Peristaltic Transport of Two Immiscible Jeffrey Fluids in an Inclined Circular Tube

S. Sreenadh, D. Venkateswarulu Naidu and P. Devaki

Abstract— **The flow of a two immiscible Jeffrey fluids in an inclined circular tube is investigated. The motion is caused by the movement of peristaltic waves on the flexible walls of the tube. The equation for the interface is obtained as sixth degree polynomial. Moreover when the Jeffrey parameters, $\lambda'_1, \lambda''_1 \to 0$ our results agree well with those of Rao and Usha[12] for the peristaltic transport of two immiscible Newtonian fluids in a circular tube. Furthermore, the results obtained for the flow characteristics reveal many interesting behaviors that warrant further study of the peristaltic transport models with physiological fluids.**

Index Terms— Peristaltic wave, fluid flux, interface, immiscible fluids.

***MSC 2010 Codes* —76Txx, 76Zxx.**

I. INTRODUCTION

PERISTALTIC pumping is a form of fluid transport, generally from a region of lower to higher pressure, by means of a progressive wave of area contraction or expansion which propagates along the length of tube-like structure. Peristalsis occurs naturally as a means of pumping physiological fluids from one place in the body to another. Some electro-chemical reactions are held responsible for this phenomenon. This mechanism occurs in swallowing of food through the escophagus, the movement of chyme through the small intestine, the colonic transport in the large intestine, the passage of urine from the kidneys to the urinary bladder through the ureter, the spermatic flows in the ductus efferentes of the male reproductive tract, the vas deferens and the cervical canal, the movement of ovum in the fallopian tube and the vasomotion of small blood vessels. Even some worms move persistaltically. The major industrial application of this principle is in designing the roller pumps which are useful in pumping machinery. For example biomechanical pumps are fabricated to save blood or similar fluids from any possible contaminations arising out of contact with the pumping machinery while pumping the fluid.

The fluids present in the ducts of a living body are called biofluids. Some of the well known biofluids are Interstitial fluid, Lymph, Cerebrospinal fluid, Saliva, Mother Milk, Sweat and Gastric juice. It may not be possible to explain the behavior of these fluids by modeling them as Newtonian fluids. So there is a need to model such fluids as non-Newtonian fluids. Casson, Herschel-Bulkley, Bingham, Power-law, etc. are some of the non-Newtonian models usually accepted by researchers for the description of these biofluids. Among several non-Newtonian fluid models Jeffrey model is the simplest non-Newtonian fluid model. The assumption of Newtonian behavior of blood is acceptable for high shear rate flow through larger arteries . But, blood, being a suspension of cells in plasma, exhibits non-Newtonian behavior at low shear rate ($\dot{\gamma}$ < 10/sec) in small diameter arteries. Many researchers studied the non-Newtonian behavior of blood, Vajravelu et al. [1,2].

Bugliarello and Sevilla [3] and Cokelet[[4] have shown experimentally that for blood flowing through narrow blood vessels, there exists a peripheral layer of plasma and a core region of suspension of all the erythrocytes. Thus, for a realistic description of the blood flow, it is appropriate to treat blood as a two-fluid model with the suspension of all the erythrocytes in the core region as a non-Newtonian fluid and plasma in the peripheral region as a Newtonian fluid. Jeffrey fluid model is preferred by many authors to describe peristaltic transport of physiological fluids. Vajravelu et al. [5] studied the influence of heat transfer on peristaltic transport of Jeffrey fluid in a vertical porous stratum and many authors are now concentrating on this Jeffrey model as it is the simplest non-Newtonian fluid model describing some physiological and industrial fluids, [6-8].

As the blood can be described as two-fluid model based on its rheology, many authors have given considerable attension to two fluid models [9-15]. Recently Vajravelu et.al. [16] studied Peristaltic transport of Casson fluid in contact with Newtonian fluid in a circular tube with permeable walls., Narahari and Sreenadh [17] studied the peristaltic transport of Bingham fluid in contact with Newtonian fluid.

