Robustness of Complete Diallel Cross Plan using Partially Balanced Block Designs against the Unavailability of Two Blocks

R. Shanmugathai and M.R. Srinivasan

Abstract—**Mating designs are the study of progenies developed through various methods like Diallel Cross plans which are subjected to Incomplete Block Designs. The concept of robustness in a design has been studied and available in the literature. The effects of missing blocks on Complete Diallel Cross designs are examined in this study. A-efficiencies based on non-zero eigenvalues suggest that these designs are fairly robust. The investigation shows that Partially Balanced Incomplete Block Designs are fairly robust in terms of efficiency. In this paper, the robustness of Partially Balanced Incomplete Block Design when two blocks are lost has also been discussed.**

Index Terms— **Partially Balanced Incomplete Block Design; Efficiency of residual design; Mating Design; Youden Square Design; Latin Square Design.**

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I. INTRODUCTION

DIALLEL mating design has become most popular among the breeders and geneticists. It consists of set of all possible single crosses among a given set of n lines. Griffing [14] made further contributions to Diallel Mating Design by developing suitable models and methods of analysis. Schmidt [20] introduced the concept of Diallel crossing as a means of comparing the breeding values of parents. It was further adopted in other situation by Comstock and Robinson [4]. Curnow [3] discussed the construction of Diallel crossed experiment using Randomized Block Design yields a large number of crosses various other authors develop the construction of Diallel Crossed experiment using Balanced Incomplete Block Design, Partially Balanced Incomplete Block Design and so on. After getting the cross, one has to test and verify the best or promising variety of crop.

Diallel Cross experiment is said to be a Complete Diallel Cross (CDC) design if all potential crosses occur at least once in the design, even though they need not be replicated the same number of times, Ghosh and Desai [9]. When the number of lines is increased, the number of crosses becomes so large that there may not be enough experimental material to accommodate a Complete Diallel Cross design. In these situations, a Partial Diallel Cross (PDC) design involving fewer crosses may be chosen to estimate the general combining ability of the p lines, as considered by Gilbert [13] and Kempthorne and Curnow [17].

Diallel crossing is a very useful method for conducting plant and animal breeding experiments, especially for estimating combining ability effects of lines. Diallel crosses, in which all possible distinct crosses in pairs among the available lines are taken, are called Complete Diallel Crosses (CDC). Diallel crosses in which only a fraction of all possible crosses among the available lines are taken are called Partial Diallel Crosses (PDC) Ghosh and Divecha, [11]. Partial Diallel Cross designs have been constructed by several authors, Curnow [3], Fyfe and Gilbert [7], Das and Sivaram [5], Ghosh and Biswas [8], Hinkelmann and Kempthorne [15], Das and Kageyama [6], Ghosh and Divecha [11], Kaushik, Puri and Mehta [16], Arya [1], Bose and Nair [2], Ghosh, Divecha and Das [12]. Most of these plans are laid out as Incomplete Block Designs and these Incomplete Block Designs require special analysis. Partial Diallel Crossing is discussed in detail by Kempthorne and Curnow [17].

Ghosh and Desai [9] and [10] obtained the robustness of Complete Diallel Crosses Plan against the unavailability of one block and also for those plans, which have unequal number of crosses in a block. Further, Ghosh and Biswas [8] also pointed out the robustness for Complete Diallel Crosses Plan, which are binary, balanced against the loss of one block. Bose and Nair [2] introduced a class of binary, equireplicate and proper designs that are called Partially Balanced Incomplete Block designs (PBIBD). In these designs, the variance of every estimated elementary contrast among treatment effects is not the same and hence the name PBIBD. The definition of PBIBD is based on the association schemes. Hinkelmann and Kempthorne [15] have presented a modified definition of PBIBD.

Lattice designs have the drawback that they are available only for number of treatments which are perfect squares or cubes. The limitation was removed considerably by Bose and Nair [2] by evolving Partially Balanced Incomplete Block (PBIB) designs. Das and Kageyama [6] showed that Balanced Incomplete Block Designs and extended Balanced Incomplete Block Designs are fairly robust against the unavailability of

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$s(s \leq k)$ observations in any block, while any Youden Design and Latin Square Designs are found to be fairly robust against the loss of any one column.

Prescott and Mansson [19] investigated the effect of missing observations on complete diallel cross designs. They examined the robustness of CDC using BIBD and PBIBD. A-efficiencies, based on average variances of the elementary contrasts of the line effects, suggest that Complete Diallel Cross Design is fairly robust against the unavailability of observations. Mansson and Prescott [18] examined the robustness of CDC against the loss of a block of observations using BIBD and PBIBD. They found a simple generalized inverse for the information matrix of the line effects, which allows evaluation of expressions for the variances of the line-effects differences with and without the missing block. A-efficiencies, based on average variances of the elementary contrasts of the line-effects, suggest that CDC is fairly robust against the unavailability of a block.

