Correlation between Hepatitis and Cancer: A Mathematical Model

M. Agarwal and A.S. Bhadauria

Abstract— **In this paper, a nonlinear mathematical model is proposed and analyzed to demonstrate the relation between hepatitis and cancer in a homogeneous population with constant immigration of cancer patients in the community. Both the horizontal and vertical mode of transmission of hepatitis in the population is considered. The model is analyzed for stability of equilibria using stability theory of differential equations. Sensitivity analysis of the endemic equilibrium to changes in the value of the different parameters associated with the system is done. Moreover, the numerical simulation of the proposed model is also performed by using fourth order Runge - Kutta method. Modeling the effect of hepatitis virus infections among cancer patients and their impact on the increase in spread of hepatitis among the population is the novel feature of our model. It is concluded from the analysis that if the rate of transmission of hepatitis infection increases, the endemic level of infective population increases which can further be enhanced if risk of hepatitis infection among cancer patients also increases. Further, it is found from our analysis that hepatitis infection leads to an increase in number of cancer patients in the population because of the progression of hepatitis *B* and *C* infection to liver cancer.**

Index Terms—**Cancer, Equilibria, Hepatitis, Sensitivity Analysis.**

MSC 2010 Codes —**93D20, 37C75**

I. Introduction

THE word 'hepatitis' simply means an inflammation of the liver without pinpointing a specific cause. It is mainly caused by three viruses namely *A* , *B* and *C*. The role of hepatitis virus specially *B* and *C* in causing liver cancer is well established. It is found that the frequency of liver cancer relates to (correlates with) the frequency of chronic hepatitis *B* virus infection. In addition, the patients with hepatitis *B* virus who are at greatest risk for liver cancer are men with hepatitis *B* virus cirrhosis (scarring of the liver) and a family history of liver cancer. Perhaps the most convincing evidence, however, comes from a prospective study done in the 1970's in Taiwan involving male government employees over the age of 40. In this study, the investigators found that the risk of developing liver cancer was 200 times higher among employees who had chronic hepatitis *B* virus as compared to employees without chronic hepatitis *B* virus. Similarly, Hepatitis *C* virus infection is also associated with the development of liver cancer. In fact, in Japan, hepatitis *C* virus is present in up to 75% of cases of liver cancer. As with hepatitis *B* virus, the majority of hepatitis *C* virus patients with liver cancer have associated cirrhosis (liver scarring) [1, 2].

Hepatitis *B* (also called serum hepatitis) is caused by the hepatitis *B* virus *HBV*. *HBV* can cause a wide spectrum of symptoms ranging from general malaise to chronic liver disease that can lead to liver cancer. *HBV* spreads through the following ways: infected body fluids (such as blood, saliva, semen, tears, and urine),a contaminated blood transfusion, shared contaminated needles or syringes for injecting drugs , sexual activity with an *HBV* - infected person, transmission from *HBV* - infected mothers to their newborn babies. The hepatitis *C* virus *HCV* spreads by direct contact with an infected person's blood. The symptoms of the hepatitis *C* virus can be very similar to those of the hepatitis *B* virus. Infection with *HCV* can lead to chronic liver disease.

Apart from the role of hepatitis *B* and *C* virus infections to cirrhosis and primary liver cancer worldwide, individuals with cancer may be at elevated risk of liver failure from *HBV* [3]. It may be because of their frequent hospitalizations or immunological changes in patients with tumors that may lower their threshold for infection [4]. Cancer is, in general, more common in industrialized nations, but there has been a growth in cancer rates in developing countries, particularly as these nations adopt the diet and lifestyle habits of industrialized countries. Anyone can get cancer at any age; however, about 80 percent of all cancers occur in people over the age of fifty-five. Cancer population acquires hepatitis infection by contaminated blood transfusion, shared contaminated needles or syringes for injecting drugs etc.

The mathematical models on ecological systems [5, 6] received great attention to researchers but the role of disease factors in ecological system cannot be ignored. Some of the basic mathematical models on epidemics may be found in literature [7, 8]. Not among all diseases, hazardous effect of Hepatitis can be ignored. Primarily because of hepatitis significance as a global public health threat, the virus and its associated diseases have attracted considerable attention from mathematical and theoretical biologists [9 - 13]. The models

This work was supported in part by the University Grants Commission, New Delhi, India.

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typically used to study hepatitis dynamics within the host and tend to focus on healthy cells, free virus, and infected cells. It may be noted that although above-mentioned studies demonstrate the transmission of hepatitis within the host yet could not take into account the disease dynamics of hepatitis on population. Here a model is constructed to describe the dynamics of the spread of hepatitis in a community which was not taken up by any mathematical model in this direction. In addition to this, keeping in mind, the hazardous effects of hepatitis virus infection on the cancer patients, an attempt is also made in the present study to model the effect of hepatitis virus infections among cancer patients and their impact on the spread of hepatitis among the population.