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In the present paper, peristaltic transport of two immiscible Jeffrey fluids in an inclined circular tube is studied. Here we observe that the interface is a sixth degree polynomial and it is a function of non-Newtonian Jeffrey parameter. The results are deduced and discussed.

II. Formulation of the Problem

Consider the peristaltic transport of a bio-fluid consisting of two immiscible and incompressible Jeffrey fluids of different viscosities μ_1 and μ_2 occupying the core and peripheral layers in a circular tube of radius 'a'. The axisymmetric geometry facilities the choice of the cylindrical polar coordinates system (R, θ, Z) to study the problem. The wall deformation due to the propagation of an infinite train of peristaltic waves is given by

$$R = H(Z,t) = a + b\sin\frac{2\pi}{\lambda}(Z - ct)$$

where b is the amplitude, λ is the wavelength and c is the wave speed. The subsequent deformation of the interface separating the core and the peripheral-layer is denoted by $R = H\ (Z,t)$ which is not known a priori.

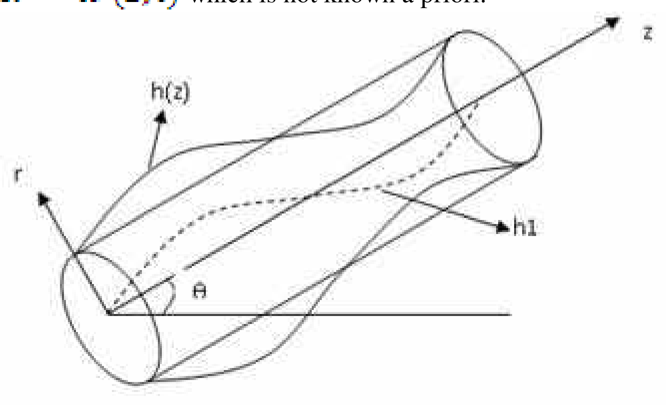

Figure 1. Physical Model

Under the assumptions that the tube length is an integral multiple of the wavelength λ and the pressure difference across the ends off the tube is a constant, together with an additional condition of periodicity of the interface with the same period as the peristaltic wave, the flow becomes steady in a frame (r, θ, z) moving with velocity c away from the fixed frame (R, θ, Z) and the transformation is given by

$$r = R, \qquad \theta = \Theta, \qquad z = Z - ct,$$

$$\psi = \Psi - \frac{R^2}{2} \qquad and \qquad p(Z,t) = p(z) \tag{1.1}$$

In equation (1.1), Ψ and ψ are the stream functions in the moving and stationary frames respectively. The assumptions of negligible surface tension on the interface makes the pressure p to remain a constant in any cross-section of the tube given by z = constant. Using the non-dimensional quantities.

$$\bar{r} = \frac{r}{a};\ \bar{z} = \frac{z}{\lambda};\ \bar{t} = \frac{ct}{\lambda},\ h = \frac{H}{a};\ h_1 = \frac{H_1}{a}; \in = \frac{b}{a},$$

$$\psi = \frac{\psi}{\pi a^2 c}; \bar{w} = \frac{1}{r}\frac{\partial\psi}{\partial\bar{r}}; \bar{u} = -\frac{1}{r}\frac{\partial\psi}{\partial\bar{z}},$$

$$\bar{\mu}_r = 1, \qquad 0 \le \bar{r} \le h_1,$$

$$= \mu\left(= \frac{\mu_2}{\mu_1}\right); h_1 \le \bar{r} \le h,$$

and
$$p = \frac{a^2}{\pi\lambda\mu_1 c}p, \tag{1.2}$$

where $\bar{u}$, and $\bar{w}$ are the radial and axial velocities in the wave frame, in the equations governing the motion, under the lubrication approach, we get (dropping the bars):

