A Novel Method for Poor Household Identification by Grey Theory

S. Mukherjee and S. Kar

Abstract— **A grey based technique is proposed here to solve the problem of identifying poor households in rural areas. The problem has been treated from the viewpoint of multi attribute decision making. The proposed approach is based on grey theory and a suitable case study is provided to illustrate the methodology.**

Index Terms— **Poor Household Identification, Fuzzy Logic, Grey Numbers, Grey Possibility Theory**

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I. Introduction

THE problem of identifying poor household in rural areas is really very critical and a concrete model of this problem satisfying all the socio economic parameters is somehow missing in the literature. Different authors attained their views on poverty from different angles both theoretically and practically. The main problem is that the theoretical possibilities face such type of problems that cannot be defined or structured in mathematical framework. So the error is there and we are unable to limit it. We always confront some questions while measuring poverty in rural areas. They are:

1. what should be the procedure of measuring the standard of living?
2. how do we aggregate data on welfare into a poverty measure? And finally
3. when do we say that someone is poor?

The authors got an emergence to observe this problem from the viewpoint of uncertainty. The various aspects of the status of a rural household are not completely deterministic. For example, consider the attribute: income. Normally in Indian rural areas the total income of an individual household varies in a certain range because of the unavailability of monetary engagements at different times. So when a household is asked to state their status corresponding to this attribute, it is quiet unnatural to have a numeric answer, rather we should seek for a linguistic termed answer, e.g., Very Poor, Poor, Medium Poor, Fair, Medium Good, Good, Very Good. The uncertainty is thus considered for each attribute.

Among the existing approaches towards this problem, two classifications are popular in the literature: traditional head count ratio approach and totally fuzzy relative approach. Some poverty models are also based on these approaches, e.g., logit model, conditional income distribution model, Dagum model, fuzzy set theoretic model, etc.

In Dagum poverty model the same evaluation is followed by the distribution function $F(y)=\frac{1}{(1+\lambda y^{-a})^{b}}$ where a and b are the shape parameters and λ is a scale parameter.

F. Bourguignon and S.R. Chakravarty proposed a simpler way to identify a poor household in a community by the poverty indicator variable

$\rho(x_i;z) = 1$ if there exists $j \in (1,2,\dots,m)$: $x_{ij} < z_j$

$= 0$ otherwise

where z_j is the line of separation for the jth attribute and hence the total number of poor households is simply given by H = $\sum_i \rho(x_i;z)$.

S.R. Chakravarty contributed another important method ([1]) in the context of the third type. In this method the membership function $\mu_j(x_{ij})$ depends on the quantity m_j at or above which a household is regarded as non poor with certainty with respect to the jth attribute, i.e., $\mu_j(x_{ij}) = 1$ if $x_{ij} = 0$

$= 0$ if $x_{ij} \geq m_j$.

In the mid values the membership function is $\mu_j(x_{ij}) = \left(\frac{m_j - x_{ij}}{m_j}\right)^{\theta_j}$ where $\theta_j \geq 1$ is a parameter. In [2] Verro has also presented an important work in this context. In some methods, the evaluation of the membership function of the attribute 'income' has been proposed in other non linear forms. In [3], an interpolation based method has been demonstrated to determine the membership function of a fuzzy set, where some statistical data is present. Based on this methodology, Mukherjee et al. has presented another way to specify the membership function of the attribute 'income' of a certain household. Some other methods are also there in the literature ([4], [5], [6], [7], [8], [9], [10]). An elaborate study on the existing approaches is present in [11].

In section II, some preliminary facts on grey theory are demonstrated. In section III, the proposed grey based approach is stepwise discussed and in section IV, a case study is talked about. Section V concludes the paper.