This paper looks into the robustness of CDC plan using Partially Balanced Incomplete Block Design when two blocks are lost from a design. The robustness criteria against the unavailability of data are: (*i*) to get the connectedness of the residual design; (*ii*) to have the variance balance of the residual design; (*iii*) to consider the A-efficiency of residual design for the robustness study. So far, robustness of Incomplete Block Designs and complete block designs are carried out against loss of either $s(s \leq k)$ observations in one block.

This investigation, consider a connected Complete Diallel Crosses Plan *D*. Let *D** be the residual design obtained when two blocks are lost. Assume *D** to be connected. In this case, the criterion of robustness against the unavailability of two blocks is the overall A-efficiency, of the residual design *D**, is given by,

$$e(s) = \frac{\varphi_2(s)}{\varphi_1(s)} \tag{1}$$

Efficiencies of 63 Partially Balanced Incomplete Block Designs are computed. Thus, it shows that design is fairly robust against loss of two blocks. *C** matrix and its non-zero eigenvalues are also computed with corresponding multiplicity.

II. C-Matrix of CDC Plan

We know that for any block design *C* matrix can be defined as,

$$C = rI_v - \frac{NN'}{k}$$

Now *C* matrix is given as,

$$C = \begin{bmatrix} r(k-1) & \lambda_1 & \lambda_1 \\ \lambda_1 & r(k-1) & \lambda_1 \\ \lambda_1 & \lambda_1 & r(k-1) \end{bmatrix} - \begin{bmatrix} r(k-1)^2 & \lambda_1(k-1)^2 & \lambda_1(k-1)^2 \\ \lambda_1(k-1)^2 & r(k-1)^2 & \lambda_1(k-1)^2 \\ \lambda_1(k-1)^2 & \lambda_1(k-1)^2 & r(k-1)^2 \end{bmatrix} * \left(\frac{2}{k(k-1)}\right)$$

$$C = \Theta\left[I_v - \frac{E_{vv}}{v}\right]$$

So, $\Theta = \left[\frac{\lambda_1 v(k-2)}{k}\right]$

The non - zero eigenvalues of P_d matrix and its corresponding multiplicity of Partial Diallel Cross Design can be given by, $\Theta = \left[\frac{\lambda_1 v(k-2)}{k}\right]$ with multiplicity (v-1), respectively.

III. Robustness of CDC Plan using PBIBD Against the Unavailability of Two Blocks

Consider a Partially Balanced Incomplete Block Design D having parameters $v = p, b, r, k, \lambda_1, \lambda_2, \delta, m, n.$ Suppose two blocks of a Partially Balanced Incomplete Block Design are lost, under this situation, the following three cases are considered:

Case (*i*): Unavailability of two blocks where the number of common lines between two blocks is zero.

Case (*ii*): Unavailability of two blocks where the number of common lines between two blocks are one.

Case (*iii*): Unavailability of two blocks where the number of common lines between two blocks are two.

For all the three cases, when two blocks are lost in a Partially Balanced Incomplete Block Design. The efficiency factor depends upon the common number of lines between two lost blocks. The efficiency for all the three cases, when the common number of lines between two lost blocks are 0, 1, 2, 3, . . . , (k-1), k respectively are studied. Here, the robustness criterion of Partially Balanced Incomplete Block Design further discussed for the different value of common number of lines between two blocks.

Case (*i*): Unavailability of two blocks where the number of common lines between two blocks are zero.

Consider a Partially Balanced Incomplete Block Design *D* with parameters $v = p, b, r, k, \lambda_1, \lambda_2, \delta, m, n$. The *C* matrix of the design is given by

$$C = \theta\left[I_v - \frac{E_{vv}}{v}\right],$$

where $\theta = \left[\frac{\lambda_1 v\ (k-2)}{k}\right]$ is the eigenvalues of *C* matrix of design *D* with multiplicity (v-1).

Let us consider that two blocks be lost. Call this design as a residual design assuming a residual design D^* is a connected design. Let the blocks be b_i and b_j and their zero line be common between two lost blocks that is $\eta(b_i \cap b_j) = 0$. Each line that is present in the two lost blocks will be replicated (r-1) times. All remaining lines will be replicated r times in design. Let C^* be the information matrix of design D^*. For this design D^*, the diagonal element of C^* matrix are as follows,

1. $C_{jj} = \frac{(r-1)(k-1)}{k}$, where j denotes those lines which are present in both the lost blocks but are distinct.
2. $C_{ll} = \frac{r(k-1)}{k}$, where l denotes the remaining lines.