This paper is organized as follows: in section 2, we introduce mathematical model. Section 3 focuses on the analysis of model that includes the study of region of attraction and equilibria of the system. The local stability of the equilibrium points are also established in this section. Section 4 deals with the sensitivity analysis of the endemic equilibrium corresponding to changes in the value of the different parameters associated with the system. Section 5 highlights the results of our analysis using numerical simulation. Section 6 presents the relevance of the results presented in the manuscript.

II. MATHEMATICAL MODEL

A system of differential equations is introduced to model the correlation between the diseases hepatitis and cancer in a homogeneous population with constant immigration of cancer patients. The total population $N(t)$ is divided into four classes, namely, susceptible class $S(t)$, exposed class $E(t)$, infective population $I(t)$ and cancer population $C(t)$. Susceptible population gets infection by direct exposure to the virus. Both the horizontal and vertical mode of transmission of hepatitis infection among population is considered. After getting infection susceptible population move to the exposed class and further after a certain latent period of time enters into the class of infectives. We assume that mothers in exposed and infective class give birth to exposed and infective children respectively. In the infective class, some of the individuals acquire liver cancer and move to the cancer population. It is assumed that there is a constant recruitment of cancer population in the system. Cancer population acquires hepatitis infection by contaminated blood transfusion, shared contaminated needles or syringes for injecting drugs etc. Disease related mortality in cancer population is considered. Administration of vaccination among the susceptible individuals is considered as a preventive measure to control the spread of virus infection. Keeping the above in mind and by considering simple mass interaction, a mathematical model is presented as follows:

$$\frac{dS}{dt} = \mu - \beta IS - p\mu E - q\mu I - (\mu + \gamma)S,$$

$$\frac{dE}{dt} = \beta IS + p\mu E + \theta\beta_1 CI - (\varepsilon + \mu)E,$$

$$\frac{dI}{dt} = \varepsilon E - (\mu + v)I + q\mu I,$$

$$\frac{dC}{dt} = \sigma + rvI - \beta_1 CI - (\mu + \alpha)C, \quad (1)$$

With initial conditions, $S(0) > 0$, $E(0) \geq 0$, $I(0) \geq 0$, $C(0) > 0$. The total population size denoted by N can be determined by $N = S + E + I + C$.

Here natural birth and death rate of host population is assumed to be same μ, for the sake of simplicity of calculation. σ is the constant recruitment rate of cancer population in the system. We consider the of vertical transmission of hepatitis infection by assuming that fractions p of the offsprings of the exposed and fraction q of the offsprings of infective host are infected at birth and enter the exposed and infective compartment respectively. Consequently there is the influx $q\mu E$ and $p\mu I$ of the newborns in the exposed and infective compartment respectively and thus influx of the new born into the susceptibles compartment is $\mu - q\mu E - p\mu I$ where $0 \leq p, q \leq 1$. β and β_1 are the transmission coefficient of hepatitis infection among susceptible population with and without cancer respectively. It is assumed that vaccination is administered among susceptible population without cancer at the rate γ. θ is the conversion rate of cancer population to hepatitis-exposed population after getting hepatitis infection from virus-contaminated articles. ε is the rate of transfer of exposed population to infective population. v is the rate at which hepatitis infected population develop cirrhosis and acquire liver cancer. r is the fraction of hepatitis infected population acquiring liver cancer. α is the disease related death rate constant of cancer population.

III. ANALYSIS OF THE MODEL

A. Region of Attraction

To analyse the existence and stability of the equilibrium points we need bounds on dependent variables involved, for this, we find the region of attraction in the following lemma.

Lemma 1: The set

$$\Omega = \left\{ (S, E, I, C) \in R^4 : 0 < S + E + I + C \leq N \leq \frac{\mu + \sigma}{\mu} \right\}$$

attracts all solutions initiating in the interior of positive orthant.

Proof. On adding all the four equations of system (1), we get

$$\frac{dN}{dt} = \mu + \sigma - \mu N - (1 - \theta)\beta_1 CI - \gamma S - \alpha C - v(1 - r)I,$$

$$\frac{dN}{dt} \leq \mu + \sigma - \mu N,$$

this implies that, $\limsup_{t \to \infty} N \leq \frac{\mu + \sigma}{\mu}$.

Thus, solutions of (1) starting in the positive orthant R_+^4 approach, enter or remain in the subset of R_+^4 defined by $\Omega = \left\{ (S, E, I, C) \in R^4 : 0 < S + E + I + C \leq N \leq \frac{\mu + \sigma}{\mu} \right\}$ where

R_+^4 denote the non-negative cone of R^4 including its lower dimensional faces. Thus, it suffices to consider solutions in the region Ω. Solutions of the initial value problem starting in Ω and defined by (1) exist and are unique on a maximal interval. Since solutions remain bounded in the positively invariant region Ω, the maximal interval is well posed both mathematically and epidemiologically.

