Multi-Objective Interval-Valued Transportation Probabilistic Problem Involving Log-Normal

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***Abstract*—This paper deals with the interval coefficients to the multi-objective stochastic transportation problem. In this paper, we concentrate on our attention to multi-objective stochastic transportation problem involving an inequality type of constraints in which all parameters (supply and demand) are log-normal random variables and the coefficients of the objectives are interval numbers. The minimization interval valued multi-objective transportation problem is also converted in optimization of bi-objective functions using the order relation which represent the decision makers preference between the interval costs have been defined by the right limits, left limits, the center and the width of an interval. At first we convert the proposed probabilistic constraints into an equivalent deterministic constraints by chance constrained programming technique. Then the equivalent transformed problem of original multi-objective transportation problem has been solved by the weighted sum method and we obtained the optimal compromise solution. Lastly a numerical example is provided for the sake of illustrate the methodology.**

***Index Terms*—Transportation Problem, Multi-objective Programming, Stochastic Programming, Interval Programming, Log-normal Random Variable.**

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I. Introduction

THE linear programming technique can be fruitfully applied to transportation problem. The basic transportation problem involves the transportation or physical distribution of goods from several supply points to demand points and it also involves minimization of cost for distribution of product from factories to the number of cities. We should consider the requirement of goods at each demand points, variety of shipping routes and associated cost of distribution or transportation of goods or products from each origin to each destination. In conventional programming method requires the parameters to be known as constants. In practical situation, however, the parameters are seldom known exactly and have to be estimated. The imprecision may follow from the inexact environment or may be a consequence of a certain flexibility for the given enterprize associated with the cost of objective function. A frequently used means to express the imprecision information generalizes the concepts of multiple states of affairs and the concepts of the interval parameters. The probability and fuzzy approaches are frequently used to describe the uncertain element and treat imprecise present in a decision variable. The probability is widely used for characterizing uncertainty in physical system, when estimates of input probability distributions are available. It is describe uncertainty arising from stochastic distribution, variability condition and risk consideration.

In the typical problem, a product is transported from m sources to n destinations and their supply $(a_1, a_2, \cdots, a_m)$ and demand $(b_1, b_2, \cdots, b_n)$ are respectively. The coefficient C_{ij} of the objective function could represent the transportation cost, delivery time, number of goods transported, unfulfilled supply and demand, and others, are provided with transporting a unit of product from sources i to destinations j. Thus multi-objective penalty criteria may exist concurrently which lead to the research work on multi-objective transportation problem. In most of the real life problems of practical importance are modeled with multi-objective stochastic transportation problem which are measured in different scales and in the same type in conflict.

The mathematical model of the multi-objective transportation problem is presented as follows:

$$\text{Min} : z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k x_{ij}, \qquad k = 1, 2, \cdots, K \tag{1}$$

$$\text{subject to} \qquad \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \cdots, m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \cdots, n \tag{3}$$

$$x_{ij} \geq 0, \quad \forall\ i \text{ and } j. \tag{4}$$

We assume that $C_{ij}^k \quad (k = 1, 2, \cdots, K)$ are the cost coefficients in the objective function and $a_i \quad (i = 1, 2, \cdots, m)$ and $b_j \quad (j = 1, 2, \cdots, n)$ may be random variables in multi-objective stochastic transportation problem. Here z^k represents the minimum value of k-th objective function and it is assumed that $a_i > 0, \quad b_j > 0$, and $C_{ij}^k > 0$ and $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ (for unbalanced transportation problem)

Dantzig [1] was first to formulate the mathematical model of probabilistic programming in which he has suggested a two stage programming technique for solving a stochastic programming problem by converting the said problem into deterministic model. Charnes et al. [2] first introduced the chance constrained programming model known as probabilistic programming with suggestion of three models with different types of objective functions, such as the E-model, V-model and P-model. The E-model minimizes the expected value of objective functions, the V-model minimizes the generalized mean square of the objective functions and the

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P-model maximizes the probability of aspiration levels of the objective functions. Goicoechea et al. [3] described the deterministic equivalents for some probabilistic programming involving normal and other distributions. Stancu-Minasion et al. [4] has been presented a review paper on stochastic programming with a single objective function. Song et al. [5] developed the optimal compromise solution of the transportation problem with several objective functions. Sahoo et al. [6] discussed the probabilistic linear programming problem with random variables and they [7] are developed the computation of probabilistic linear programming problem involving normal and log-normal to some random variables with joint constraints and obtaining its solution by fuzzy programming technique. Kambo [8] discussed the chance constrained and two stage programming methods for solving a stochastic linear programming problem. Bit et al. [9] in 1992 have been presented the multi-objective transportation problem using the fuzzy programming technique on probabilistic constraints. Mahapatra et al. [10], have presented the fuzzy programming technique to obtained the compromise solution of multi-objective functions of stochastic transportation problem. Also they have discussed the conversion procedure from stochastic constraints to deterministic constraints when the sources and destination parameters involved randomness.