Similarly, in the residual design, pair of lines occurs together in two ways, which is given as λ_1, λ_2. Pattern of λ_i (i = 1, 2) are as follows,

1. $\lambda_1 = (\lambda_1 - 1)$, for those lines, which are present in two lost blocks.
2. $\lambda_2 = \lambda_1$, for remaining pair of lines.

The C^* matrix of design D^* can be written as,

$$\text{k(k-2)C*} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \qquad (*)$$

where

$$\varepsilon_{11} = (k-2)(\lambda v - k)I_k - (k-2)(\lambda - 1)J_{kk}$$
$$\varepsilon_{12} = -(k-2)\lambda J_{kk}$$
$$\varepsilon_{13} = -(k-2)\lambda J_{k(v-2k)}$$
$$\varepsilon_{21} = -(k-2)\lambda J_{kk}$$
$$\varepsilon_{22} = (\lambda v - k)(k-2)I_k - (k-2)(\lambda - 1)J_{kk}$$
$$\varepsilon_{23} = -(k-2)\lambda J_{k(v-2k)}$$
$$\varepsilon_{31} = -(k-2)\lambda J_{(v-2k)k}$$
$$\varepsilon_{32} = -(k-2)\lambda J_{(v-2k)k}$$
$$\varepsilon_{33} = (k-2)\lambda v(I_{(v-2k)} - v^{-1}(k-2)J_{(v-2k)(v-2k)})$$

The non – zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(k-2)(\lambda v-k)}{k}$, with multiplicity 2($k$-1).
2. $\frac{(k-2)(\lambda v)}{k}$, with multiplicity ($v-2k$+1).

Theorem 1: Partially Balanced Incomplete Block Designs with parameters $v, b, r, k, \lambda_1, \lambda_2, m, n$ are fairly robust against the unavailability of two blocks, where the number of common line between two lost blocks are zero, provided the overall efficiency of the residual design is given by,

$$\text{e(s)} = \frac{(\lambda_1 v - k)(v-1)}{(\lambda_1 v - k)(v - 2k + 1) + 2\lambda_1 v(k-1)} \qquad (2)$$

Proof: Without loss of generality, let two blocks be lost from design D where number of common line between two blocks is zero that is. $\eta(b_i \cap b_j) = 0$, C^* matrix of the residual design is given by Eq. (*).

The non-zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1., $\frac{(k-2)(\lambda_1 v-k)}{k}$ with multiplicity 2(k-1).

2., $\frac{(k-2)(\lambda_1 v)}{k}$ with multiplicity (v-2k+1).

Further, overall A-efficiency is calculated as,

$$\text{e(s)} = \frac{\varphi_2(s)}{\varphi_1(s)} \qquad (3)$$

That is,

$$\varphi_2(s) = \frac{k(v-1)}{\lambda_1 v(k-2)} \qquad (4)$$

$$\varphi_1(s) = \frac{k(\text{v-2k+1})}{(k-2)(\lambda_1 v)} + \frac{2(k-1)k}{(k-2)(\lambda_1 v - k)} \qquad (5)$$

Finally, A- efficiency is given by,

$$\text{e(s)} = \frac{(\lambda_1 v - k)(v-1)}{(\lambda_1 v - k)(v - 2k + 1) + 2\lambda_1 v(k-1)} \qquad (6)$$

Example 1: Let D represent the Partially Balanced Incomplete Block Design with parameters $v = p = 9$, $b = 9$, $r = 3$, $k = 3$, $\lambda_1 = 1$, $\lambda_2 = 0$, $m = 6$, $n = 2$. Design D is given by,

Table 1: 9 lines of PBIB design of Das and Giri (1979)

Block	PBIBD			Crosses in the PBIB design		
1	1	2	3	1 × 2	1 × 3	2 × 3
2	1	6	4	1 × 6	1 × 4	4 × 6
3	1	7	5	1 × 7	1 × 5	5 × 7
4	6	8	3	6 × 8	3 × 6	3 × 8
5	6	9	5	6 × 9	5 × 6	5 × 9
6	7	8	4	7 × 8	4 × 7	4 × 8
7	**7**	**9**	**3**	**7 × 9**	**3 × 7**	**3 × 9**
8	**2**	**8**	**5**	**2 × 8**	**2 × 5**	**5 × 8**
9	2	9	4	2 × 9	2 × 4	4 × 9

When two blocks containing block 7 and block 8 are lost, and number of common lines between two blocks is zero, C^* matrix of the residual design is given by,

$$3C^* = \begin{bmatrix} 6 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 6 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 6 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 6 & 0 & 0 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 & 6 & 0 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 & 0 & 6 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 8 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 8 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 8 \end{bmatrix}$$

The non-zero eigenvalues with their corresponding multiplicities are,

1. $\frac{6}{3}$, with multiplicities 4.
2. $\frac{9}{3}$, with multiplicities 4.

