Numerical treatment of MHD Powell-Eyring Fluid Flow using the Method of Satisfaction of Asymptotic Boundary Conditions

M. Patel and M.G.Timol

***Abstract*— The steady, laminar, incompressible boundary layer flow of an electrically conducting non-Newtonian fluids past a two-dimensional surface and flowing under the influence of transverse magnetic field is studied. Group–theoretic method is applied to search out similarity transformations. The proper form of free stream velocity and imposed transverse magnetic field strength are systematically derived from similarity requirement. The important conclusion drawn from this analysis is that, in case of flow past any body shape, similarity solution exists only for all those non–Newtonian fluids whose shearing stress is homogeneous explicit function of rate of strain. On the other hand, the similarity solutions for all those non–Newtonian fluids whose shearing stress is either non–homogeneous explicit function of rate of strain or composite function of rate of strain or implicit function of rate of strain exist only for the flow past 90^o wedge. The graphical presentation along with the numerical solution using MSABC for Powell-Eyring fluids is presented.**

***Index Terms*—Group–theoretic method, MSABC, non–Newtonian fluids, Ostwald-de-Waele power-law fluids, Powell-Eyring Fluid**

***MSC 2010 Codes* —34BXX, 76XX**

NOMENCLATURE

u' - velocity in x direction
v' - velocity in y direction
ρ - fluid density
$\tau_{y'x'}$ - shearing stressing x direction and normal to y direction
σ - electrical conductivity
$U_e'(x')$ - main stream velocity
F - arbitrary continuous function
Rn - Reynold's number
$S'(x')$ – magnetic parameter
$\psi(x, y)$ - stream function
α_i, A_i, p, q- real constants
n - flow consistency index and m is fluid constant
f, g- similarity variables
A, B, C - fluid constants

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I. INTRODUCTION

FOR a long time, there has been considerable interest in non-Newtonian fluids [1-3]. This is because non-Newtonian fluids are found to be of great commercial importance. Examples of such fluids includes slurries, shampoo, toothpaste, paint, clay coating and suspensions, grease, cosmetic products, custard, blood and many others. Non-Newtonian fluids are handled extensively by chemical industries namely plastics and polymer. Thus wide usages of these fluids have prompted modern researchers to explore extensively, the field of non-Newtonian fluids [4-9].

When we consider electrically conducting non-Newtonian fluids flowing under the influence of external magnetic field, the study becomes interesting. This is because in such situation magnetic forces produced in it could influence the motion of the fluids in significant way and hence such interaction problems have great practical applications. The problem of two-dimensional magneto hydrodynamic boundary layer equation for laminar incompressible flow past flat plate has been investigated by Rossow [10] and Greenspan et al [11]. Rossow [10] has considered transverse magnetic field where as Greenspan et al [11] have considered longitudinal magnetic fields on the velocity and temperature distributions. Timol et al [12] have investigated three-dimensional magneto hydrodynamic boundary layer flow with pressure gradient and fluid injection. Similarity transformation for both steady and unsteady three-dimensional MHD boundary layer flow of purely viscous non-Newtonian fluid has been derived by Manisha et al [13].

They have also derived Similarity Analysis in MHD Heat and Mass Transfer of Non-Newtonian Power Law Fluids Past a Semi-infinite Flat Plate [14].

The two-dimensional magneto hydrodynamic boundary layer flow of electrically conducting non-Newtonian fluids has been studied by many investigators. But all these cases are found to be limited to Ostwald-de-Waele power-law fluids. This is because it is assumed that non-Newtonian behavior can be described mostly by power-law model. This assumption was probably made due to mathematical simplicity of this model. But it is well known fact that there are certain shortcomings of power-law fluids. For example, this model is derived from empirical relation; hence dimension of one parameter depends on that of the others. Further this model indicates infinite effective viscosities for low share rates. These shortcomings reduce it range of applicability.

The Powell-Eyring model, although mathematically more complex, deserves our attention because it has certain advantages over the power-law model. Firstly it is deduced from kinetic theory of liquid rather than the empirical relation as in the case of power-law model. Secondly it correctly reduces to Newtonian behavior for law and high shear stress.

In our previous paper [15], the numerical study is made of the laminar, incompressible flow of a non-Newtonian fluid past 90^o wedge, where the fluid under consideration is Powell-Eyring. The non-MHD case was considered in it. In the present paper similarity analysis is made of two-dimensional steady, laminar, incompressible MHD boundary layer flow of electrically conducting non-Newtonian fluids. The different non-Newtonian fluids considered are characterized by the property that its shearing stress tensor is either explicit function (homogeneous) or non-homogeneous or composite or implicit function of rate of deformation tensor .Some important fluids belonging to these categories are given in Table–1. From present analysis it is observed that for non-Newtonian fluids past any body shape, only non-similar solution can exist. Expect for power-law fluids for which similarity solution is available. That is for all non-Newtonian fluids similarity solution exists only for the flow past 90^o wedge.