As the cost coefficients of the objective function, an interval signifies the extent a region that the parameters can take possibility. Each interval represents a set of possible values that particular entity may assume, without any a priori assumption about exact value. In practical situation, intervals of possibility should be used whenever decision variable can assume different values, but the probability measure in the field of economies, engineering and natural science branches on these values is not available.

Chanas et al. [11] presented the solution procedure of the linear programming problem with interval coefficient in the objective function based on preference relation between intervals. Oliveira et al. [12] provided an illustrated over view of the state of all interval programming in the context of multi-objective linear programming models. Suprajitno et al. [13] presented the interval linear programming, where the coefficients and variables are in the form of intervals and solved this problem by the modified simplex method that can be used in real interval. Dutta et al. [14] have studied the transportation problem with addition impurity restriction where the costs are subnormal fuzzy interval with strictly increasing membership function and this problem treated as a mixed integer nonlinear programming problem which is simplified into a linear fractional programming problem. Ishibuchi et al. [15] have proposed the simple decision making method under uncertainty by considering the coefficients in mathematical programming problem as intervals. Das et al. [16] presented a method to solve the multi-objective transportation problem in which the coefficients of the objective function as well as the source and destination parameters are in the form of interval. Steuer [17] developed a concept for optimization of multi-objective programming problem with interval objective function. Sengupta et al. [18] proposed a methodology for solving an interval valued transportation problem where multiple but a synchronized set of penalty function is involved .

Most of the researchers [[6],[16]], such as first authors have presented the fuzzy model in the objective function and the order relation of interval form approaches in the case of constraints involving sources, destinations for the classical transportation problem and the second author have followed by the conventional fuzzy approach for the solution to multi-objective linear programming problem with constraints having only one deterministic and other probabilistic.

In this paper, we study the multi-objective stochastic transportation problem involving interval coefficients in the objective function with two probabilistic constraints to the practical sense of real life problem. Both the probabilistic constraints are inequality type and the parameters (supply and demand) are followed log-normal random variables.

II. Basic Operations on Interval Programming and Order Relations

A. Interval Arithmetic

Definition: Denote the set of all real numbers by $R^{'}$. An order pair in a bracket defines an interval as

$$C_{ij} = \left[C_{L_{ij}}, C_{R_{ij}}\right] \quad = \quad \left[C_{L_{ij}} \le x \le C_{R_{ij}}, x \in R^{'}\right] \tag{5}$$

where $C_{L_{ij}}$ and $C_{R_{ij}}$ are left and right limits respectively of the interval in the coefficients of the objective function represented by its center or half width $w(C_{ij})$ and mean or mid point $m(C_{ij})$ of an interval may be calculated as $C_{ij} =< m(C_{ij}),\ w(C_{ij}) >$ where

$$m(C_{ij}) = \frac{1}{2}\left(C_{R_{ij}} + C_{L_{ij}}\right) \tag{6}$$

$$\text{and} \quad w(C_{ij}) = \frac{1}{2}\left(C_{R_{ij}} - C_{L_{ij}}\right) \tag{7}$$

Also $m(C_{ij})$ is referred as mean or center or central value or expected value of interval C_{ij}. And $w(C_{ij})$ is also referred as the spread or width or range or level of uncertainty or the extent of uncertainty of interval C_{ij}.

Definition: Let $o \in \{+, -, ., \div\}$ be the binary operation on $R^{'}$. If a and b are two intervals, then

$$aob \quad = \quad \{xoy \ : \ x \in a, \ y \in b\} \tag{8}$$

In the case of division it is always assumed that $0 \notin b$. The operation on intervals used in this paper are as follows:

$$a + b = [a_L, \ a_R] + [b_L, \ b_R] = [a_L + b_L, \ a_R + b_R]$$

$$a + b =< a_m, \ a_w > + < b_m, \ b_w >=< a_m + b_m, \ a_w + b_w >$$

$$a - b = [a_L, \ a_R] - [b_L, \ b_R] = [a_L - b_R, \ a_R - b_L]$$

$$a - b =< a_m, \ a_w > - < b_m, \ b_w >=< a_m - b_w \ , \ a_w - b_m >$$

$$\frac{a}{b} = [a_L, \quad a_R][\frac{1}{b_L}, \quad \frac{1}{b_R}] \quad \text{for} \quad 0 \notin b$$

$$\begin{aligned} ab \quad = \quad & [\min\{a_Lb_L, \ a_Lb_R, \ a_Rb_L, \ a_Rb_R\}, \\ & \max\{a_Lb_L, \ a_Lb_R, \ a_Rb_L, \ a_Rb_R\}] \end{aligned}$$

$$ka = \begin{cases} [ka_L, \ \ ka_R], & \text{for} \quad k \ge 0 \\ [ka_R, \ \ ka_L], & \text{for} \quad k < 0 \end{cases}$$

$$ka = k \left\langle a_m \ , \ a_w \right\rangle = \left\langle ka_m \ , \ \mid k \mid a_w \right\rangle, \quad k > 0$$

where k is a real number.