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II. PRELIMINARIES

Let X is the universal set of considerations. Then a Grey set G of X is defined by its two mappings $\overline{\mu}_G(x)$ and $\underline{\mu}_G(x)$: $\overline{\mu}_G(x): X \to [0,1]$ and $\underline{\mu}_G(x): X \to [0,1]$ such that $\overline{\mu}_G(x) \geq \underline{\mu}_G(x), x \in X$. The Grey set G becomes a fuzzy set when the upper and lower membership functions in G are equal to each other, i.e., when $\overline{\mu}_G(x) = \underline{\mu}_G(x)$. When the lower and upper limits of any information can be estimated by real numbers, we certainly are able to express it by an interval Grey number $\otimes G = [\underline{G}, \overline{G}] = \{\theta \in \otimes G : \underline{G} \leq \theta \leq \overline{G}\}$ where θ is an information and $\underline{G}$, $\overline{G}$ are respectively the lower and upper limits of the information's existence.

The degree of greyness, denoted by $\tilde{g}(\otimes G)$ is defined by a function of the two ends of the interval, i.e., $\tilde{g}(\otimes G) = f(\underline{G}, \overline{G})$.

According to Wary and Wu's approach ([12]) we now define some basic grey number operations:

$$\otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 - \overline{G}_2]$$

$$\otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 - \overline{G}_2]$$

$$\otimes G_1 \times \otimes G_2 = [\min(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2), \max(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2)]$$

$$\otimes G_1 \div \otimes G_2 = [\underline{G}_1, \overline{G}_1] \times \left[\frac{1}{\underline{G}_2}, \frac{1}{\overline{G}_2}\right]$$

We cite [15] to obtain the Grey Possibility Degree of $\otimes G_1 \leq \otimes G_2$ as

$$P\{\otimes G_1 \leq \otimes G_2\} = \frac{\max(0, L(\otimes G_1) + L(\otimes G_2) - \max(0, \overline{G}_1 - \underline{G}_2))}{L(\otimes G_1) + L(\otimes G_2)},$$ where $L(\otimes G) = \overline{G}_1 - \underline{G}_2$.

It is clear from the concept of possibility, that

i) when $\otimes G_1 = \otimes G_2$, then $P\{\otimes G_1 \leq \otimes G_2\} = 0.5$,

ii) when $\overline{G}_1 < \underline{G}_2$, then $P\{\otimes G_1 \leq \otimes G_2\} = 1$, and

iii) when $\overline{G}_1 > \underline{G}_2$, then $P\{\otimes G_1 \leq \otimes G_2\} = 0$.

III. METHODOLOGY

In this paper, a new approach based on grey theory is proposed for ranking poor households by evaluating their socio economic status. This is somehow equivalent to multi-criteria decision making in uncertain environment. Let us consider a discrete set of m households, H= {X_1, X_2, …, X_m} and a set of n attributes, T= {T_1, T_2, …, T_n}. These attributes are additively independent.

Also consider $\otimes w = \{\otimes w_1, \otimes w_2, ..., \otimes w_n\}$ as an attribute weight vector of the attribute weights which are realized to be linguistic variables. Now these linguistic weights and attribute ratings can be expressed in grey numbers as shown in table 2.

Table 2. Expression of linguistic terms in grey numbers

Linguistic Attribute weights		Attribute ratings	
Linguistic terms	Grey numbers	Linguistic terms	Grey numbers
Very Low	[0.0,0.1]	Very Poor	[0,1]
Low	[0.1,0.3]	Poor	[1,3]
Medium Low	[0.3,0.4]	Medium Poor	[3,4]
Medium	[0.4,0.5]	Fair	[4,5]
Medium High	[0.5,0.6]	Medium Good	[5,6]
High	[0.6,0.9]	Good	[6,9]
Very High	[0.9,1.0]	Very Good	[9,10]

Step1: A group of k Decision Makers D= {D_1, D_2, …, D_k} is formed. The attribute weight $\otimes w_j$ of the jth attribute A_j is calculated as

$$\otimes w_j = \frac{1}{k}\left[\otimes w_j^1 + \otimes w_j^2 + ... + \otimes w_j^k\right] \cdots (3.1)$$ where $\otimes w_j^k$ is the attribute weight given by the kth Decision Maker, described by the grey number [$\underline{w}_j^k$, $\overline{w}_j^k$].