B. Equilibrium Analysis

The equilibrium points must satisfy the following equations:

$$\mu - \beta IS - p\mu E - q\mu I - (\mu+\gamma)S = 0,$$
$$\beta IS + p\mu E + \theta\beta_1 CI - (\varepsilon+\mu)E = 0,$$
$$\varepsilon E - (\mu+\nu)I + q\mu I = 0, \quad (3.1)$$
$$\sigma + r\nu I - \beta_1 CI - (\mu+\alpha)C = 0, \quad (2)$$

From first and fourth equation of system (2) we get

$$C = \frac{\sigma + r\nu I}{\mu+\alpha+\beta_1 I},$$

$$S = \frac{\mu - p\mu E - q\mu I}{\mu+\gamma+\beta I},$$

using value of S and C in second equation of (2) we get

$$E = \frac{\theta\beta_1 I\left(\frac{\sigma+r\nu I}{\mu+\alpha+\beta_1 I}\right)(\mu+\gamma+\beta I) + \varepsilon\beta I(\mu - q\mu I)}{(\varepsilon+\mu)(\mu+\gamma+\beta I) - p\mu(\mu+\gamma)},$$

putting value of E in third equation of (2) we get

$I = 0$

and the quadratic equation

$$F(I) = AI^2 + BI + C = 0, \quad (3)$$

where

$$A = \beta\beta_1\{(\mu+\nu)(\mu+\varepsilon) - q\mu^2 - \varepsilon\theta r\nu\},$$
$$B = q\mu(\mu+\alpha)\varepsilon\beta + (\mu+\nu-q\mu)(\varepsilon+\mu)(\alpha+\mu)\beta$$
$$+ (\mu+\nu-q\mu)(\varepsilon+\mu-p\mu)(\mu+\gamma)\beta_1$$
$$- \varepsilon\beta_1(\beta(\theta\sigma+\mu) + \theta\, r\nu(\mu+\gamma)),$$
$$C = (\mu+\gamma)(\mu+\alpha)(\mu+\nu-q\mu)(\mu+\varepsilon-p\mu)(1-R_0)$$

and $R_0 = \dfrac{\varepsilon\beta\mu(\mu+\alpha) + \varepsilon\theta\beta_1\sigma(\mu+\gamma)}{(\mu+\alpha)(\mu+\gamma)(\varepsilon+\mu-p\mu)(\mu+\nu-q\mu)}$.

Taking $I = 0$ and substituting into value of S, E and C we get the hepatitis-free equilibrium point as

$$P_0 = (\frac{\mu}{\mu+\gamma}, 0, 0, \frac{\sigma}{\mu+\alpha}).$$

A solution I of (3) corresponds to endemic equilibrium solution. From (3) it is easy to observe that the system has a unique endemic solution if and only if $R_0 > 1$ and $A, B > 0$.

C. Local Stability Analysis of Hepatitis-Free Equilibria

The local stability of the system (1) can be checked by computing the variational matrix of the given system. The signs of the real parts of the eigenvalues of the variational matrix evaluated at a given equilibrium point determine its stability. The entries of general variational matrix $V(P)$ are given by differentiating the right hand side of the system (1) with respect to S, E, I, C i.e.

$$V(P) = \begin{bmatrix} -\beta I - \mu - \gamma & -p\mu & \beta S - q\mu & 0 \\ \beta I & p\mu - \varepsilon - \mu & \theta\beta_1 C + \beta S & \theta\beta_1 I \\ 0 & \varepsilon & -\mu - \nu + q\mu & 0 \\ 0 & 0 & r\nu - \beta_1 C & -\mu - \alpha - \beta_1 I \end{bmatrix}$$

In the following theorem, it is shown that the hepatitis free equilibrium point $P_0\left(\frac{\mu}{\mu+\gamma}, 0, 0, \frac{\sigma}{\mu+\alpha}\right)$ is locally asymptotically stable.

Theorem1: If $R_0 < 1$ the equilibrium point P_0 of the system (1) is locally asymptotically stable and it is unstable if $R_0 \geq 1$.