The overall A- efficiency of the design is,
$e(s) = 0.8$

Case (*ii*): Unavailability of two blocks where number of common lines between two blocks are one.

Consider a Partially Balanced Incomplete Block design with parameters $v, b, r, k, \lambda_1, \lambda_2, m, n.$ Let two blocks be lost. Call this design as a residual design and assume that the residual design D^* is a connected design. Let the blocks be b_i and b_j and one line be common between two lost blocks i.e. $\eta\left(b_i \cap b_j\right) = 1.$ Here this line is repeated (r-2) times. Similarly those lines which are present in the two lost blocks but are not common will be replicated (r-1) times. The remaining lines will be replicated r times in design. Let C^* be the information matrix of design D^*. For this design D^*, the diagonal element of C^* matrix are as follows,

1. $C_{ii} = \frac{(r-2)(k-1)}{k}$ where i denotes those lines which are present in both the lost blocks but are distinct.
2. $C_{jj} = \frac{(r-1)(k-1)}{k}$ where j denotes those lines which are presents in both the lost blocks but are distinct.
3. $C_{ll} = \frac{r(k-1)}{k}$, where l denotes the remaining lines.

Similarly, in the residual design, pair of lines occurs together in three ways, which is given as
$\lambda_1, \lambda_2, \lambda_3.$ Pattern of λ_i, $(i = 1, 2, 3)$ are as follows,

1. $\lambda_1 = (\lambda_1$-1), for those lines, which are present in two lost blocks.
2. $\lambda_2 = (\lambda_1$-1), for those lines, which are present in two lost blocks.
3. $\lambda_3 = \lambda_1$, for remaining lines.

The C^* matrix of design D^* can be written as,

$$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & \varepsilon_{14} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & \varepsilon_{24} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & \varepsilon_{34} \\ \varepsilon_{41} & \varepsilon_{42} & \varepsilon_{43} & \varepsilon_{44} \end{bmatrix} \qquad (**)$$

where

$\varepsilon_{11} = (k-2)(\lambda_1 v - 2k + 1)I_1 - (k-2)(\lambda_1 - 1)J_{11}$

$\varepsilon_{12} = -(k-2)(\lambda_1 - 1)J_{1(k-1)}$

$\varepsilon_{13} = -(k-2)(\lambda_1 - 1)J_{1(k-1)}$

$\varepsilon_{14} = -(k-2)\lambda_1 J_{1(v-2k+1)}$

$\varepsilon_{21} = -(k-2)(\lambda_1 - 1)J_{1(k-1)}$

$\varepsilon_{22} = (k-2)(\lambda_1 v - k)I_{(k-1)} - (k-2)(\lambda_1 - 1)J_{(k-1)(k-1)}$

$\varepsilon_{23} = -(k-2)\lambda_1 J_{(k-1)(k-1)}$

$\varepsilon_{24} = -(k-2)\lambda_1 J_{(k-1)(v-2k+1)}$

$\varepsilon_{31} = -(k-2)(\lambda_1 - 1)J_{(k-1)1}$

$\varepsilon_{32} = -(k-2)\lambda_1 J_{(k-1)(k-1)}$

$\varepsilon_{33} = (k-2)(\lambda_1 v - k)I_{(k-1)} - (k-2)(\lambda_1 - 1)J_{(k-1)(k-1)}$

$\varepsilon_{34} = -(k-2)\lambda_1 J_{(k-1)(v-2k+1)}$

$\varepsilon_{41} = -(k-2)\lambda_1 J_{(v-2k+1)1}$

$\varepsilon_{42} = -(k-2)\lambda_1 J_{(v-2k+1)(k-1)}$

$\varepsilon_{43} = -(k-2)\lambda_1 J_{(v-2k+1)(k-1)}$

$\varepsilon_{44} = (k-2)\lambda_1 v I_{(v-2k+1)} - (k-2)v^{-1}J_{(v-2k+1)(v-2k+1)}$

The non–zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(k-2)(\lambda_1 v - 2k + 1)}{k}$, with multiplicity 1.
2. $\frac{(k-2)(\lambda_1 v - k)}{k}$, with multiplicity 2($k$-1).
3. $\frac{(k-2)(\lambda_1 v)}{k}$, with multiplicity ($v-2k$+1).
4. $\frac{(k-2)(\lambda_1 v - 1)}{k}$, with multiplicity 1.