Step2: We denote the linguistic variables for the ratings to make an attribute rating value as $\otimes G_{ij}$, defined by the grey numbers $\left[\otimes \underline{G}_{ij}, \otimes \overline{G}_{ij}\right]$. Thus the Grey Decision Matrix (GDM) is established. This matrix is normalized and the Normalized Grey Decision Matrix (NGDM) is constituted by the elements $\otimes G_{ij}{}' = \left[\frac{\underline{G}_{ij}}{G_j^{\max}}, \frac{\overline{G}_{ij}}{G_j^{\max}}\right] \cdots (3.2)$ where $G_j^{\max} = \max_{1 \leq i \leq m}\left\{\overline{G}_{ij}\right\} \cdots (3.3)$.

Step3: Now each normalized elements are multiplied by their corresponding weights and we retrace the weighted NGDM by the elements $\otimes T_{ij} = \otimes G_{ij}{}' \times \otimes w_j \cdots (3.4)$.

Step4: At the final stage of this method, a pseudo alternative, called Comparatively Richest Household (CRH) is constructed by CRH =

$$\left\{\otimes G_1^{\max} = \left[\max_i \underline{T}_{i1}, \max_i \overline{T}_{i1}\right], ..., \otimes G_n^{\max} = \left[\max_i \underline{T}_{in}, \max_i \overline{T}_{in}\right]\right\} \cdots (3.5).$$

We use this pseudo alternative to evaluate the ordering. Each household is compared with the CRH by the grey-possibility degree $P\left\{X_i \leq CRH\right\} = \frac{1}{n}\sum_{j=1}^{n} P\left\{\otimes T_{ij} \leq \otimes G_j^{\max}\right\} \cdots (3.6)$.

The ranking is done according to this possibility value and higher possible alternative gets greater rank.

Based on this rank the Government or the policy makers can establish a poverty line separation algorithm. The next section exemplifies the methodology with an alternative proposition.

IV. EXAMPLE

We now collect a real life example to show the effectiveness of the above method. There are few methods for multidimensional poverty evaluation in literature. Some of them are based on classical two valued logic, i.e., only black and white cases are considered there. The problem in these approaches is that some of the linguistic terms used to construct the attributes are fuzzy in the sense that 0 or 1 answers to them really do not help to solve the issue fully. For example, consider the case of 'education'. A person may have completed his education up to a certain level or he is in the middle of his educational life. The degree of membership of belongingness to the fuzzy set 'educated persons' cannot be only 0 and 1 in any sense. So the concept of Grey interval numbers is here introduced.

The attributes we consider here in this example are

T_1: Housing Condition

T_2: Clothing Status

T_3: Educational Status

T_4: Average monthly income per capita

T_5: Food Security

From an Indian rural area, we have surveyed five household alternatives. On the basis of the five attributes, the grey decision matrix GDM is constructed and displayed in table 3. The entries are linguistic terms.

Table 3. Grey Decision Matrix

GDM	T_1	T_2	T_3	T_4	T_5
X_1	P	MP	VP	MP	MP
X_2	MP	F	F	MP	MG
X_3	VP	P	VP	P	F
X_4	F	F	MG	MG	P
X_5	MP	MP	G	F	VP

A group of four decision makers is constructed. Table 4 is constructed by the weights of the attributes T_1, T_2, T_3, T_4 and T_5 given by the decision makers. These weights are also linguistic terms.

Table 4. Attribute Weight Matrix

$\otimes w_j$	D_1	D_2	D_3	D_4
T_1	MH	M	M	M
T_2	L	ML	L	L
T_3	L	L	VL	L
T_4	VH	VH	VH	VH
T_5	M	M	MH	M

Using (3.1) the average attribute weights are calculated as: $\otimes w_1$ = [0.425, 0.525], $\otimes w_2$ = [0.15, 0.325], $\otimes w_3$ = [0.075, 0.25], $\otimes w_4$ = [0.9, 1], $\otimes w_5$ = [0.425, 0.525].