It is worth to note that the study of MHD boundary layer flow of non-Newtonian Powell-Eyring fluids treated here is a topic which seems to have been rarely available in the literature. This may be due to the mathematical complexity of this particular model. The governing equations of motion are solved by the satisfaction of asymptotic boundary conditions. The method is based upon the least-square convergence criterion, which leads to the unique solution. The description of the method is very lengthy so we have not discussed separately here but it is discussed in detailed by Nachtsheim and Swigert [16]. The graphical presentation along with the numerical solution of similarity equation for Powell-Eyring fluids is presented.

II. PROBLEM FORMULATION

The equation of motion for incompressible electrically conducting non-Newtonian fluids past external surface under the influence of transverse magnetic field can be written as

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \qquad \text{------------------} \qquad (1)$$

$$u'\frac{\partial u'}{\partial x'} + v\frac{\partial u'}{\partial y'} = \frac{1}{\rho}\frac{\partial}{\partial y'}\left(\tau_{y'x'}\right) + \frac{\sigma B_{y'}^{2}(x')}{\rho}(U_e' - u') + U_e'\frac{dU_e'}{dx'} \qquad \text{---------} \qquad (2)$$

With boundary conditions

$$y' = 0 \Rightarrow u' = 0 \,.\, v' = 0 \qquad \text{-------------------} \qquad (3)$$

$$y' \to \infty \Rightarrow u' \to U_e' \qquad \text{-------------------} \qquad (4)$$

Here, the shearing stress $\tau_{y'x'}$ related to the rate of strain $\frac{\partial u'}{\partial y'}$ by an arbitrary continuous function F given by

$$F\left[\tau_{y'x'}; \frac{\partial u'}{\partial y'}\right] = 0 \qquad \text{----------------------} \qquad (5)$$

This stress strain relationship will differ for different fluid models as shown in Table – 1.
Introduction following dimensionless quantities:

$$\left.\begin{aligned} & x = \frac{x'}{L}, \quad y = \frac{y'}{L}\sqrt{Rn} \\ & u = \frac{u'}{U_\infty}, \quad v = \frac{v'}{U_\infty}\sqrt{Rn} \\ & T_{yx} = \frac{T_{y'x'}}{\rho U_\infty^{2}}\sqrt{Rn}, \\ & S(x) = \frac{L}{U_\infty} S'(x'), \; U_e = \frac{U_e'}{U_\infty} \end{aligned}\right\} \qquad \text{————————} \qquad (6)$$

where $Rn = \frac{U_\infty L}{\gamma}$ magnetic Reynolds number and

$$S'(x') = \frac{\sigma B_{y'}^{2}(x')}{\rho} \qquad \text{magnetic parameter}$$

Equation (1) – (4) becomes

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y}=\frac{\partial}{\partial y}(\tau_{yx})+ S(x)\,U_e-\frac{\partial\psi}{\partial y}+U_e\frac{dU_e}{d_x}$$

------------------- (7)

with boundary condition

$$y=0\Rightarrow\frac{\partial\psi}{\partial y}=0\,,\ \frac{\partial\psi}{\partial x}=0$$ ------------------- (8)

$$y\to\infty\Rightarrow\frac{\partial\psi}{\partial y}=U_e$$ ------------------- (9)

where $\psi(x,y)$ is a stream function such that

$$u=\frac{\partial\psi}{\partial y}\quad,\quad v=-\frac{\partial\psi}{\partial x}$$ -------------------- (10)

Solutions (7)–(9) represent system of non-linear partial differential equations which are quite difficult to solve. If we convert above system of non-linear partial differential equations to the system of non-linear ordinary differential equations with the help of some suitable transformations, known as similarity transformations. Then we can solve this simplified system with the help of some numerical method. In fact, similarity transformations available for present situation for special form of main stream velocity and imposed transverse magnetic field parameter S(x).We have used group theoretic method to derive similarity solution of equations (7)–(10).