Theorem 2: Partially Balanced Incomplete Block Designs with parameters v, b, r, k, λ_1, λ_2, m, n are fairly robust against the unavailability of two blocks, where number of common line between two blocks is one, provided the overall efficiency of the residual design is given by

$$e(s) = \frac{L}{M}, \qquad (7)$$

where

$$L=(v-1)(\lambda_1 v-2k)(\lambda_1 v-1)(\lambda_1 v-2k+1)$$

and

$$M=(\lambda_1 v-2k)(\lambda_1 v-1)((v-2k+1)(\lambda_1 v-2k+1)+\lambda_1 v) + (\lambda_1 v-2k+1)\lambda_1 v(2(k-2)(\lambda_1 v-1)+(\lambda_1 v-2k))$$

Proof: Without loss of generality, let two blocks be lost from design D where number of common line between two blocks is one i.e. $\eta(b_i \cap b_j)=1$, C^* matrix of the residual design is given in Equation (**).

The non–zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(k-2)(\lambda_1 v-2k+1)}{k}$, with multiplicity 1.
2. $\frac{(k-2)(\lambda_1 v-k)}{k}$, with multiplicity 2($k$-1).
3. $\frac{(k-2)(\lambda_1 v)}{k}$, with multiplicity ($v-2k$+1)
4. $\frac{(k-2)(\lambda_1 v-1)}{k}$, with multiplicity 1.

Further, overall A-efficiency is calculated as,

$$e(s)=\frac{\varphi_2(s)}{\varphi_1(s)} \tag{8}$$

That is,

$$\varphi_2(s)=\frac{k(v-1)}{\lambda_1 v(k-2)} \tag{9}$$

$$\varphi_1(s)=\frac{k(v-2k+1)}{(k-2)\ (\lambda_1 v)}+\frac{k}{(k-2)(\lambda_1 v\text{-}2k+1)}+\frac{2(k-2)k}{(k-2)(\lambda_1 v-k)}+\frac{k}{(k-2)(\lambda_1 v\text{-}1)} \tag{10}$$

Finally, A- efficiency is given by,

$$e(s)=\frac{L}{M} \tag{11}$$

Example 2: Let D represent the Partially Balanced Incomplete Block Design with parameters $v=p=9$, $b=9$, $r=3$, $k=3$, $\lambda_1=1$, $\lambda_2=0$, $m=6$, $n=2$. Design D is given by,

Table 2: 9 lines of PBIB design of Das and Giri (1979)

Block	PBIBD			Crosses in the PBIB design		
1	1	2	3	1 × 2	1 × 3	2 × 3
2	1	6	4	1 × 6	1 × 4	4 × 6
3	1	7	5	1 × 7	1 × 5	5 × 7
4	6	8	3	6 × 8	3 × 6	3 × 8
5	6	9	5	6 × 9	5 × 6	5 × 9
6	**7**	**8**	**4**	**7 × 8**	**4 × 7**	**4 × 8**
7	**7**	**9**	**3**	**7 × 9**	**3 × 7**	**3 × 9**
8	2	8	5	2 × 8	2 × 5	5 × 8
9	2	9	4	2 × 9	2 × 4	4 × 9

When two blocks containing block 6 and block 7 are lost, and number of common lines between two blocks is one, C^* matrix of the residual design is given by,

$$3C^*=\begin{bmatrix} 4 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \\ 0 & 6 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 6 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & 6 & 0 & -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & 0 & 6 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 8 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 8 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 8 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 8 \end{bmatrix}$$

The non-zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{4}{3}$, with multiplicities 1.
2. $\frac{6}{3}$, with multiplicities 2.
3. $\frac{8}{3}$, with multiplicities 2.
4. $\frac{9}{3}$, with multiplicities 4.

The overall A- efficiency of the design is,
$e(s)=0.598131$

Case (*iii*): Unavailability of two blocks where number of common lines between two blocks are two.

Consider a Partially Balanced Incomplete Block Design D with parameters v, b, r, k, λ_1, λ_2, m, n. Let two blocks be lost. Call this design as a residual design and assume that the residual design D^* is a connected design. Let the blocks be b_i and b_j and number of common lines between two blocks be two, i.e. $\eta(b_i \cap b_j)=2$. Similarly those lines which are present in the two lost blocks but are not common will be replicated (r-1) times. The remaining lines will be replicated r times in design. Let C^* be the information matrix of design D^*. For this design D^*, the diagonal element of C^* matrix are as follows,

1. $C_{ii}=\frac{(r-2)(k-1)}{k}$, where i denotes those lines which are present in both the lost blocks but are distinct.
2. $C_{jj}=\frac{(r-1)(k-1)}{k}$, where j denotes those lines which are presents in both the lost blocks but are distinct.
3. $C_{ll}=\frac{r(k-1)}{k}$, where l denotes the remaining lines.