B. Relation Arithmetic

In this subsection, the order relation which represent the decision maker's preference between interval costs are defined for minimization problem. Let the uncertain costs from two alternatives be represented by interval A and B respectively. It is assumed that the cost of each alternative is known only to lie in the corresponding interval.

Definition: The order relation by the left and right limits of interval consider as the order relation $\leq_{LR}$ between $A = [a_L,\ a_R]$ and $B = [b_L,\ b_R]$ is defined as

$$\begin{aligned} A \leq_{LR} B \quad &\text{iff} \quad a_L \leq b_L \text{ and } a_R \leq b_R \\ A <_{LR} B \quad &\text{iff} \quad A \leq_{LR} B \text{ and } A \neq B \end{aligned} \tag{9}$$

The order relation represents the decision maker's preference for the alternative with the lower minimum cost and upper maximum cost, i.e, if $A \leq_{LR} B$, then A is preferred to B.

Definition: The order relation defines the center. The order relation $\leq_{mw}$ between $A = \langle a_m,\ a_w \rangle$ and $B = \langle b_m,\ b_w \rangle$ is defined as

$$\begin{aligned} A \leq_{mw} B \quad &\text{and} \quad a_m \leq b_m \text{ and } a_w \leq b_w \\ A <_{mw} B \quad &\text{and} \quad A \leq_{mw} B \text{ and } A \neq B \end{aligned} \tag{10}$$

The order relation $\leq_{mw}$ represents the decision maker's preference for the alternative with lower expected cost and less uncertainty, i.e, if $A \leq_{mw} B$, then A is preferred to B.

C. Formulation of the crisp objective functions

In this section, we have presented the formulation of the original interval valued objective function (1) is an equivalent form of the specified problem.

Definition: If $x^* \in S$ is an optimal solution of the specified problem if there is no other solution $x \in S$ which satisfied $z^k(x) <_{LR} z^k(x^*)$ or $z^k(x) <_{mw} z^k(x^*)$, where S is the set of feasible solution.

The central limit $m\left(z^k x\right)$and half width $w\left(z^k x\right)$ of the coefficient C_{ij}^k of the interval minimum objective function may be established, when $x_{ij} \geq 0, \ \ \forall\ i$ and j, i.e, $m\left(z^k x\right)$ and $w\left(z^k x\right)$ can be modified as the mean and half width of the objective function $z^k(x)$ can be established as follows:

$$m(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C_{R_{ij}}^k + \sum_{i=1}^{m}\sum_{j=1}^{n} C_{L_{ij}}^k\right] x_{ij} \tag{11}$$

$$w(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C_{R_{ij}}^k - \sum_{i=1}^{m}\sum_{j=1}^{n} C_{L_{ij}}^k\right] x_{ij} \tag{12}$$

where $C_{L_{ij}}^k$ and $C_{R_{ij}}^k$ are left and right limits respectively of the coefficient C_{ij}^k of $z^k(x)$.

III. Mathematical Model

In this paper, we have considered the mathematical model for multi-objective stochastic transportation problem with interval valued cost coefficients of the objective functions based on preference relation between the intervals and the parameters of all constraints are in log-normal random variable involving in the constraints. The interval $\left[C_{L_{ij}}^k, C_{R_{ij}}^k\right]$, $k = 1, 2, \cdots, K$ is represented as an interval inexact cost components for the stochastic transportation problem, it can be transportation cost, delivery time, unfulfilled demand, quantity of good delivered. The cost coefficient lies between the left limits $C_{L_{ij}}^k$ and right limits $C_{R_{ij}}^k$ as follows:

Model 1:

$$\min : z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[C_{L_{ij}}^k, C_{R_{ij}}^k\right] x_{ij},\ k = 1, 2, \cdots, K \tag{13}$$

$$\text{subject to } \Pr\left(\sum_{j=1}^{n} x_{ij} \leq a_i\right) \geq 1 - \alpha_i,\ i = 1, 2, \cdots, m \tag{14}$$

$$\Pr\left(\sum_{i=1}^{m} x_{ij} \geq b_j\right) \geq 1 - \beta_j,\ j = 1, 2, \cdots, n \tag{15}$$

$$x_{ij} \geq 0, \quad \forall \ \ i \text{ and } \ j.$$

where $0 < \alpha_i < 1,\ \forall \ \ i$ and $0 < \beta_j < 1,\ \forall \ \ j$. We assumed that $a_i \ \ (i = 1, 2, \cdots, m)$ and $b_j \ \ (j = 1, 2, \cdots, n)$ are specified with log-normal random variables. Now the following cases are to be considered.