Core layer

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{1}{1+\lambda'_1} r\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial\psi_1}{\partial r}\right\}\right] = \frac{\partial p}{\partial z} - \eta Sin\beta$$

$$0 = \frac{\partial p}{\partial r} \tag{1.3a}$$

Peripheral layer

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{1}{1+\lambda''_1} r\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial\psi_2}{\partial r}\right\}\right] = \frac{\partial p}{\partial z} - \eta Sin\beta$$

$$0 = \frac{\partial p}{\partial r} \tag{1.3b}$$

The non dimensional boundary conditions are

$$\psi_1 = 0 \qquad \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi_1}{\partial r}\right) = 0 \qquad \text{at } r = 0 \tag{1.4}$$

$$\psi_2 = \frac{q}{2} \qquad \frac{1}{r}\frac{\partial\psi_2}{\partial r} = -1 \qquad \text{at } r = h \tag{1.5}$$

$$\psi_1 = \psi_2 = \frac{q_1}{2} \qquad \text{at } r = h_1 \tag{1.6}$$

$$\frac{\partial\psi_1}{\partial r} = \frac{\partial\psi_2}{\partial r} \qquad \text{at } r = h_1 \tag{1.7}$$

where q and q_1 are the total and the core fluxes respectively across my cross-section in the wave frame. Further, the velocity and the shear stress are continuous across the interface. The peripheral-layer flux is given by $q_2 = q - q_1$. It follows from the incompressibility of the fluids and the lubrication theory that q, q_1 and q_2 are independent of z.

$$\frac{1}{1+\lambda'_1}\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial\psi_1}{\partial r}\right\} = \frac{\mu}{1+\lambda''_1}\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial\psi_2}{\partial r}\right\} at \quad r = h_1 \tag{1.8}$$

The stream function is determined using the boundary conditions mentioned earlier together with the boundary conditions at the ends of the pipe given by specifying $\bar{Q}$ or the pressure difference Δp across one wavelength.

III. SOLUTION

Solving equation (1.3) together with the boundary conditions (1.4)-(1.6), we obtain the stream function in the core and the peripheral-layer as

$$\psi_1 = -\frac{r^2}{2} + \left(\frac{q+h^2}{2}\right)\frac{\mu r^2(2h_1^2 - r^2)(1+\lambda'_1) + 2r^2(h^2-h_1^2)(1+\lambda''_1)}{h^4(1+\lambda''_1) + h_1^4(\mu(1+\lambda'_1) - (1+\lambda''_1))}$$

for $0 \le r \le h_1$ (1.9a)

$$\psi_2 = -\frac{r^2}{2} + \left(\frac{q+h^2}{2}\right)\frac{\mu h_1^4(1+\lambda'_1) + (1+\lambda''_1)(2h_1^2(h^2-h_1^2) + (r^2-h_1^2)(2h^2-r^2-h_1^2)}{h^4(1+\lambda''_1) + h_1^4(\mu(1+\lambda'_1) - (1+\lambda''_1))}$$

for $h_1 \le r \le h$ (1.9b)

And the velocity functions are given by

$$w_1 = -1 - \frac{2\mu(q+h^2)((r^2-h_1^2)(1+\lambda'_1) + (1+\lambda''_1)(h_1^2-h^2)}{h^4(1+\lambda''_1) + h_1^4(\mu(1+\lambda'_1) - (1+\lambda''_1))}$$

for $0 \le r \le h_1$

$$w_2 = -1 - \frac{2\mu(q+h^2)((r^2-h^2)(1+\lambda''_1)}{h^4(1+\lambda''_1) + h_1^4(\mu(1+\lambda'_1) - (1+\lambda''_1))}$$

for $h_1 \le r \le h$

The stream function for the case of a single fluid is obtained by putting $\mu = 1$ in either (1.9a) or (1.9b). The pressure gradient is obtained by using equation (1.9) in (1.3) as

$$\frac{dp}{dz} = \frac{-8\mu(q+h^2)}{h_1^4(\mu(1+\lambda'_1) - (1+\lambda''_1)) + h^4(1+\lambda''_1)} + \eta Sin\beta \tag{1.10}$$