III. GROUP THEORETIC METHOD:

The group theoretic method which is used to find the similarity transformation is based on the concepts derived from continuous transformation of groups. Recently, this technique is found to give most adequate treatment of boundary layer equations. The basic concept of this method was first introduced by Lie [17] in the later part of the century. For the present problem we introduce one parameter group of transformation $\lceil 1$ given below:

$$\lceil 1=\begin{bmatrix} x=A^{\alpha_1}\overline{x} & y=A^{\alpha_2}\overline{y}\\ \psi=A^{\alpha_3}\overline{\psi} & T=A^{\alpha_4}\overline{T}\\ U_e=A^{\alpha_5}\overline{U_e} & S(x)=A^{\alpha_6}\overline{S}\end{bmatrix}$$

----------------- (11)

Where α_i (i=1 to 6) and A are real constants introducing transformation $\lceil 1$ in equations (7)-(9) and these equations remaining invariant under the following relations among x's

$$2\alpha_3-2\alpha_2-\alpha_1=\alpha_4-\alpha_2=\alpha_6\alpha_5 =\alpha_6(\alpha_3-\alpha_2)=2\alpha_5-\alpha_1$$

------------------- (12)
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and $$\alpha_3-\alpha_2=\alpha_5$$ -------------------- (13)

solving equations (12) and (13) under the assumption that

$$\frac{\alpha_4}{\alpha_1}=p\quad and\quad\frac{\alpha_5}{\alpha_1}=q$$ -------------------- (14)

We get

$$\frac{\alpha_2}{\alpha_1}=1+p-2q\ ,\ \frac{\alpha_3}{\alpha_1}=1+p-q\ and\ \alpha_6=q-1$$

-------------------- (15)

Above values of x's gives following similarity variables [18]

$$\eta=\frac{y}{x^{(1+p-2q)}}\ ;\ f(\eta)=\frac{\psi}{x^{(1+p-q)}}\ ;\ g(\eta)=\frac{\tau}{x^{p}}$$ --------------------- (16)

$$U_e=\frac{U_0}{x^q}\ ;\ S(x)=\frac{S_0}{x^{(q-1)}}$$ ---------------------- (17)

Introducing equations (16) and (17) in equation (7) and (9), we get following similarity equation

$$q\,f'^2-(1-p-q)\,f\,f''=g'+S_0\,(U_0-f')+q$$

----------------------- (18)

where prime denote derivative with respect to η .

The boundary conditions are

$$\eta=0\to\ f=0\,,\ f'=0$$ --------------------- (19)

$$\eta=\alpha\ \to\ f'=U_0$$ --------------------- (20)

Now for general non-Newtonian fluid of any model (given by Table-1) similarity solution can be obtained by the method given below:

IV. SIMILARITY SOLUTION FOR POWER-LAW FLUIDS:

The mathematical expression of strain-stress relation for this fluid is given by (Table 1)

$$\tau_{y'x'} = m \left| \frac{\partial u'}{\partial y'} \right|^n \quad \text{------------------------- (21)}$$

where n is flow consistency index and m is fluid constant.

Note that here shearing stress $\tau_{y'x'}$ is explicit homogeneous function of strain $\frac{\partial u'}{\partial y'}$ introducing non-dimensional quantities defined by equation (6) and stream functions (10)
in equation (21) we get

$$\tau_{y'x'} = \left| \frac{\partial^2 \psi}{\partial y^2} \right| \quad \text{------------------- (22)}$$

The invariance of equation (22) under group transformation (11) gives following relation

$$\alpha_4 = n \left| \alpha_3 - 2\alpha_2 \right| \quad \text{------------------- (23)}$$

Combining (15) and (23) get

$$p = \frac{n(3q-1)}{n+1} \quad \text{------------------- (24)}$$

Substituting this value of p in equation (18) we get

$$q f'^2 - \left[\frac{(2n-1)q+1}{n+1} \right] f f'' = n \left| f'' \right|^{n-1} f''' + S_0 (U_0 - f') + q \quad \text{------------------ (25)}$$

With $$g' = n \left| f'' \right|^{n-1} f''' \quad \text{------------------- (26)}$$

It is interesting to note that for the value of constant p given by equation (24) the similarity transformations (16) will reduce to

$$y = \eta\, x^{\left(\frac{(n-2)q+1}{n+1} \right)}, \psi = f(\eta)\, x^{\left(\frac{(2n-1)q+1}{n+1} \right)} \quad \text{-------------------- (27)}$$

V. ANALYSIS FOR POWELL-EYRING FLUIDS:

Among different model proposed in Table-1,we have particularly Powell-Eyring fluids because of two main reasons. Firstly, this model is deduced from kinetic theory of liquids rather than empirical relations. Secondly, it correctly reduces to Newtonian behavior for low and high shear rates.