Case [1.] Only $a_i, (i = 1, 2, \cdots, m)$ has assumed as log-normal random variable.

Case [2.] Only $b_j, (j = 1, 2, \cdots, n)$ has assumed as log-normal random variable.

Case [3.] Both $a_i, (i = 1, 2, \cdots, m)$ and $b_j, (j = 1, 2, \cdots, n)$ have assumed as log-normal random variables.

A. Only $a_i, i = 1, 2, \cdots, m$ has assumed as log-normal random variable

It is assumed that $a_i, \quad i = 1, 2, \cdots, m$ are independent log-normal random variables with mean = $\mu_{a_i} = E(\ln a_i)$ and variance = $Var(\ln a_i) = \sigma_{a_i}^2$, which are known in our problem, i.e.,

$$\text{mean of } a_i = E(a_i) = \exp\left(\mu_{a_i} + \frac{\sigma_{a_i}^2}{2}\right), \quad i = 1, 2, \cdots, m \tag{16}$$

$$\text{variance of } a_i = Var(a_i) = \exp\left(2\mu_{a_i} + \sigma_{a_i}^2\right) \exp(\sigma_{a_i}^2 - 1),\ i = 1, 2, \cdots, m \tag{17}$$

The probability density function of i-th random variable $a_i,\ i = 1, 2, \cdots, m$ is

$$f(a_i) = \frac{1}{\sqrt{(}2\pi)\sigma_i a_i} \exp\left[-\frac{1}{2}\left(\frac{\ln a_i - \mu_i}{\sigma_i}\right)^2\right], \quad 0 < a_i < \infty, \sigma_i > 0. \tag{18}$$

As $a_i, (i = 1, 2, \cdots, m)$ is log-normal random variable, so the equation (14) can be represented as follows:

$$\Pr\left(\ln\sum_{j=1}^{n} x_{ij} \leq \ln a_i\right) \geq 1 - \alpha_i, \quad i = 1, 2, \cdots, m \tag{19}$$

The above constraints can be expressed as:

$$\Pr\left[\frac{\ln(\sum_{j=1}^{n} x_{ij}) - E(\ln a_i)}{\sqrt{V}ar(\ln a_i)} \leq \frac{\ln a_i - E(\ln a_i)}{\sqrt{V}ar(\ln a_i)}\right] \geq (1 - \alpha_i),\ i = 1, 2, \cdots, m \tag{20}$$

On rearranging, we get

$$1-\Phi\left[\frac{\ln(\sum_{j=1}^{n} x_{ij})-E(\ln a_i)}{\sqrt{V}ar(\ln a_i)} \geq \frac{\ln a_i - E(\ln a_i)}{\sqrt{V}ar(\ln a_i)}\right] \geq (1-\alpha_i), \quad i=1,2,\cdots,m \quad (21)$$

$$\text{or, } \Phi\left[\frac{\ln\sum_{j=1}^{n} x_{ij}-E(\ln a_i)}{\sqrt{V}ar(\ln a_i)} \geq \frac{\ln a_i - E(\ln a_i)}{\sqrt{V}ar(\ln a_i)}\right] \leq \alpha_i, \quad i=1,2,\cdots,m \quad (22)$$

where $\frac{\ln a_i - E(\ln a_i)}{\sqrt{V}ar(\ln a_i)}$ is a standard normal random variable with zero mean and unit variance and $\sigma_{a_i} = \sqrt{V}ar(ln(a_i))$. Here $\Phi(.)$ represents the cumulative density function of the standard normal random variable and if K_{α_i} denotes the value of the standard normal variable then we have $\alpha_i = \Phi(-K_{\alpha_i})$.
Then the constraint (19) can be stated as:

$$\Phi\left(\frac{\ln\sum_{j=1}^{n} x_{ij}-\mu_{a_i}}{\sqrt{V}ar(\ln a_i)}\right) \leq \Phi(-K_{\alpha_i}), \quad i=1,2,\cdots,m \quad (23)$$

This inequality will be satisfied only if

$$\frac{\ln\sum_{j=1}^{n} x_{ij}-\mu_{a_i}}{\sqrt{V}ar(\ln a_i)} \leq -K_{\alpha_i}, \quad i=1,2,\cdots,m \quad (24)$$

$$\text{or, } \ln\sum_{j=1}^{n} x_{ij} \leq \mu_{a_i} - K_{\alpha_i}\sqrt{V}ar(\ln a_i), \ i=1,2,\cdots,m \quad (25)$$