Proof: The variational matrix corresponding to hepatitis free equilibrium point $P_0\left(\frac{\mu}{\mu+\gamma}, 0, 0, \frac{\sigma}{\mu+\alpha}\right)$ can be written as:

$$V(P_0) = \begin{bmatrix} -\mu - \gamma & -p\mu & \beta\frac{\mu}{\mu+\gamma} - q\mu & 0 \\ 0 & p\mu - \varepsilon - \mu & \theta\beta_1\frac{\sigma}{\mu+\alpha} + \beta\frac{\mu}{\mu+\gamma} & 0 \\ 0 & \varepsilon & -\mu - \nu + q\mu & 0 \\ 0 & 0 & r\nu - \beta_1\frac{\sigma}{\mu+\alpha} & -\mu - \alpha \end{bmatrix}$$

The eigenvalues of variational matrix corresponding to hepatitis free equilibria are: $-\mu-\gamma$, $-\mu-\alpha$ and roots of the quadratic equation

$$\lambda^2 + \lambda(\varepsilon + 2\mu + \nu - p\mu - q\mu)$$
$$+ (\varepsilon + \mu - p\mu)(\mu + \nu - q\mu)(1 - R_0) = 0$$

Thus, hepatitis free equilibrium point is locally asymptotically stable (by Routh test) if $R_0 < 1$ and unstable if $R_0 \geq 1$.

D. Stability Analysis of Endemic Equilibria

Theorem 2: The endemic equilibrium point P^* is locally asymptotically stable if $M_1, M_3, M_4 > 0$, and $M_1M_2M_3 - M_3^2 - M_1^2M_4 > 0$ where M_1, M_2, M_3 and M_4 are given by (3-6).

Proof: The variational matrix $V(P^*)$ about equilibrium point P^* is given by

$$V(P^*) = \begin{bmatrix} -\beta I^* - \mu - \gamma & -p\mu & \beta S^* - q\mu & 0 \\ \beta I^* & p\mu - \varepsilon - \mu & \theta\beta_1 C^* + \beta S^* & \theta\beta_1 I^* \\ 0 & \varepsilon & -\mu - \nu + q\mu & 0 \\ 0 & 0 & r\nu - \beta_1 C^* & -\mu - \alpha - \beta_1 I^* \end{bmatrix}$$

The characteristic equation corresponding to $V(P^*)$ is given by

$$\lambda^4 + M_1\lambda^3 + M_2\lambda^2 + M_3\lambda + M_4 = 0,$$

where

$$M_1 = 4\mu + v - q\mu + \alpha + 2\beta I^* + \beta_1 I^* + \varepsilon - p\mu + \gamma, \tag{3}$$

$$\begin{aligned} M_2 &= (\mu + v - q\mu)(\mu + \alpha + \beta_1 I^*) \\ &\quad + (\beta I^* + \mu + \gamma)(\varepsilon + \mu - p\mu) \\ &\quad - \varepsilon(\theta\beta_1 C^* + \beta S^*) + \beta I^* p\mu \\ &\quad + (\varepsilon + 2\mu - p\mu + \beta I^* + \gamma)(2\mu + v - q\mu + \alpha + \beta_1 I^*), \end{aligned} \tag{4}$$

$$\begin{aligned} M_3 &= (\varepsilon + 2\mu - p\mu + \beta I^* + \gamma) \\ &\quad \mathrm{X}(\mu + v - q\mu)(\mu + \alpha + \beta_1 I^*) \\ &\quad + (2\mu + v - q\mu + \alpha + \beta_1 I^*) \\ &\quad \mathrm{X}(\beta I^* + \mu + \gamma)(\varepsilon + \mu - p\mu) \\ &\quad + \beta I^* p\mu(2\mu + \alpha + \beta_1 I^* + v - q\mu) \\ &\quad + \beta \mathrm{I}^* \varepsilon(\beta S^* + q\mu) - \varepsilon(2\mu + \alpha + \gamma \\ &\quad + \beta I^* + \beta_1 I^*)(\theta\beta_1 C^* + \beta S^*) \\ &\quad - \varepsilon\theta\beta_1 I^*(rv - \beta_1 C^*), \end{aligned} \tag{5}$$

$$\begin{aligned} M_4 &= (\beta I^* + \mu + \gamma)(\varepsilon + \mu - p\mu) \\ &\quad (\mu + v - q\mu)(\mu + \alpha + \beta_1 I^*) \\ &\quad + \beta I^* p\mu(\mu + v - q\mu)(\mu + \alpha + \beta_1 I^*) \\ &\quad + \beta \mathrm{I}^* \varepsilon(\beta S^* + q\mu)(\mu + \alpha + \beta_1 I^*) \\ &\quad - \varepsilon(\theta\beta_1 C^* + \beta S^*)(\mu + \gamma + \beta I^*)(\mu + \alpha + \beta_1 I^*) \\ &\quad - \varepsilon\theta\beta_1 I^*(\mu + \gamma + \beta I^*)(rv - \beta_1 C^*). \end{aligned} \tag{6}$$

Thus, by Routh-Hurwitz criterion, the endemic equilibrium P^* is local asymptotically stable for $M_i > 0$, i=1, 3, 4 and $M_1 M_2 M_3 - M_3^2 - M_1^2 M_4 > 0$.