Mathematically, Powell-Eyring model can be written as

$$\tau_{y'x'} = A \frac{\partial u'}{\partial y'} + \frac{1}{B} \sinh^{-1} \left[\frac{1}{c} \frac{\partial u'}{\partial y'} \right] \quad \text{------------------ (28)}$$

where A, B, C are fluid constants.

For these fluids, clearly shearing stress is $\tau_{y'x'}$ is composite function of rate of strain $\frac{\partial u'}{\partial y'}$ as pointed out earlier, here too, introducing (6)–(10) and then group transformation (11) in equation (28) the invariance of equation (28) gives following relations.

$$\alpha_4 = 0 \; and \; \alpha_3 - 2\alpha_2 = 0 \quad \text{------------------- (29)}$$

Using (29) in (15) ,we get

$$p = 0 \;\; and \;\; q = \frac{1}{3} \quad \text{------------------- (30)}$$

Substituting these values of p and q in (18) , we get

$$\frac{1}{3} f'^2 \frac{2}{3} ff'' = g' + S_0 (U_0 - f') + \frac{1}{3} \quad \text{-------------------- (31)}$$

$$g' = \alpha f'' + \frac{1}{\left[1 + \beta f''^2 \right]^{\frac{1}{2}}} f''' \quad \text{-------------------- (32)}$$

where α = ABC and $\beta = \frac{\rho U_0{}^3 x^3}{3AC^2 L}$ are non dimensional numbers.

Substituting (32) in (31) and simplifying we get

$$f''' = \frac{\left[\beta f''^2 + 1 \right]^{\frac{1}{2}} \left[f'^2 - 2ff'' - 1 + 3S(f' - 1) \right]}{\frac{1}{\alpha} + \left[\beta f''^2 + 1 \right]^{\frac{1}{2}}} \quad \text{------------------- (33)}$$

Again substituting (30) into (16) – (17) we get

$$\eta = \frac{y}{x^{\frac{1}{3}}} \;\; ; \;\; f(\eta) = \frac{\psi}{x^{\frac{2}{3}}} \quad \text{------------------- (34)}$$

$$U(x) = U_0\, x^{\frac{1}{3}} \quad \text{------------------ (35)}$$

$$S(x) = S_0\, x^{\frac{-2}{3}} \quad \text{------------------- (36)}$$

The main stream velocity U(x), given in equation (35) corresponds to the boundary layer flow over a wedge of 90° [19].Thus similarity solution for MHD boundary layer flow of Powell-Eyring fluids (and hence for all non-Newtonian fluids characterized by the same type of stress strain relationship) exists only for the case of a 90° wedge flow. Further, in this case, magnetic parameter S(x) will vary inversely with $2/3^{rd}$ power of x-coordinate. In the absence of external magnetic fields (i.e. when S = 0) the equation (33) reduced that of derived by Hansen et al [19], Sirohi et al [20].

VI. Numerical Solution

Equation (33) with boundary conditions (19)–(20) is solved numerically by the method of satisfaction of asymptotic boundary conditions due to Nachtsheim et al [16]. The method is briefly described below:

Here to solve equation (33) with boundary condition (19) – (20) is equivalent to the problem of finding a value of f'' (0) for which the boundary condition at the edge of boundary layer is to be satisfied . That is the solution of non-linear equation

$$f'_{edge}\left[f''(0) \right] = 1 \qquad \text{-------------------- (37)}$$

Where, $f'_{edge} = f'(\eta_{edge})$ is to be determined.

Denoting $f''(0) = x$, a small change Δx changes f'' by the amount

$$\frac{\partial f'}{\partial x}\Delta x \qquad \text{-------------------- (38)}$$

So that the correlation for the first approximation of x is given by the equation

$$1 = f' + \frac{\partial f'}{\partial x}\Delta x \qquad \text{-------------- (39)}$$

At $\eta = \eta_{edge}$

Here, Δx can be obtained by forming the perturbation equation for the function f. The necessary perturbation equation can be formed by differentiating the function f' with respect to x.

Firstly writing,

$$z = \left(1 + \beta f''^2\right)^{\frac{1}{2}} \qquad \text{--------------------- (40)}$$

The equation (33) will be

$$f''' = \frac{z\left[(f'^2 - 1) + 3S_0 (f' - 1) - 2 f f'' \right]}{z + \frac{1}{\alpha}} \qquad \text{------------------- (41)}$$

Differentiating equations (40) and (41) with respect to x we get

$$z_x = \frac{\beta f'' f''_x}{z} \qquad \text{------------------- (42)}$$

and

$$f_x''' = \frac{z_x\left[(f'^2 - 1) + 3S_0 (f' - 1) - 2 f' f'' \right] + z\left[2f' f'_x + 3S_0 f'_x - 2 f_x f'' - 2 f f''_x \right]}{z + \frac{1}{\alpha}} \qquad \text{------------- (43)}$$