Finally, the probabilistic constraint (14) can be transformed into deterministic constraints as follows:

$$\sum_{j=1}^{n} x_{ij} \quad \leq \quad \exp[\mu_{a_i} - K_{\alpha_i}\sigma_{a_i}], \quad i=1,2,\cdots,m \quad (26)$$

Therefore, for the Case 1, we have obtained the multi-objective interval valued deterministic transportation problem denoted by **Model 2** instead of multi-objective interval valued probabilistic transportation problem as described in **Model 1** is follows:

Model 2:

$$\min : z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[C^k_{L_{ij}}, C^k_{R_{ij}}\right] x_{ij}, \ k=1,2,\cdots,K \quad (27)$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij} \leq \exp[\mu_{a_i} - K_{\alpha_i}\sigma_{a_i}], \ i=1,2,\cdots,m \quad (28)$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, \ j=1,2,\cdots,n \quad (29)$$

$$x_{ij} \geq 0, \quad \forall \ i \ \text{ and } \ j.$$

As described in subsection $II.C$, so the multi-objective interval valued functions can be defined as follows:

$$m(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{R_{ij}} + \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{L_{ij}}\right] x_{ij}$$

$$w(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{R_{ij}} - \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{L_{ij}}\right] x_{ij}$$

where $C^k_{L_{ij}}$ and $C^k_{R_{ij}}$ are the left and right limits of the coefficient C^k_{ij} of $z^k(x)$, where $k=1,2,3,\cdots,K$.

B. Only $b_j, (j=1,2,\cdots,n)$ has assumed as log-normal random variable

Assume that $b_j, \ j=1,2,\cdots,n$ is independent log-normal random variable with mean = $E(\ln b_j) = \mu_{b_j}$, variance = $Var(\ln b_j) = \sigma^2_{b_j}$, which are known in our problem, i.e.,

$$\text{mean of } b_j = E(b_j) = \exp\left(\mu_{b_j} + \frac{\sigma^2_{b_j}}{2}\right), \quad j=1,2,\cdots,n \quad (30)$$

$$\text{variance of } b_j = Var(b_j) = \exp\left(2\mu_{b_j} + \sigma^2_{b_j}\right) \exp(\sigma^2_{b_j} - 1), \ j=1,2,\cdots,n \quad (31)$$

As $b_j, (j=1,2,\cdots,n)$ is a log-normal random variable, so the equation (15) can be represented as follows:

$$\Pr[\ln\sum_{i=1}^{m} x_{ij} \geq \ln b_j] \geq 1-\beta_j, \quad j=1,2,\cdots,n \quad (32)$$

The above constraints can be expressed as:

$$\Pr\left[\frac{\ln(\sum_{i=1}^{m} x_{ij})-E(\ln b_j)}{\sqrt{V}ar(\ln b_j)} \geq \frac{\ln b_j - E(\ln b_j)}{\sqrt{V}ar(\ln b_j)}\right] \geq 1-\beta_j, \ j=1,2,\cdots,n \quad (33)$$

On rearranging, we get

$$\Pr\left[\frac{\ln(\sum_{i=1}^{m} x_{ij})-\mu_{b_j}}{\sqrt{V}ar(\ln b_j)} \geq \frac{\ln b_j - \mu_{b_j}}{\sqrt{V}ar(\ln b_j)}\right] \geq 1-\beta_j, \quad j=1,2,\cdots,n \quad (34)$$

But, $\frac{\ln b_j - E(\ln b_j)}{\sqrt{V}ar(\ln b_j)}$ is a standard normal random variable with zero mean and unit variance. Here $\Phi(.)$ represents the cumulative density function of the standard normal random variable and if K_{β_j} denotes the value of the standard normal variable then we have $\Phi\left(K_{\beta_j}\right) = 1-\beta_j$
Therefore, the constraint (32) can be stated as:

$$\Phi\left(\frac{\ln\sum_{i=1}^{m} x_{ij}-\mu_{b_j}}{\sqrt{V}ar(\ln b_j)}\right) \geq \Phi(K_{\beta_j}), \ j=1,2,\cdots,n \quad (35)$$

The inequality will be satisfied only if

$$\frac{\ln\sum_{i=1}^{m} x_{ij}-\mu_{b_j}}{\sqrt{V}ar(\ln b_j)} \geq K_{\beta_j}, \quad j=1,2,\cdots,n \quad (36)$$

$$\text{or, } \ln\sum_{i=1}^{m} x_{ij} \geq \mu_{b_j} + K_{\beta_j}\sigma_{b_j}, \quad j=1,2,\cdots,n \quad (37)$$

Finally, the probabilistic constraints (15) can be transformed into deterministic constraints as follows:

$$\sum_{i=1}^{m} x_{ij} \quad \geq \quad \exp[\mu_{b_j} + K_{\beta_j}\sigma_{b_j}], \quad j=1,2,\cdots,n \quad (38)$$