IV. SENSITIVITY ANALYSIS

We now study sensitivity of the endemic equilibrium to changes in the value of the different parameters associated with the system. The results are shown in table 1. The purpose of this analysis is to identify the parameters, which are sensitive; estimation of these parameters in the field studies is to be done with sufficient care.

Sensitivity of the solution to changes in the parameter values is described in Table 1.

Regarding sensitivity of the endemic equilibrium level of susceptible population $S(t)$, the following features are observed:

1. It is less sensitive to changes in the value of the parameters β_1, p, q, α γ, v, r.
2. It is fairly sensitive to changes in the value of the parameters μ, β, θ, σ, ε.

The equilibrium level of exposed population $E(t)$ exhibit the following characteristics:

1. It is less sensitive to the changes in the parameters μ, β_1, p, q, γ, v, r, α.
2. It has high sensitivity to β, θ, ε, σ.

The endemic hepatitis infective class size $I(t)$ behaves as follows:

1. It is less sensitive to β_1, p, q, γ, r, α.
2. It is fairly sensitive to changes in the value μ, β, θ, v, ε, σ.

The equilibrium level of cancer population $C(t)$ exhibit the following characteristics:

1. It is less sensitive to changes in the values of parameters p, q, γ, r, v, α
2. It is fairly sensitive to changes in the parameters μ, β, β_1, θ, ε, σ.

Since the spread of epidemic in the population is direct outcome of endemic infective population size, determination of the equilibrium level of the infective population size is the primary problem and more attention needs to be given to the estimation of those parameters to which infective class size is more sensitive. In this context, more care should be taken to estimate the parameters μ, β, β_1, θ, v, ε, σ.

TABLE 1
PERCENTAGE CHANGES IN THE ENDEMIC EQUILIBRIUM CORRESPONDING TO DIFFERENT PERCENTAGE CHANGES IN THE PARAMETERS.

S.No.	Parameter	% Change in parameter	% Change in $S(t)$	% Change in $E(t)$	% Change in $I(t)$	% Change in $C(t)$
1	μ	+50	+70.84	-4.91	-28.53	+17.55
		+20	+26.64	-0.33	-11.96	+6.75
		-20	-24.34	-2.16	+12.80	-6.21
		-50	-56.72	-10.67	+33.55	-14.67
2	β	+50	-31.26	10.37	+10.38	-6.93
		+20	-15.36	+5.09	+5.10	-3.51
		-20	+21.89	-7.29	-7.29	+5.70
		-50	+78.23	-26.29	-26.28	+24.84
3	β_1	+50	-1.45	+2.07	+2.07	-31.05
		+20	-0.72	+1.00	+1.03	-15.30
		-20	+1.05	-1.49	-1.46	+22.05
		-50	+3.62	-4.88	-4.87	+72.18
4	p	+50	-1.12	+0.36	+0.38	-0.18
		+20	-0.46	+0.12	+0.15	-0.09
		-20	+0.46	-0.15	-0.13	+0.18
		-50	+1.12	-0.36	-0.36	+0.27
		+50	-0.79	-0.18	+0.51	-0.36
		+20	-0.32	-0.06	+0.20	-0.09
		-20	+0.32	+0.06	-0.20	+0.18

5	q	-50	+0.79	+0.15	-0.49	+0.45
6	γ	+50	-2.50	-7.29	-7.29	+5.76
		+20	-0.98	-2.98	-2.95	-2.25
		-20	+1.84	+6.13	+6.14	-4.23
		-50	+2.30	+7.68	+7.72	-5.22
7	θ	+50	-13.32	+21.35	+21.35	-13.14
		+20	-5.73	+8.51	+8.51	-5.76
		-20	+6.46	-8.51	-8.48	+6.75
		-50	+17.81	-21.29	-21.26	+18.99
8	ε	+50	-10.55	-21.65	+17.49	-11.07
		+20	-5.27	-9.82	+8.21	-5.58
		-20	+7.84	+11.65	-10.65	+8.64
		-50	+30.73	+31.26	-34.36	+35.55
9	ν	+50	+14.64	-4.42	-18.22	+18.06
		+20	+5.87	-1.70	-7.92	+8.82
		-20	-5.93	+1.58	+8.96	-8.64
		-50	-14.84	+3.72	+24.83	-21.51
10	σ	+50	-11.54	+18.15	+18.17	+26.28
		+20	-5.01	+7.32	+7.36	+11.43
		-20	+5.60	-7.47	-7.47	-12.78
		-50	+15.63	-19.00	-18.98	-35.46
11	r	+50	-1.97	+2.77	+2.79	+4.50
		+20	-0.79	+1.09	+1.10	+1.80
		-20	+0.79	-1.09	-1.08	-1.71
		-50	+1.97	-2.68	-2.66	-4.41
12	α	+50	+0.19	-0.27	-0.27	-0.36
		+20	+0.06	-0.12	-0.11	-0.09
		-20	-0.06	+0.09	+0.11	+0.18
		-50	-0.19	+0.27	+0.29	+0.45