Integrating (1.10) over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\begin{aligned}\Delta p &= -8\mu I_1 - 8\mu I_2 + \eta Sin\beta \\ &= -8\mu(\overline{Q} - + -\frac{\varepsilon^2}{2})I_1 - 8\mu I_2 + \eta Sin\beta\end{aligned} \tag{1.11}$$

where

$$I_1 = \int_0^1 \frac{dz}{h^4(1+\lambda''_1) + h_1^4(\mu(1+\lambda'_1) - (1+\lambda''_1)} \text{ and}$$

$$I_1 = \int_0^1 \frac{h^2}{h^4(1+\lambda''_1) + h_1^4(\mu(1+\lambda'_1) - (1+\lambda''_1)}dz$$

The time averaged flux at zero pressure rise is denoted by $\overline{Q}_0$ and the pressure rise required to produce zero average flow rate is denoted by Δp_0.

IV. THE EQUATION FOR THE INTERFACE

The interface is also a streamline as seen from the boundary condition (1.7). For a given geometry of the wave and the time averaged flux $\overline{Q}$, the unknown interface $h_1(z)$ is solved from (1.9) using the boundary condition (1.7). Substituting (1.7) in (1.9), we get the algebraic equation governing the interface $h_1(z)$ as

$$\begin{aligned}&h_1^6(\mu(1+\lambda'_1) - (1+\lambda''_1)) + h_1^4[(q+h^2)(2(1+\lambda''_1) - \\ &\mu(1+\lambda'_1) + q_1(\mu(1+\lambda'_1) - (1+\lambda''_1))] + h_1^2h^2(1+\lambda''_1) \\ &(-2q-h^2) + q_1h^4(1+\lambda''_1) = 0\end{aligned} \tag{1.12}$$

$$\begin{aligned}q_1 = &\frac{\alpha^2}{\alpha^4(\mu(1+\lambda'_1) - (1+\lambda'_2)) + (1+\lambda'_2)}[\alpha^4((1+\lambda''_1) \\ &-\mu(1+\lambda'_1)) + \alpha^2(q+1)(\mu(1+\lambda'_1) - 2(1+\lambda''_1)) \\ &+(1+2q)(1+\lambda''_1)\end{aligned} \tag{1.13}$$

V. RESULTS AND DISCUSSIONS

The peristaltic transport of two immiscible Jeffery fluids in an inclined circular tube is investigated. The effect of different parameters on the velocity profiles , the shape of the interface and the pressure rise are observed. Moreover when the Jeffrey parameters $\lambda'_1, \lambda''_1 \to 0$ our results agree well with those of Rao and Usha [12].

The velocity profiles are shown in figs(2)-(5). It is observed from fig(2) that as the amplitude ratio, φ increases, the velocity is increasing. From fig(3), it is observed that as the viscosity ratio increases, the velocity is decreasing. It is shown in fig (4) that as the Jeffrey parameter of the fluid in the peripheral region increases the velocity is decreasing. The same behavior is observed with the change in the Jeffrey parameter of the fluid in the core region, which is depicted in fig (5). The shape of the interface for different amplitude ratios is shown in fig(6). High amplitude gives rise to a thicker core layer in the first half wave length of the tube region and high amplitude gives rise to a thinner core layer in the second half wave length region of the tube. The shape of the interface for different viscosity ratios is given in fig(7). The shape of interface is similar to Hagen-Poisuelle flow of two Newtonian Immiscible fluids studied by Rao and Usha[12], but there is a difference in the magnitude due to the Jeffrey parameters, λ'_1, λ''_1. High viscosity gives rise to a thicker core layer in the first half wave length of the tube region and high viscosity gives rise to a thinner core layer in the second half wave length region of the tube

The variation of pressure rise with time averaged flux is calculated for different values of amplitude ratio which is shown in fig(8). It is observed that observed that as the amplitude increases, the pressure rise increases for a given flux.

VI. CONCLUSIONS

The effects of different parameters on the velocity profiles, the shape of the interface and the pressure rise are observed with are explained in brief:

1. As the Jeffrey parameter increases the velocity decreases in the core and the peripheral regions;

2. The interface is a sixth degree equation and it is a function of Jeffrey parameters, λ'_1, λ''_1
3. Velocity increases as the amplitude ratio increases;
4. Velocity decreases with increasing viscosity ratio;
5. High viscosity gives thicker core layer in the first half wave length region of the tube, where as it gives thinner core layer in the second half wave length region of the tube ;
6. High amplitude gives thicker core layer in the first half wave length region of the tube, where as it gives thinner core layer in the Second half wave length region of the tube ; and
7. Pressure rise increases as the amplitude increases.