With initial condition

$$\eta = 0 \rightarrow f_x = f'_x = 0 \;, f''_x = 1. \qquad \text{------------- (44)}$$

Here Δx is so chosen that both the equations

$$1 = f' + f'_x \Delta x \qquad \text{-------------------- (45)}$$

$$0 = f'' + f''_x \Delta x \qquad \text{--------------------- (46)}$$

are satisfied at $\eta = \eta_{stop}$

Following the principal of least squares, Δx is given by

$$\Delta x = \frac{f'_x (1 - f') - f'' f''_x}{f'^2_x + f''^2_x} \qquad \text{--------------------- (47)}$$

The error E between the asymptotic conditions and the computed values at $\eta = \eta_{stop}$ is given by

$$E = (1 - f')^2 + f''^2 \qquad \text{-------------------- (48)}$$

The value of x which gives $\Delta x = 0$ corresponds to the minimum value of E with respect to x. Once E minimum is attained no more change in the initial condition is required and this ensures the satisfaction of asymptotic boundary conditions at a finite value of n. Further the variation of $f''(0)$ with respect to $\frac{1}{\alpha}$ and β can similarity be discussed. For details one may refer Sirohi et al [20].

VII. The Computation Scheme

Assuming $x = 1.5$, the equations (33) and (43) along with their boundary conditions (19) – (20) and (44) respectively are integrated using the Adams-Moulton procedure with the step size 0.05 up to 3 steps the fourth order Runge-Kutta method is used. For each value of η the tolerance value of $\frac{\Delta x}{x}$ is specified. Starting from η = 0, h = 0.05 integration is done until η_{stop} is equal to specified value. At this step, the value of $\frac{\Delta x}{x}$ is checked. If it is found to be more than the specified tolerance the procedure is repeated with $x = x + \Delta x$ when value of $\Delta x / x$ is within the tolerance range, error value E is computed and tested against the tolerance value specified which is taken to be 1.0×10^{-10}. If the value of computed E exceeds the specified value, η_{stop} is increased and the complete procedure is repeated. For the present case, the tolerance value specified is satisfied for $\eta_{stop} = 10$.

VIII. RESULTS AND DISCUSSION

Numerical solution is generated for non-Newtonian Powell-Eyring fluids flowing over a 90^{o} wedge under the influence of transverse magnetic field. Here the physical quantity of interest is the shearing stress on the plate which is given by the equation

$$\tau_w = f''(0) + \frac{1}{\alpha\sqrt{\beta}}\sinh^{-1}\left[\sqrt{\beta f''(0)}\right] \quad \text{------------------- (49)}$$

The numerical solutions of this skin friction expression for a range of $\frac{1}{\alpha}$ and β for external magnetic field strength S_0 = 0.0 to 3.0 are shown in Fig-1 .The effect of both the parameters $\frac{1}{\alpha}$ and β simultaneously on skin friction is shown by three dimensional plot given in Fig-2. The slope of velocity profile $f''(0)$ on the surface of 90^{o} is given in Fig-3. For various values of $\frac{1}{\alpha}$, β and S_0. The effect of two parameters $\frac{1}{\alpha}$ and β , at the same time on $f''(0)$ is given in Fig-4. For non-magnetic case, that is for S_0 = 0.0, the value of skin friction is quite consistent with Powell-Eyring fluids compared with the Newtonian fluids which are constant at $f''(0)$ = 1.3149. This confirms that Powell-Eyring fluid correctly reduces to Newtonian fluid. It is interesting to note that as magnetic field strength S_0 increasing the value of skin friction increases with increasing $1/\alpha$ and with decreasing β. But at the same time slope of velocity profile $f''(0)$ decreases rapidly. These facts illustrate a very interesting feature of the magneto hydrodynamic boundary layer flow of Powell-Eyring fluid, namely that the skin friction shows an increase although the slope on the surface of the plate exhibits a decrease. Thus the effect of non-Newtoniality on the slope of velocity and skin friction is seen to be opposite. This conclusion is quite consistent with the study made by Sirohi et al [20]. The velocity profiles for different values of magnetic field strength S_0 and for fixed value of $1/\alpha = 10$ and $\beta = 5\times10^{4}$ are given in Fig-5 and Fig-6. Clearly the velocity of magneto hydrodynamic Powell-Eyring fluids increases with the increase in the magnetic field strength S_0.

All Figures 1-6 are plotted in terms of dimensionless parameters and hence represent all magneto hydrodynamic Powell-Eyring fluids.