Similarly, in the residual design, pair of lines occurs together in the three ways, which is given as λ_1, λ_2, λ_3. Pattern of λ_i (i = 1, 2, 3) are as follows,

1. $\lambda_1 = (\lambda_1 - 2)$, for those lines, which are present in two lost blocks.
2. $\lambda_2 = (\lambda_1 - 1)$, for those lines, which are present in two lost blocks.
3. $\lambda_3 = \lambda_1$, for remaining lines.

The C^* matrix of design D^* can be written as,

$$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \qquad (***)$$

where

$$\varepsilon_{11} = (k-2)(\lambda_1 v - k + 1)I_2 - (k-2)(\lambda_1)J_{22}$$
$$\varepsilon_{12} = -(k-2)(\lambda_1 - 1)J_{2(k-1)}$$
$$\varepsilon_{13} = -(k-2)\lambda_1 J_{2(v-k-1)}$$
$$\varepsilon_{21} = -(k-2)(\lambda_1 - 1)J_{(k-1)2}$$
$$\varepsilon_{22} = (k-2)(\lambda_1 v - 2k)I_{(k-1)} - (k-2)(\lambda_1 - 2)J_{(k-1)(k-1)}$$
$$\varepsilon_{23} = -(k-2)\lambda_1 J_{(k-1)(v-k-1)}$$
$$\varepsilon_{31} = -(k-2)\lambda_1 J_{(v-k-1)2}$$
$$\varepsilon_{32} = -(k-2)\lambda_1 J_{(v-k-1)(k-1)}$$
$$\varepsilon_{33} = (k-2)\lambda_1 v(I_{v-k-1} - v^{-1}(k-2)J_{(v-k-1)(v-k-1)})$$

The non-zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(k-2)(\lambda_1 v-k+1)}{k}$, with multiplicity 1.
2. $\frac{(k-2)(\lambda_1 v-2k)}{k}$, with multiplicity ($k$-2).
3. $\frac{(k-2)(\lambda_1 v-k-1)}{k}$, with multiplicity 1.
4. $\frac{(k-2)(\lambda_1 v)}{k}$, with multiplicity ($v$-$k$-1).

Theorem 2: Partially Balanced Incomplete Block Design with parameters $v, b, r, k, \lambda_1, \lambda_2, m, n$ are fairly robust against the unavailability of two blocks, where number of common lines between two blocks are two, i.e. $\eta(b_i \cap b_j) = 2$, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{L}{M}, \qquad (12)$$

where

$$L = (v-1)(\lambda_1 v - 2k)(\lambda_1 v - k + 1)(\lambda_1 v - k - 1)$$
$$M = (\lambda_1 v - k + 1)(\lambda_1 v - k - 1)((v-k-1)(\lambda_1 v - 2k) + \lambda_1 v(k-2)) + 2\lambda_1 v(\lambda_1 v - k)(\lambda_1 v - 2k)$$

Proof: Without loss of generality, let two blocks be lost from design D where number of common line between two blocks is two i.e. $\eta(b_i \cap b_j) = 2$, C^* matrix of the residual design is given by,

$$k(k-2)C^* = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

where

$$\varepsilon_{11} = (k-2)(\lambda_1 v - k + 1)I_2 - (k-2)(\lambda_1)J_{22}$$
$$\varepsilon_{12} = -(k-2)(\lambda_1 - 1)J_{2(k-1)}$$
$$\varepsilon_{13} = -(k-2)\lambda_1 J_{2(v-k-1)}$$
$$\varepsilon_{21} = -(k-2)(\lambda_1 - 1)J_{(k-1)2}$$
$$\varepsilon_{22} = (k-2)(\lambda_1 v - 2k)I_{(k-1)} - (k-2)(\lambda_1 - 2)J_{(k-1)(k-1)}$$
$$\varepsilon_{23} = -(k-2)\lambda_1 J_{(k-1)(v-k-1)}$$
$$\varepsilon_{31} = -(k-2)\lambda_1 J_{(v-k-1)2}$$
$$\varepsilon_{32} = -(k-2)\lambda_1 J_{(v-k-1)(k-1)}$$
$$\varepsilon_{33} = (k-2)\lambda_1 v(I_{v-k-1} - v^{-1}(k-2)J_{(v-k-1)(v-k-1)})$$

The non-zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(k-2)(\lambda_1 v-k+1)}{k}$, with multiplicity 1.
2. $\frac{(k-2)(\lambda_1 v-2k)}{k}$, with multiplicity ($k$-2).
3. $\frac{(k-2)(\lambda_1 v-k-1)}{k}$, with multiplicity 1.
4. $\frac{(k-2)(\lambda_1 v)}{k}$, with multiplicity ($v$-$k$-1).