V. NUMERICAL ANALYSIS

In the previous sections, the qualitative analysis of the model is presented. The conditions for stability are determined. We now see the dynamical behavior of the model system. System (1) is integrated numerically by fourth order Runge-Kutta method using the following set of hypothetical parameters:

$\mu = 0.02,\ \beta = 0.2,\ \beta_1 = 0.3,\quad p = 0.05,\ q = 0.02,\ \theta = 0.4,$
$\gamma = 0.02,\ \varepsilon = 0.04,\ \nu = 0.01,\ \sigma = 0.015,\ \alpha = 0.002,\ r = 0.5$.

With the above parameter values, we get following equilibrium values of population in the presence of disease:

$$S^* = 0.1516,\ E^* = 0.3278,\ I^* = 0.4429,\ C^* = 0.1112.$$

Eigenvalues corresponding to endemic equilibria $P^*(S^*, E^*, I^*, C^*)$ are obtained as:

$$-0.1500 \pm 0.0078i,\quad -0.0556 \quad \text{and} \quad -0.0164.$$

Since all the eigenvalues corresponding to P^* have negative real parts, therefore P^* is locally asymptotically stable. Further, to illustrate the global stability of the equilibrium point, numerical simulation is performed for different initial starts and the results are displayed in Figures 1 and 2 for $S-I$ and $S-C$ planes respectively. From the solution curves obtained in these figures, we can infer that the system is globally stable about the interior (endemic) equilibrium point $P^*(S^*, E^*, I^*, C^*)$.

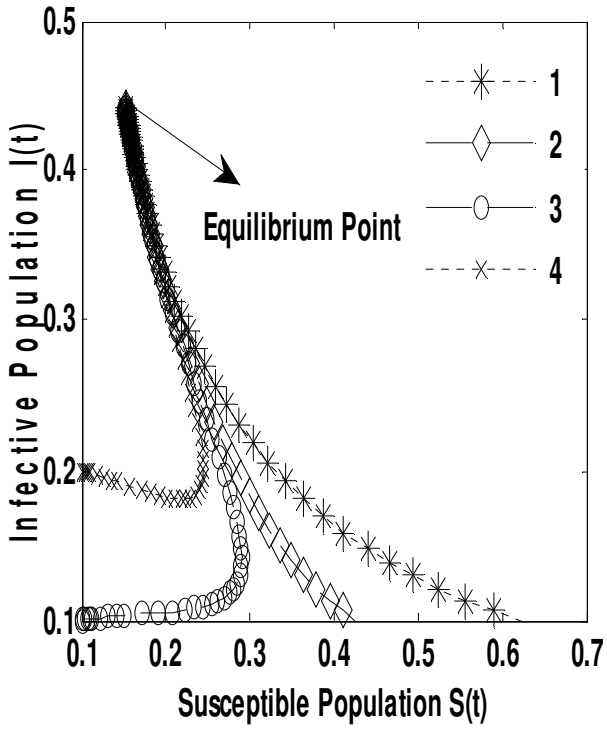

Fig 1(a). Variation of infective population with susceptible population

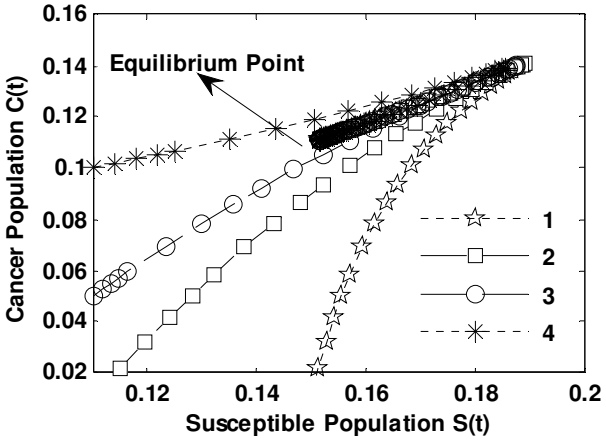

Fig. 1 (b). Variation of cancer population with susceptible population

Figures 3-7 are drawn to display the relationship of various parameters involved in the system with the equilibrium level of population. Figure 3 depicts the variation of infective population with time for different transmission coefficient of hepatitis infection in cancer patients, $\theta\beta_1$. It is found from the graph that equilibrium level of infective population size increases with the increase in $\theta\beta_1$. This implies that hepatitis infection among cancer patients plays a considerable role in the spread of disease and hence in increasing the number of hepatitis infectives in the population.