REFERENCES

[1] A.H.P. Skelland, Non-Newtonian Flow and Heat Transfer, John Wiley, 1967.

[2] W.L. Wilkinson, Non-Newtonian Fluids , Peragaman , 1960.

[3] R.B. Bird, W.E. Stewert W.E and F.W. Lightfoot, Transport Phenomena, John Wiley, 1960.

[4] M. Patel and M.G. Timol, Similarity solutions of three-dimensional boundary layer small-cross flows of power law fluid, International Journal of Mathematical Sciences and Engineering Application, vol.2 no. 1, pp.167-174, 2008.

[5] M. Patel and M.G. Timol, Numerical solution of the equation for unsteady boundary layer flow of non-Newtonian fluids past semi-infinite plate, International Journal of Applied Mechanics and Mathematics, vol. 5, no 3, 22-29(2009).

[6] M. Patel and M.G. Timol, The general stress–strain relationship for some different visco-inelastic non-Newtonian fluids, International Journal of Applied Mechanics and Mathematics, vol. 6, no. 12, pp. 79-93, 2010.

[7] C. Fetecau, J. Zierep, R. Bohning and C. Fetecauc, On the energetic balance for the flow of an Oldroyd-B fluid due to a flat plate subject to a time-dependent shear-stress , Comput. Math. Appl., vol. 60, pp. 74-82, 2010.

[8] T. Hayat and M. Qasim, Influence of thermal radiation and Joule heating on MHD flow of a Maxwell fluid in the presence of thermophoresis, Int.J. Heat Mass Transfer, vol. 53, 4780-4788, 2010.

[9] H.C. Surati and M.G. Timol, Numerical study of force convection wedge flow of some non-Newtonian fluids, Int. J. of Appl. Math and Mech. vol. 6, no. 18, pp. 50-65, 2010.

[10] V.J. Rossow, On the flow of electrically conducting fluids over a flat plate in the presence of transverse magnetic field, NASA NT 3971, 1957.

[11] H.P. Greenspan and G.F. Carrier, Magneto – fluid dynamic flow past a flat plate, JFM, vol. 6, 77-82, 1959.

[12] M.G. Timol and A.G. Timol, Three dimensional magneto – fluid dynamic flow with pressure gradient and fluid injection , Ind. J. pure and App. Maths, vol. 19, no. 2, 208-212, 1988.

[13] M. Patel and M.G. Timol, Similarity solutions of three-dimensional MHD boundary layer flow of non-Newtonian fluids, ISH Journal of Hydraulic Engineering, Special Issue, Vol.14, no.3. pp. 82-93, 2008.

[14] M. Patel and M.G. Timol, Similarity Analysis in MHD Heat and Mass Transfer of Non-Newtonian Power Law Fluids Past a Semi-infinite Flate Plate, International Journal of Applied Mechanics and Engineering, vol.15, pp 1205-1214, 2010.

[15] M. Patel and M.G. Timol, Numerical treatment of Powell-Eyring fluid flow using method of satisfaction of asymptotic boundary conditions, Applied Numerical Mathematics, vol. 59, 2584-2592, 2009.

[16] P.R. Nachsheim and P .Swigert, Satisfication of asymptotic boundary conditions in numerical solution of system of non-linear equation of boundary layer type, NASA, TND – 3004.

[17] S. Lie, Uber differential invarianten . Ann. Vol. 24. p. 52. 1932.

[18] A.G. Hansen, Similarity Analysis of Boundary Value Problem in Engineering , Princeton Univ. Press. New Jersey, 1964.

[19] A.G. Hansen and T.Y. Na, Similarity solution of laminar incompressible boundary layer equation on non-Newtonian fluids, J. Basic Engn. Trans. ASME. Vol. 90. Series-D, 71-75, 1968.

[20] V. Sirohi, M.G. Timol and N.L. Kalathia, Numerical treatment of Powell-Eyring fluid flow past a 90 wedge , Reg J. Energy Heat & Mass Trans., vol. 6, no. 3, 219-223, 1984.