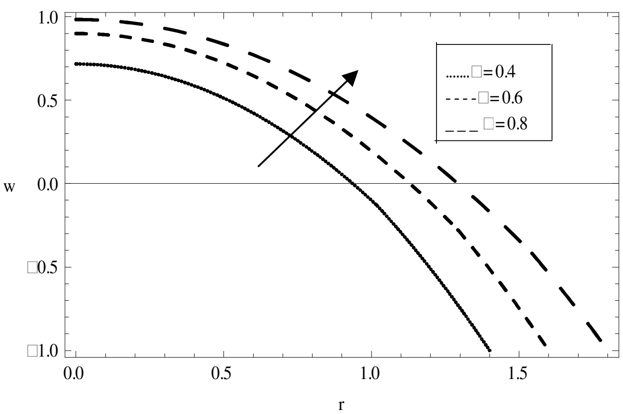

Fig2: The velocity profiles for different amplitude with $\lambda'_1 = 0.3\, and \quad \lambda''_1 = 0.5$

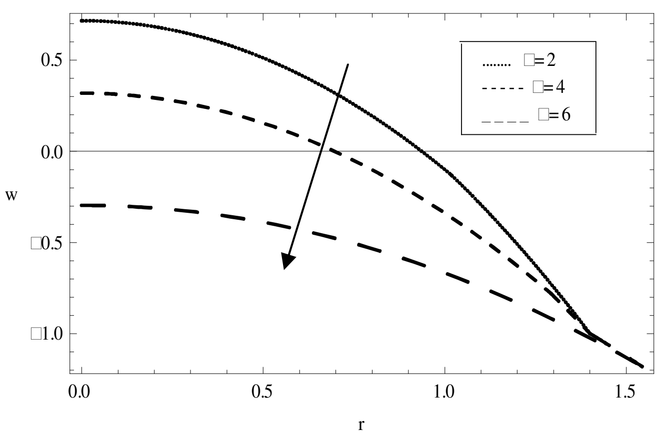

Fig 3: The velocity profiles for different μ with $\lambda'_1 = 0.3\, and \quad \lambda''_1 = 0.5$

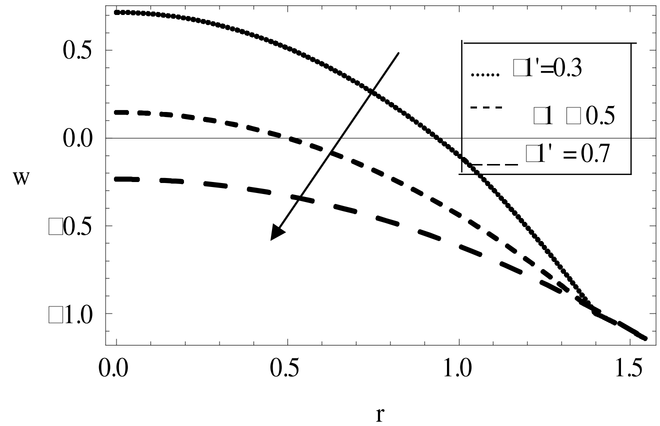

Fig 4: Velocity profile for different Jeffrey parameter λ'_1 With $\lambda''_1 = 0.5$.