Now using (3.2) and (3.3) normalized grey decision matrix is constructed and shown in table 5.

Table 5. Normalized Grey Decision Matrix

NGDM	T_1	T_2	T_3	T_4	T_5
X_1	[0.11, 0.33]	[0.33, 0.44]	[0, 0.11]	[0.33, 0.44]	[0.33, 0.44]
X_2	[0.33, 0.44]	[0.44, 0.55]	[0.44, 0.55]	[0.33, 0.44]	[0.55, 0.67]
X_3	[0, 0.11]	[0.11, 0.33]	[0, 0.11]	[0.11, 0.33]	[0.44, 0.55]
X_4	[0.44, 0.55]	[0.44, 0.55]	[0.55, 0.67]	[0.55, 0.67]	[0.11, 0.33]
X_5	[0.33, 0.44]	[0.33, 0.44]	[0.67, 1]	[0.44, 0.55]	[0.33, 0.44]

The entries in the NGDM are multiplied by the corresponding attribute weights. Now using (3.4) the elements of the weighted normalized grey decision matrix are calculated and the matrix is formed in table 6.

Table 6. Weighted Normalized Grey Decision Matrix

WNGDM	T_1	T_2	T_3	T_4	T_5
X_1	[0.047, 0.173]	[0.140, 0.231]	[0, 0.058]	[0.14, 0.231]	[0.14, 0.231]
X_2	[0.049, 0.143]	[0.066, 0.179]	[0.066, 0.179]	[0.049, 0.143]	[0.082, 0.218]
X_3	[0, 0.027]	[0.008, 0.082]	[0, 0.027]	[0.008, 0.082]	[0.033, 0.137]
X_4	[0.396, 0.55]	[0.396, 0.55]	[0.495, 0.0.67]	[0.495, 0.67]	[0.099, 0.33]
X_5	[0.140, 0.231]	[0.14, 0.231]	[0.285, 0.525]	[0.187, 0.289]	[0.14, 0.231]

From the above matrix, we first construct the 'Comparatively Richest Household' CRH by using (3.5) as CRH = {[0.396, 0.55], [0.396, 0.55], [0.495, 0.67], [0.495, 0.67], [0.14, 0.231]}. Finally we use (3.6) to find the grey-possibility degrees:

$$P\{X_1 \le CRH\} = 0.9,$$

$$P\{X_2 \le CRH\} = 0.92,$$

$$P\{X_3 \le CRH\} = 1,$$

$$P\{X_4 \le CRH\} = 0.48, \text{ and}$$

$$P\{X_5 \le CRH\} = 0.89.$$

Now it is the responsibility of the policy makers to draw the line of separation. We propose here to divide the range [0, 1] of possibilistic values in some intervals like [0, 0.3), [0.3, 0.6), [0.6, 0.8), [0.8, 0.9) and [0.9, 1] and separately economic policies should be applied on them. We call the household individuals of these classes as Comparatively Richest, Comparatively Medium Rich, Comparatively Rich, Comparatively Poor and Comparatively Poorest respectively. In our example, the household X_4 belongs to the Comparatively Richest class, X_5 belongs to the Comparatively Poor class and the households X_1, X_2 and X_3 belong to the Comparatively Poorest Class.

V. CONCLUSION

In literature, the problem of identification of the poor households has mainly been treated from two viewpoints: social status with respect to the socio-economic attributes and comparative position in the society. In this paper Grey theory has been applied to this problem considering the second type of approach. The difference in the possibilistic degrees reflects the socio economic positions of the household individuals. Finally the authors have proposed to subdivide the whole interval of analysis into more parts rather dividing them into just Poor and Non Poor classes. This way of evaluation certainly leads to a new scope of research in poverty evaluation.

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