Therefore, for the Case 2, we have obtained the multi-objective interval valued deterministic transportation problem denoted by **Model 3** instead of multi-objective interval valued probabilistic transportation problem as described in **Model 1** is

follows:
Model 3:

$$\min : z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[C^k_{L_{ij}}, C^k_{R_{ij}}\right] x_{ij},\ k = 1, 2, \cdots, K \quad (39)$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1, 2, \cdots, m \quad (40)$$

$$\sum_{i=1}^{m} x_{ij} \geq \exp[\mu_{b_j} + K_{\beta_j}\sigma_{b_j}], \quad j = 1, 2, \cdots, n \quad (41)$$

$$x_{ij} \geq 0, \quad \forall\ i \text{ and } j.$$

As described in subsection $II.C$, so the multi-objective interval valued functions can be defined as follows:

$$m(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{R_{ij}} + \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{L_{ij}}\right] x_{ij}$$
$$w(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{R_{ij}} - \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{L_{ij}}\right] x_{ij}$$

where $C^k_{L_{ij}}$ and $C^k_{R_{ij}}$ are the left and right limits of the coefficient C^k_{ij} of $z^k(x)$, where $k = 1, 2, 3, \cdots, K$.

C. Both $a_i, (i = 1, 2, \cdots, m)$ and $b_j, (j = 1, 2, \cdots, n)$ have assumed as log-normal random variables

The mean and variance of a_i and b_j are known and previously defined. In this case, the equivalent deterministic model for the chance constrained programming technique of the multi-objective stochastic transportation problem in which cost coefficients are defined as interval valued form representing as:
Model 4:

$$\min : z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[C^k_{L_{ij}}, C^k_{R_{ij}}\right] x_{ij},\ k = 1, 2, \cdots, K \quad (42)$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij} \leq \exp[\mu_{a_i} - K_{\alpha_i}\sigma_{a_i}],\ i = 1, 2, \cdots, m \quad (43)$$

$$\sum_{i=1}^{m} x_{ij} \geq \exp[\mu_{b_j} + K_{\beta_j}\sigma_{b_j}],\ j = 1, 2, \cdots, n \quad (44)$$

$$x_{ij} \geq 0,\ \forall\ i \text{ and } j.$$

As described in subsection $II.C$, so the multi-objective interval valued functions can be defined as follows:

$$m(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{R_{ij}} + \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{L_{ij}}\right] x_{ij}$$
$$w(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{R_{ij}} - \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{L_{ij}}\right] x_{ij}$$

where $C^k_{L_{ij}}$ and $C^k_{R_{ij}}$ are the left and right limits of the coefficient C^k_{ij} of $z^k(x)$, where $k = 1, 2, 3, \cdots, K$.

IV. Solution Procedure

The central limit $m\left(z^k x\right)$and half width $w\left(z^k x\right)$ of the co-efficient C^k_{ij} of the interval valued objective function may be established, when $x_{ij} \geq 0, \quad \forall\ i$ and j, i.e, $m\left(z^k x\right)$ and $w\left(z^k x\right)$ can be modified as the mean and half width of the coefficient of the objective function $z^k(x)$ can be reduced into bi-objective functions using the order relation as follows:

$$m(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{R_{ij}} + \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{L_{ij}}\right] x_{ij}$$
$$w(z^k x) = \frac{1}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{R_{ij}} - \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{L_{ij}}\right] x_{ij}$$

where $C^k_{L_{ij}}$ and $C^k_{R_{ij}}$ are the left and right limits of the coefficient C^k_{ij} of $z^k(x)$, where $k = 1, 2, 3, \cdots, K$.
Using the above two relations, we developed a composite single objective function of this problem (13), is in its final form as

$$\min : z^k = [\lambda_k \sum_{k=1}^{K} v_k m(z^k x) + (1 - \lambda_k) \sum_{k=1}^{K} v_k w(z^k x)] \quad (45)$$

where $k = 1, 2, \cdots, K$. We solved the specified model by using the (45) with subject to unbalanced constraints (43) and (44) and non-negative conditions (4) respectively. Now with the help of weighted sum method, we obtained the optimal compromise solution for the given values of λ_k, where $\lambda_k \in [0, 1]$, λ_k = non-negative weighted vector and weighted values $v_1, v_2 \ldots, v_K$, such that $\sum_{k=1}^{K} v_k = 1$. The decision maker defines the factors λ_k as pessimistic or optimistic bias in achieving the compromise solution. If λ_k = 1 and λ_k = 0, then the decision maker shows absolute optimistic and pessimistic bias in the solution process of this specified problem. With λ_k = 0.5, or any value-close to 0.5, a similar proportional balance between decision maker's optimistic and pessimistic preference is of rationality and ability to find a compromise solution in conflictive solution. Here the compromise solution is known as non-inferior solution.