Further, overall A-efficiency is calculated as,

$$\text{e(s)} = \frac{\varphi_2(s)}{\varphi_1(s)} \qquad (13)$$

That is,

$$\varphi_2(s) = \frac{k(v-1)}{\lambda_1 v(k-2)} \qquad (14)$$

$$\varphi_1(s) = \frac{k(\text{v-k-1})}{(k-2)(\lambda_1 v)} + \frac{k}{(k-2)(\lambda_1 v\text{-k+1})} + \frac{(k-2)k}{(k-2)(\lambda_1 v - 2k)} + \frac{k}{(k-2)(\lambda_1 v\text{-k-1})} \qquad (15)$$

Finally, A- efficiency is given by,

$$\text{e(s)} = \frac{L}{M} \qquad (16)$$

where

$$L = (v-1)(\lambda_1 v - 2k)(\lambda_1 v - k + 1)(\lambda_1 v - k - 1)$$

$$M = (\lambda_1 v - k + 1)(\lambda_1 v - k - 1)((v - k - 1)(\lambda_1 v - 2k) + \lambda_1 v(k-2)) + 2\lambda_1 v(\lambda_1 v - k)(\lambda_1 v - 2k)$$

Example 3: Let D represent the Partially Balanced Incomplete Block Design with parameters $v = p = 9$, $b = 9$, $r = 3$, $k = 3$, $\lambda_1 = 1$, $\lambda_2 = 0$, $m = 6$, $n = 2$. Design D is given by,

Table 3: 9 lines of PBIB design of Das and Giri (1979)

Block	PBIBD			Crosses in the PBIB design		
1	1	2	3	1 × 2	1 × 3	2 × 3
2	1	6	4	1 × 6	1 × 4	4 × 6
3	1	7	5	1 × 7	1 × 5	5 × 7
4	6	8	3	6 × 8	3 × 6	3 × 8
5	6	9	5	6 × 9	5 × 6	5 × 9
6	7	8	5	7 × 8	5 × 7	5 × 8
7	7	9	3	7 × 9	3 × 7	3 × 9
8	**2**	**8**	**4**	**2 × 8**	**2 × 4**	**4 × 8**
9	**2**	**9**	**4**	**2 × 9**	**2 × 4**	**4 × 9**

When two blocks containing block 8 and block 9 are lost, and number of common lines between two blocks is two, C^* matrix of the residual design is given by,

$$3C^* = \begin{bmatrix} 6 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 \\ -1 & 6 & 0 & 0 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 4 & 1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 4 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 8 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 8 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 8 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 8 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 8 \end{bmatrix}$$

The non-zero eigenvalues of C* matrix with their corresponding multiplicities are,

1. $\frac{3}{3}$, with multiplicities 1.
2. $\frac{5}{3}$, with multiplicities 1.
3. $\frac{7}{3}$, with multiplicities 1.
4. $\frac{9}{3}$, with multiplicities 5.

The overall A-efficiency of the design is

$$e(s) = 0.721649$$

IV. Conclusions

Mating designs are the study of progenies developed through various methods like Diallel Cross plans which are subjected to Balanced Incomplete Block Designs. The analysis of such plans, namely the estimation of variance components, design and genetic, is available in the literature. However, the primary interest in this study is to examine the robustness of various mating designs as it depends on the underlying experimental design. Robustness of Partial Diallel Cross plan is examined using Partially Balanced Incomplete Block Designs. There are varying Partially Balanced Incomplete Block Designs for different parametric values with unavailability of two blocks. The paper dealt with a class of 63 Partially Balanced Incomplete Block Designs for varying parametric values with unavailability of two blocks and efficiencies are calculated for each case. Efficiencies are varying based on the:

i) number of common lines between two blocks is zero
ii) number of common lines between two blocks is one
iii) number of common lines between two blocks is two or more

Results shows that the efficiencies of case (*i*) will be more than that of case (*ii*), and case (*iii*). The efficiency is obtained and the cases are compared to determine the robustness of the Partially Balanced Incomplete Block Designs. It appears that Partially Balanced Incomplete Block Designs are fairly robust against the unavailability of two blocks corresponding to the same test treatment.