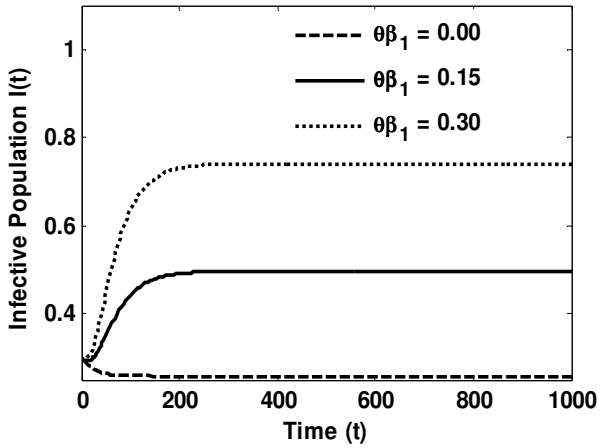

Fig. 2. Variation of infective population with time for different transmission coefficient of hepatitis infection to cancer patients

In figure 4 variation of infective population with time for different recruitment rate of cancer patients σ, in the community is studied. It is found from the graph that as recruitment rate of cancer patients increases, equilibrium value of hepatitis infective population also increases. From this graph it can be inferred that the disease spreads fast if there is continuous influx of cancer patients in the community because cancer patients acquire hepatitis infection due to frequent hospitalization or because of low threshold for infection in cancer patients due to immunological changes.

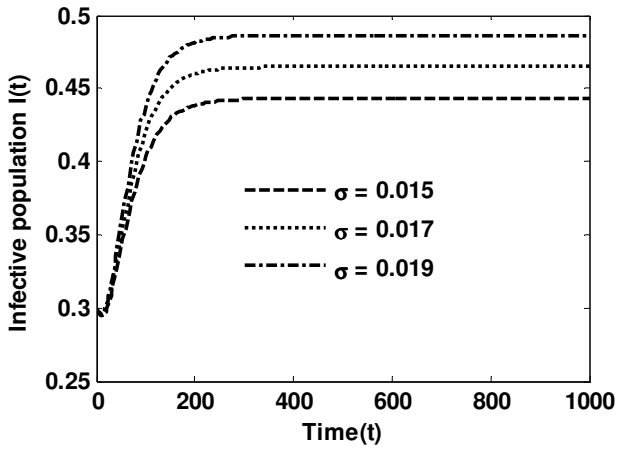

Fig. 3. Variation of infective population with time for different recruitment rates of cancer population

Figures 5 and 6 demonstrate the variation of infective population with time for different birth rate of offsprings in exposed and infective state p, q respectively with and without vaccination among population. It is found that as p and q increase, endemic infective class size increases which further enhances in absence of vaccination from which it can be concluded that, in highly endemic areas, increase in the hepatitis infective offsprings in exposed and infective state by chronically infected mothers in exposed and infective state respectively has a considerable effect in increasing endemic infective class size. Hence, attempts should be made to prevent the neonate to be prenatally infected by the chronically infected mothers. Further, it is observed from the graph that vaccination induces rapid decrease in the number of acute hepatitis virus infections and has a secondary effect of a decrease in related sequels.

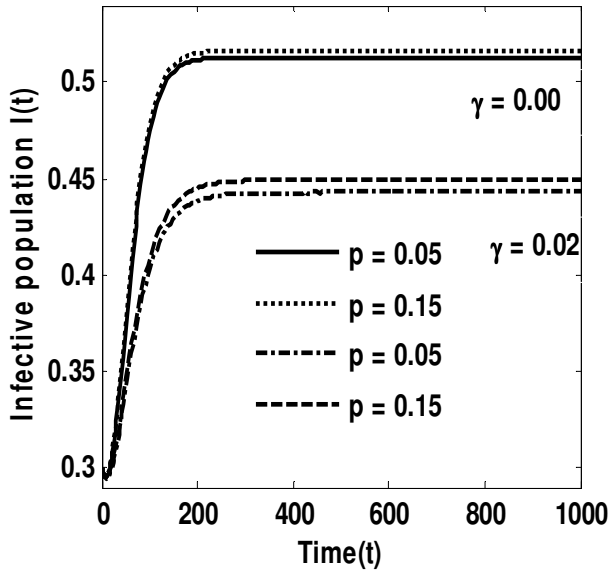

Fig. 4. Variation of infective population with time for different birth rate of offsprings in exposed state with and without vaccination