TABLE-1

Model	Stress Vs. Strain	Nature of functional relationship
Power-Law	$\tau'_{y'x'} = m \left\lvert \frac{\partial u'}{\partial y'} \right\rvert^{n-1} \frac{\partial u'}{\partial y'}$	$\tau_{y'x'}$ is homogeneous explicit function of $\partial u'/\partial y'$
Sisco	$\tau_{y'x'} = \left[a + b \left\lvert \frac{\partial u'}{\partial y'} \right\rvert^{n-1} \right] \frac{\partial u'}{\partial y'}$	$\tau_{y'x'}$ is non-homogeneous explicit function of $\partial u'/\partial y'$
William-son	$\tau'_{y'x'} = \left[\frac{A}{B + \frac{\partial u'}{\partial y'}} + \mu_\infty \right] \frac{\partial u'}{\partial y'}$	$\tau_{y'x'}$ is non-homogeneous explicit function of $\partial u'/\partial y'$
Sutterby	$\tau_{y'x'} = \mu_0 \left[\frac{\sinh^{-1}\left(B \frac{\partial u'}{\partial y'} \right)}{B\, \partial u'/\partial y'} \right] \frac{\partial u}{\partial y}$	$\tau_{y'x'}$ is composite function of $\partial u'/\partial y'$
Prandtl	$\tau_{y'x'} = A \sinh^{-1}\left(\frac{1}{C} \frac{\partial u'}{\partial y'} \right)$	$\tau_{y'x'}$ is composite function of $\partial u'/\partial y'$
Prandtl-Eyring	$\tau_{y'x'} = A \sinh^{-1}\left(\frac{1}{B} \frac{\partial u'}{\partial y'} \right)$	$\tau_{y'x'}$ is composite function of $\partial u'/\partial y'$
Powell-Eyring	$\tau_{y'x'} = A \frac{\partial u'}{\partial y'} + \frac{1}{B} \sinh^{-1}\left(\frac{1}{C} \frac{\partial u'}{\partial y'} \right)$	$\tau_{y'x'}$ is composite function of $\partial u'/\partial y'$
ReinePhi lippoff	$\tau_{y'x'} = \left[\mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + \left(\tau_{y'x'} / \tau_0 \right)^2} \right] \frac{\partial u}{\partial y}$	$\tau_{y'x'}$ is implicit function of $\partial u'/\partial y'$
Eyring	$\tau_{y'x'} = \frac{1}{B} \frac{\partial u'}{\partial y'} + \sin\left(\frac{1}{A} \tau_{y'x'} \right)$	$\tau_{y'x'}$ is implicit function of $\partial u'/\partial y'$
Ellis	$\tau_{y'x'} = \frac{\frac{\partial u'}{\partial y'}}{A + B \lvert \tau_{y'x'} \rvert^{\alpha - 1}}$	$\tau_{y'x'}$ is implicit function of $\partial u'/\partial y'$

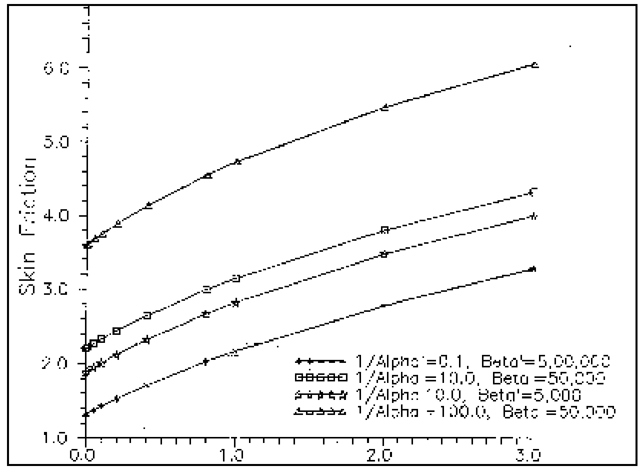


Fig.1-Skin friction versus S_o for constant $1/\alpha'$ & β'

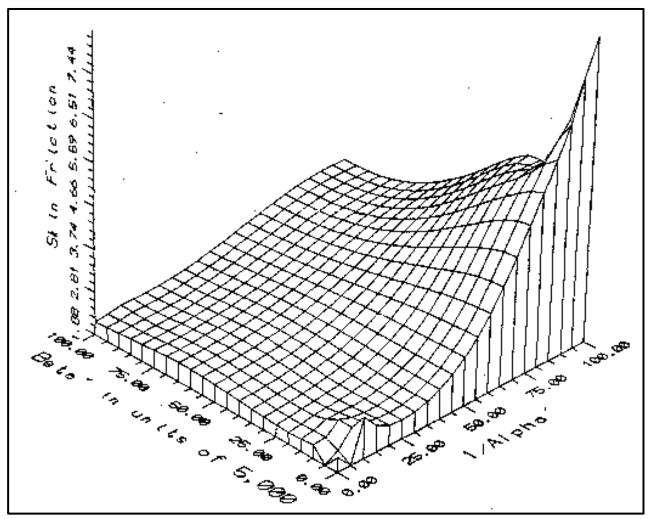


Fig.2 -Skin friction aginst $1/\alpha'$ & β'

