of the governing equations. A dispersion curve is drawn between the non-dimensional circumferential wave number versus dimensionless frequency for the different thickness parameters t*= 0.1,0.25, 0.5 with the axial wave number $t_L = 1$ and $t_L = 2$ is shown in Fig.2 and Fig.3 respectively.

From the Figs.2 and 3, it is observed that the non-dimensional frequency decreases rapidly to become linear at δ=3 for both $t_L = 1$ and $t_L = 2$ .when the thickness of the cylindrical panel is increased, the dimensionless frequency is decreases. This is the proper physical behavior of a cylindrical panel with respect to its thickness. The comparison of Fig.2 and Fig.3 shows that the non-dimensional frequency decrease exponentially for $\delta < 3$ in case of the axial wave number $t_L = 1$ and $t_L = 2$ for all value of t*,but the case when $\delta > 3$ the non-dimensional frequency is steady and slow for all values of t*.

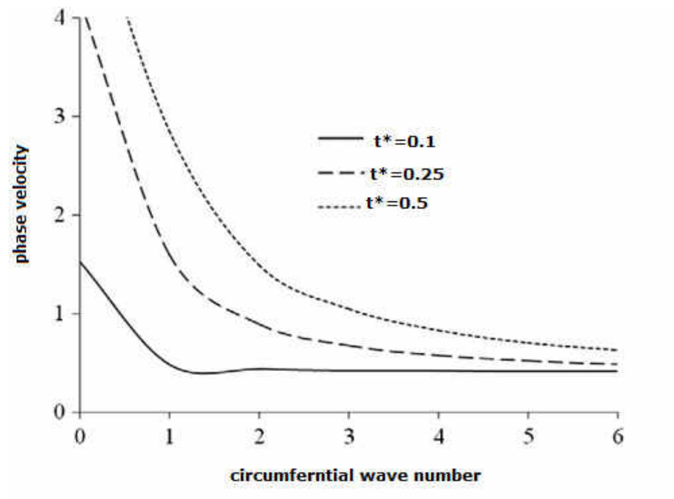

Figure 2. Variation of phase velocity with circumferential wave number of zinc cylindrical shell on elastic foundation $\left(t_L = 1, \nu = 0.3, K = 1.5\times10^7, p_2 = 0\right)$

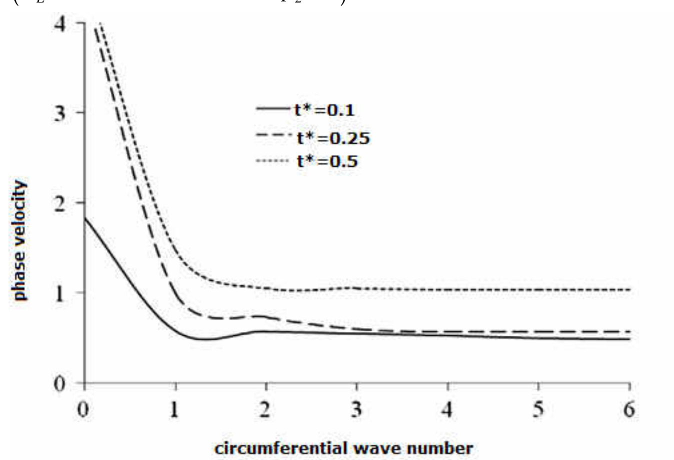

Figure 3. Variation of phase velocity with circumferential wave number of zinc cylindrical shell on elastic foundation $\left(t_L = 2, \nu = 0.3, K = 1.5\times10^7, p_2 = 0\right)$

Fig.4.Reveals that the variation of non dimensional frequency with the foundation parameter p_1 for different thickness parameter t^*. By comparing with the classical thin shell theory from [10], it is clear that the exact one agree well with the classical thin shell theory (CTST) with increase in thickness to mean radius ratio. This is identical to the well-known property of CTST for the uncoupled problem. However, for the thinner panel, when the effect of the foundation is obvious, the frequency of CTST will become smaller than the exact one. From the comparison of the dispersion curves in Fig.4 it is quite clear that due to the damping effect of the foundation on outer sides of the panel the non dimensional frequency falls significantly and become steady for $p_1 \geq 0.03$. The dispersion curves become more smoothen in this case than those in the absence of foundation

parameter because of the shock absorption nature of the foundation.

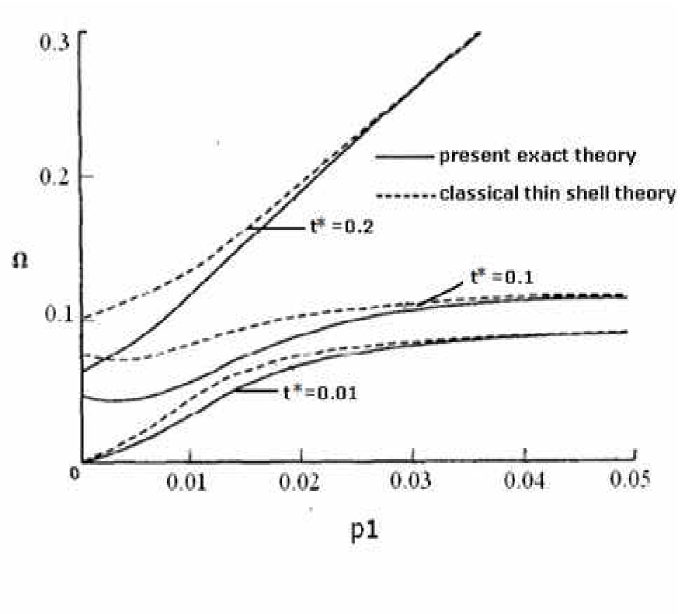

Figure 4. Variation of the foundation parameter p1 versus Non-dimensional frequency with different for $\left(t_L = 1, \nu = 0.3, n = 1, mR/L = 0.4, p_2 = 0\right)$

V. CONCLUSION

The three dimensional wave propagation of a homogenous isotropic cylindrical panel embedded on the Winkler type of elastic foundation has been considered for this paper. For this problem, the governing equations of three dimensional linear theory of elasticity have been employed and solved by the Bessel function with complex argument. Comparison of numerical results with those of related publications proves the feasibility and effectiveness of the present method. The effect of the circumferential wave number and the foundation parameter p_1 on the natural frequencies of a closed Zinc cylindrical shell is investigated and the results are presented as dispersion curves.

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