V. Numerical Example

Let us consider a real life transportation problem is defined as follows. A reputed company produces soft drink and maintain three warehouses and four retail stores in east zone of India. Each warehouse and retail store represents a potential point of supply and demand. In this real life problem, the cost coefficients of the objective function and supply and demand may not be known previously due to uncountable factors. For this reason, the cost coefficients of the objective function are of interval valued rather than real number and supply and demand are followed by log-normal random variables with known means and variances.
Then the multi-objective interval valued transportation prob-

lem is defined as follows:

$$min: z^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} C^1_{ij}x_{ij} = \sum_{i=1}^{3}\sum_{j=1}^{4}\left[C^1_{L_{ij}}, C^1_{R_{ij}}\right]x_{ij}; \quad (46)$$

$$min: z^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} C^2_{ij}x_{ij} = \sum_{i=1}^{3}\sum_{j=1}^{4}\left[C^2_{L_{ij}}, C^2_{R_{ij}}\right]x_{ij}; \quad (47)$$

$$min: z^3 = \sum_{i=1}^{3}\sum_{j=1}^{4} C^3_{ij}x_{ij} = \sum_{i=1}^{3}\sum_{j=1}^{4}\left[C^3_{L_{ij}}, C^3_{R_{ij}}\right]x_{ij}; \quad (48)$$

$$\text{subject to} \quad \Pr\left[\sum_{j=1}^{4} x_{1j} \le a_1\right] \ge 1-\alpha_1 \quad (49)$$

$$\Pr\left[\sum_{j=1}^{4} x_{2j} \le a_2\right] \ge 1-\alpha_2 \quad (50)$$

$$\Pr\left[\sum_{j=1}^{4} x_{3j} \le a_3\right] \ge 1-\alpha_3 \quad (51)$$

$$\Pr\left[\sum_{i=1}^{3} x_{i1} \ge b_1\right] \ge 1-\beta_1 \quad (52)$$

$$\Pr\left[\sum_{i=1}^{3} x_{i2} \ge b_2\right] \ge 1-\beta_2 \quad (53)$$

$$\Pr\left[\sum_{i=1}^{3} x_{i3} \ge b_3\right] \ge 1-\beta_3 \quad (54)$$

$$\Pr\left[\sum_{i=1}^{3} x_{i4} \ge b_4\right] \ge 1-\beta_4 \quad (55)$$

$$x_{ij} \ge 0, \quad i = 1,2,3 \text{ and } j = 1,2,3,4. \quad (56)$$

The transportation costs are provided to illustrate our proposed method for dealing with interval valued objective functions, they are defined as $C^1_{3\times4}$,$C^2_{3\times4}$ and $C^3_{3\times4}$ as follows:

$$C^1_{3\times4} = \begin{bmatrix} [4,13] & [3,12] & [2,6] & [5,8] \\ [4,11] & [6,14] & [7,15] & [3,10] \\ [6,10] & [4,8] & [6,12] & [8,13] \end{bmatrix},$$

$$C^2_{3\times4} = \begin{bmatrix} [1,2] & [1,3] & [5,9] & [4,8] \\ [1,2] & [7,10] & [2,6] & [3,5] \\ [7,9] & [6,10] & [3,5] & [5,7] \end{bmatrix}$$

$$C^3_{3\times4} = \begin{bmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \end{bmatrix}.$$

Assuming means and variances of log-normal random variables with specified probabilistic level of supplies i.e, a_i, $i = 1,2,3$ are expressed in the following Table -1.

Mean	Variance	Specified probability level
$E(a_1)$= 31	$V(a_1)$= 6	α_1=0.01
$E(a_2)$= 37	$V(a_2)$= 7	α_2=0.02
$E(a_3)$= 40	$V(a_3)$= 8	α_3=0.03

Table -1

Again, the means and variances of the log-normal random variables with the specified probabilistic level of demands i.e, b_j for $j = 1,2,3,4$ are represented in the following Table -2.

Mean	Variance	Specified probability level
$E(b_1)$= 10	$V(b_1)$= 2	β_1=0.04
$E(b_2)$= 15	$V(b_2)$= 3	β_2=0.05
$E(b_3)$= 21	$V(b_3)$= 4	β_3=0.06
$E(b_4)$= 26	$V(b_4)$= 5	β_4=0.07

Table-2

Now using the relations of (16) and (17) the means and standard deviations of the log-normal random variable with specified probabilistic level instead of supply i.e, a_i for $i = 1,2,3$ from the Table-1, are represented in the following Table -3.