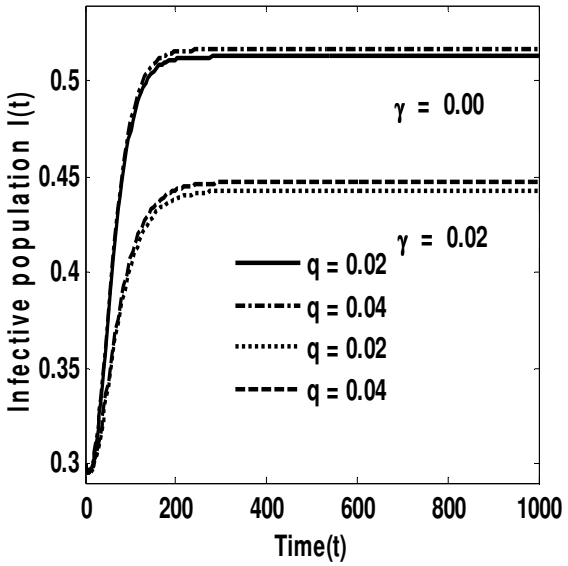

Fig. 5. Variation of infective population with time for different birth rate of offsprings in infective state with and without vaccination

In figure 7, a graph between cancer population and time for different rate of occurrence of liver cancer in hepatitis infectives rv, is drawn. It is observed from the graph that increase in rv leads to an increase in number of cancer population. Thus, hepatitis infection in a community causes an increase in the number of cancer population present in the system because of the progression of hepatitis B and C infection to liver cancer. From the figures, it can also be seen that respective populations are tending to the equilibrium level. This has also been observed for different initial values of the variables. Hence, the equilibrium P^* is globally asymptotically stable for this set of parameters.

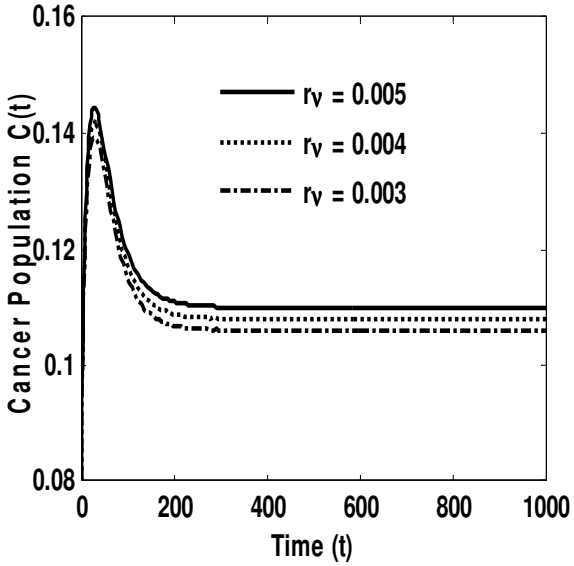

Fig. 6. Variation of cancer population with time for different rate of occurrence of liver cancer in hepatitis infectives

VI. CONCLUSION

In this paper, a nonlinear mathematical model is proposed and analyzed to demonstrate the correlation between hepatitis and cancer in a homogeneous population with constant immigration of cancer patients and population susceptible to hepatitis. It is concluded from the analysis that if the rate of transmission of hepatitis infection increases, the endemic level of infective population increases which can further be enhanced if rate of transmission of hepatitis infection among cancer patients also increases. The endemic equilibrium is found to be locally asymptotically stable. Further, to illustrate graphically the global stability of the equilibrium point, numerical simulation is performed for different initial starts and the results are displayed. Sensitivity analysis of the endemic equilibrium to changes in the value of the different parameters associated with the system is done and it is found that the parameters μ, β, β_1, θ, v, ε, σ are most sensitive to the infective population. More attention needs to be given to the estimation of these parameters. It is found from our analysis that hepatitis infection among cancer patients plays a considerable role in the spread of disease and hence in increasing the number of hepatitis infectives in the population. It is observed that as the recruitment rate of cancer patients in the population increases, equilibrium value of infective population also increases from which it can be inferred that the disease spreads fast if there is continuous influx of cancer patients in the community who are most vulnerable to acquire hepatitis infection due to frequent hospitalization or low threshold for infection because of immunological changes in them. Thus, on increasing the awareness about frequent hepatitis infection in cancer patients and thereby taking care of their treatment, the infection can be slowed down and may be kept under control. It is further concluded that in endemic areas, increase in the hepatitis infective offsprings in exposed and infective state by chronically infected mother in exposed and infective class respectively has a considerable effect in increasing endemic infective class size. Hence, attempts should be made to prevent the neonate to be prenatally infected by the chronically infected mothers. It is observed from the numerical simulation that vaccination induces rapid decrease in the number of acute hepatitis virus infections and has a secondary effect of a decrease in related sequels because of which number of infectives in the community decrease. Further, it is found from our analysis that hepatitis infection in a community causes an increase in the number of cancer population present in the system because of the progression of hepatitis B and C infection to liver cancer.

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