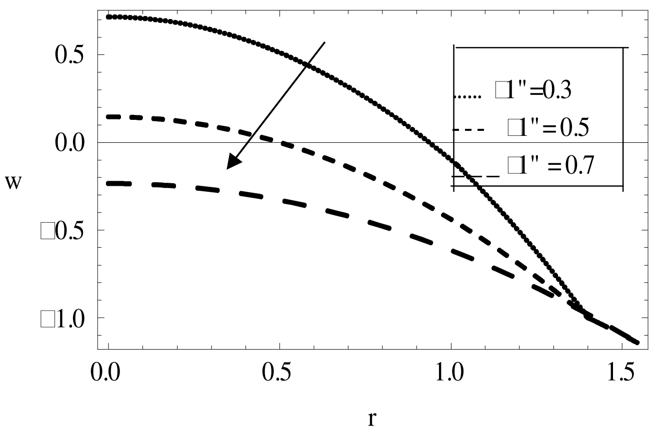

Fig 5: Velocity profile for different Jeffrey parameter λ''_1 With $\lambda'_1 = 0.3$.

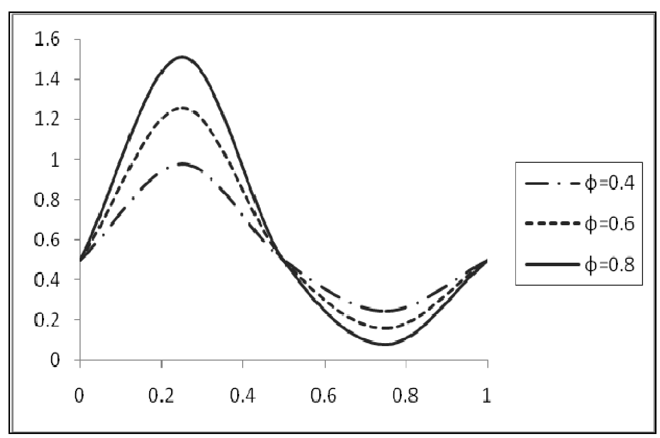

Fig 6: The shape of the interface for different amplitude ratiowith $\lambda'_1 = 0.3\, and \quad \lambda''_1 = 0.5$

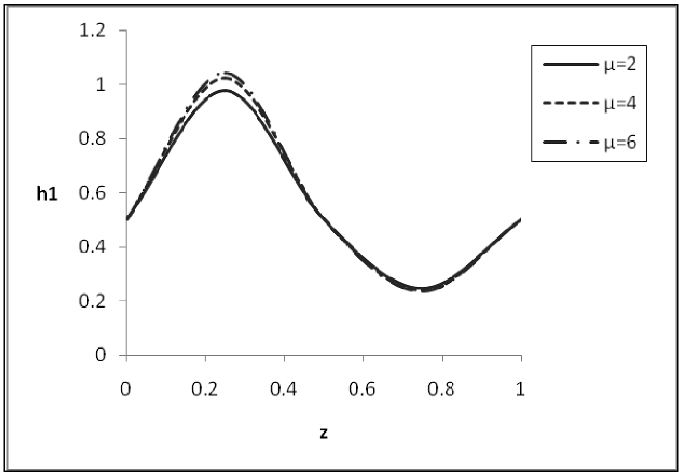

Fig 7: The shape of the interface for different viscosity ratioswith $\lambda'_1 = 0.3\, and \quad \lambda''_1 = 0.5$

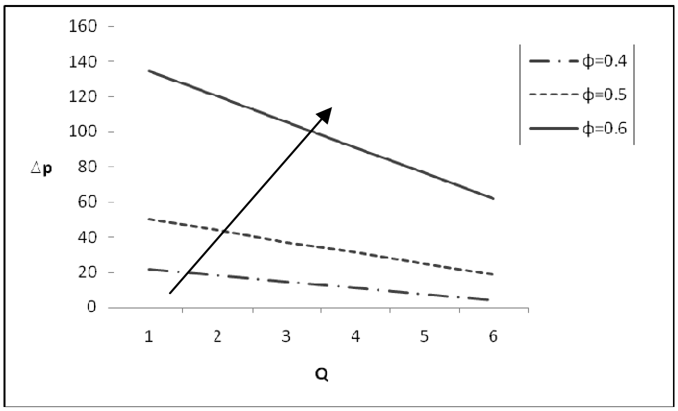

Fig 8:Variation of Δp with Q for different amplitude, φ with $\lambda'_1 = 0.3\, and \quad \lambda''_1 = 0.5$

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