Table 4: Result shown for Efficiency when two blocks is lost from a Partially Balanced Incomplete Block Design of Ghosh and Desai (1999)

D. No	v	b	r	k	λ_1	λ_2	m	n	Case *i*	Case *ii*	Case *iii*
1	6	15	10	4	5	10	3	2	0.84	0.74	0.83
2	8	12	6	4	6	2	4	2	0.93	0.88	0.92
3	8	18	9	4	3	9	4	2	0.85	0.74	0.83
4	9	9	3	3	1	0	3	3	0.80	0.60	0.72
5	10	20	8	4	2	8	5	2	0.86	0.73	0.83
6	16	28	7	4	1	7	8	2	0.88	0.76	0.85
7	18	36	8	4	1	8	9	2	0.91	0.81	0.89
8	20	45	9	4	1	9	10	2	0.93	0.85	0.91
9	22	55	10	4	1	10	11	2	0.94	0.88	0.93
10	8	12	9	6	6	9	4	2	0.83	0.70	0.81
11	9	9	6	6	3	6	3	3	0.74	0.53	0.68
12	9	12	8	6	4	8	3	3	0.80	0.64	0.77
13	9	15	10	6	5	10	3	3	0.84	0.71	0.82
14	12	30	10	4	2	10	6	2	0.90	0.82	0.89

15	12	18	9	6	3	9	4	3	0.85	0.71	0.82
16	12	20	10	6	4	10	6	2	0.89	0.79	0.87
17	14	21	9	6	3	9	7	2	0.89	0.78	0.87
18	15	20	8	6	2	8	5	3	0.85	0.70	0.82
19	18	24	8	6	2	8	9	2	0.89	0.79	0.88
20	18	30	10	6	2	10	6	3	0.89	0.79	0.88
21	20	30	9	6	2	9	10	2	0.92	0.83	0.90
22	24	28	7	6	1	7	8	3	0.87	0.72	0.83
23	26	26	6	6	1	6	13	2	0.89	0.77	0.86
24	27	36	8	6	1	8	9	3	0.90	0.78	0.87
25	30	35	7	6	1	7	15	2	0.92	0.83	0.90
26	30	45	9	6	1	9	10	3	0.92	0.83	0.90
27	33	55	10	6	1	10	11	3	0.94	0.86	0.92
28	38	57	9	6	1	9	19	2	0.95	0.90	0.94
29	42	70	10	6	1	10	21	2	0.96	0.92	0.95
30	12	16	10	8	5	10	3	4	0.84	0.70	0.82
31	12	15	10	8	6	10	6	2	0.86	0.75	0.85
32	14	14	8	8	4	8	7	2	0.85	0.71	0.83
33	18	18	8	8	3	8	9	2	0.87	0.76	0.86
34	24	30	10	8	2	10	6	4	0.89	0.78	0.87
35	26	26	8	8	2	8	13	2	0.91	0.81	0.89
36	32	20	5	8	1	5	16	2	0.87	0.71	0.82
37	32	28	7	8	1	7	8	4	0.87	0.71	0.82
38	32	40	10	8	2	10	16	2	0.94	0.88	0.93
39	36	36	8	8	1	8	9	4	0.90	0.77	0.87
40	40	45	9	8	1	9	10	4	0.92	0.82	0.90
41	44	55	10	8	1	10	11	4	0.93	0.85	0.92
42	50	50	8	8	1	8	25	2	0.95	0.89	0.94
43	56	63	9	8	1	9	28	2	0.96	0.91	0.95
44	18	20	10	9	4	10	6	3	0.88	0.77	0.87
45	21	21	9	9	3	9	7	3	0.88	0.77	0.86
46	27	24	8	9	2	8	9	3	0.89	0.78	0.87
47	30	30	9	9	2	9	10	3	0.91	0.82	0.90
48	39	26	6	9	1	6	13	3	0.89	0.75	0.85
49	45	35	7	9	1	7	15	3	0.92	0.82	0.89
50	57	37	9	9	1	9	19	3	0.95	0.89	0.94
51	63	70	10	9	1	10	21	3	0.96	0.91	0.95
52	12	12	10	10	8	10	6	2	0.84	0.71	0.82
53	18	18	10	10	5	10	9	2	0.88	0.78	0.87
54	20	18	9	10	4	9	10	2	0.88	0.77	0.87
55	22	22	10	10	4	10	11	2	0.90	0.81	0.89
56	30	30	10	10	2	10	6	5	0.89	0.77	0.87
57	42	21	5	10	1	5	21	2	0.88	0.73	0.84
58	42	42	10	10	2	10	21	2	0.94	0.89	0.94
59	45	36	8	10	1	8	9	5	0.90	0.76	0.86
60	50	10	6	10	1	6	25	2	0.92	0.81	0.89
61	50	45	9	10	1	9	10	5	0.92	0.81	0.89
62	55	55	10	10	1	10	11	5	0.93	0.85	0.91
63	82	82	10	10	1	10	41	2	0.97	0.94	0.97

It may be observed that the Partially Balanced Incomplete Block Mating Design is fairly robust for different set of parameters with unavailability of two blocks in a design.

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