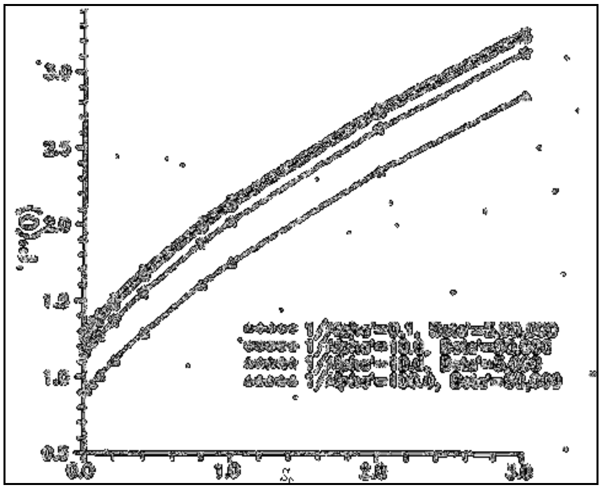


Fig.3- f''(0) versus S_o for constant $1/\alpha'$ & β'

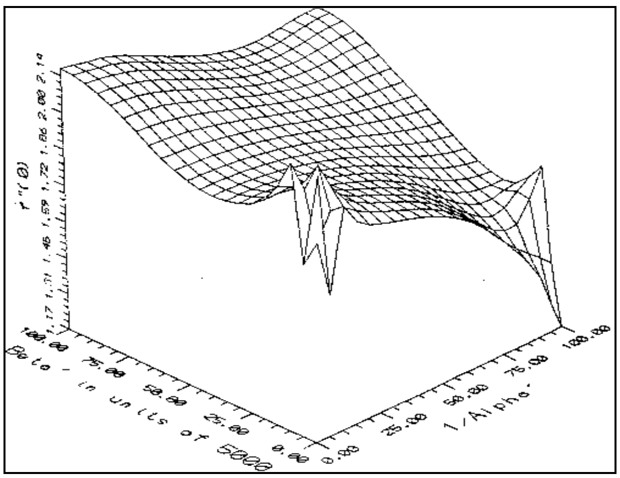
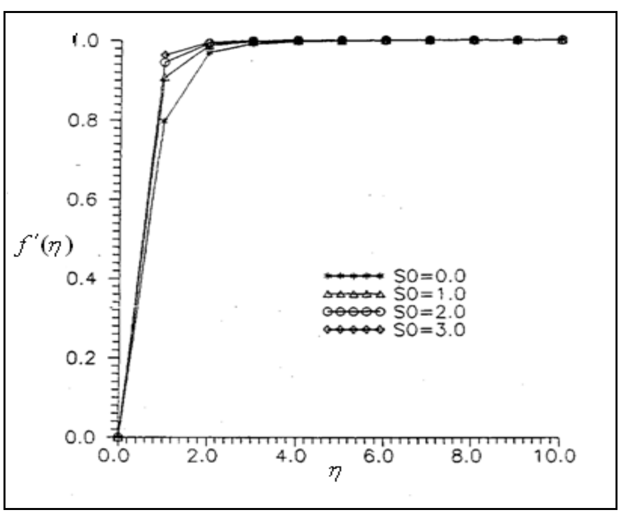
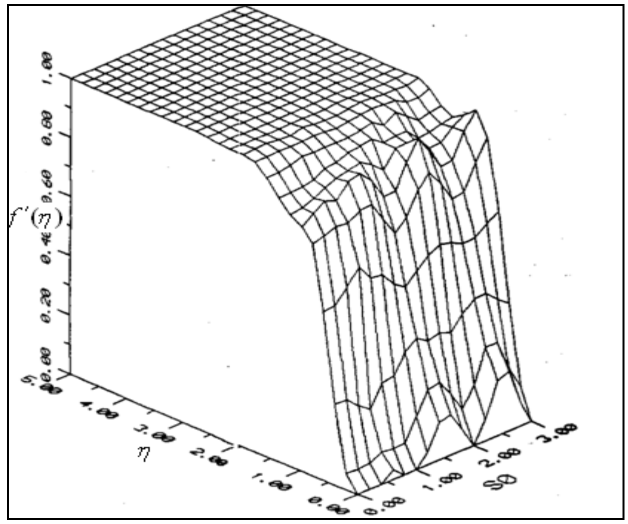


Fig.4 –f''(0) aginst $1/\alpha'$ & β'



$Fig.5 - f'(\eta)$ against η $(1/\alpha' = 10.0,\ \beta' = 50,000)$



$Fig.6 - f'(\eta)$ against η and S_0