Mean	Standard deviation	Specified prob. level
μ_{a_1}= 3.430875164	σ_{a_1}= 0.078892839	α_1=0.01
μ_{a_2}= 3.608364907	σ_{a_2}= 0.07145636	α_2=0.02
μ_{a_3}= 3.687631014	σ_{a_3}= 0.049968792	α_3=0.03

Table-3

Again, using the relation of (30) and (31) the means and standard deviations of the log-normal random variables with specified probabilistic level instead of demand i.e, b_j for $j = 1,2,3,4.$ from the Table -2, are represented in the following Table -4.

Mean	Standard deviation	Specified prob. level
μ_{b_1}= 2.292683779	σ_{b_1}= 0.0140721808	β_1=0.04
μ_{b_2}= 2.701427588	σ_{b_2}= 0.115087908	β_2=0.05
μ_{b_3}= 3.044093436	σ_{b_3}= 0.09502319	β_3=0.06
μ_{b_4}= 3.253678247	σ_{b_4}= 0.094003094	β_4=0.07

Table-4

The above three interval valued objective functions (46)-(48) are to be realized in the following way:

$$Objective^1 = \{\min: \ m(z^1), \ \min: \ w(z^1)\} \quad (57)$$

$$Objective^2 = \{\min: \ m(z^2), \ \min: \ w(z^2)\} \quad (58)$$

$$Objective^3 = \{\min: \ m(z^3), \ \min: \ w(z^3)\} \quad (59)$$

$$\text{subject to} \quad \sum_{j=1}^{4} x_{1j} \le 25.57287834 \quad (60)$$

$$\sum_{j=1}^{4} x_{2j} \le 31.31243425 \quad (61)$$

$$\sum_{j=1}^{4} x_{3j} \le 36.15059581 \quad (62)$$

$$\sum_{i=1}^{3} x_{i1} \ge 10.15548247 \quad (63)$$

$$\sum_{i=1}^{3} x_{i2} \ge 18.12110048 \quad (64)$$

$$\sum_{i=1}^{3} x_{i3} \geq 24.43778598 \quad (65)$$

$$\sum_{i=1}^{3} x_{i4} \geq 29.80520039 \quad (66)$$

$$x_{ij} \geq 0, \quad i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4. \quad (67)$$

where

$$\begin{aligned}
min : m(z^1x) &= 8.5x_{11} + 7.5x_{12} + 4x_{13} + 6.5x_{14} \\
&+ 7.5x_{21} + 10x_{22} + 11x_{23} + 6.5x_{24} \\
&+ 8x_{31} + 6x_{32} + 9x_{33} + 10.5x_{34} \quad (68) \\
min : m(z^2x) &= 1.5x_{11} + 2x_{12} + 7x_{13} + 6x_{14} \\
&+ 1.5x_{21} + 8.5x_{22} + 4x_{23} + 4x_{24} \\
&+ 8x_{31} + 8x_{32} + 4x_{33} + 6x_{34} \quad (69) \\
min : m(z^3x) &= 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} \\
&+ 5x_{21} + 8x_{22} + 8.5x_{23} + 10x_{24} \\
&+ 6x_{31} + 2x_{32} + 4.5x_{33} + 1.5x_{34} \quad (70) \\
min : w(z^1x) &= 4.5x_{11} + 4.5x_{12} + 2x_{13} + 1.5x_{14} \\
&+ 3.5x_{21} + 4x_{22} + 4x_{23} + 3.5x_{24} \\
&+ 2x_{31} + 2x_{32} + 3x_{33} + 2.5x_{34} \quad (71) \\
min : w(z^2x) &= 0.5x_{11} + x_{12} + 2x_{13} + 2x_{14} \\
&+ 0.5x_{21} + 1.5x_{22} + 2x_{23} + x_{24} \\
&+ x_{31} + 2x_{32} + x_{33} + x_{34} \quad (72) \\
min : w(z^3x) &= x_{11} + 2x_{12} + x_{13} + 2x_{14} \\
&+ x_{21} + x_{22} + 1.5x_{23} + x_{24} \\
&+ 2x_{31} + x_{32} + 1.5x_{33} + 0.5x_{34} \quad (73)
\end{aligned}$$

Now, using the above six equations i.e., (68) to (73), we construct a composite single objective function as follows:
Model

$$min : z = \lambda\{v_1 m(z^1) + v_2 m(z^2) + v_3 m(z^3)\} + (1-\lambda) \{v_1 w(z^1) + v_2 w(z^2) + v_3 w(z^3)\} \quad (74)$$

subject to the constraints i.e., (60) to (67),
where $\lambda \in [0, 1]$ represents the decision maker's pessimistic and optimistic attitude and weights v_i $(i = 1, 2, 3)$, such that $\sum_{i=1}^{3} v_i = 1$, are attached to the objective functions to facilitate the decision making with more control over the decision making process.
Solving the above model, we have obtained the optimal solution which shown in the next section.

VI. Results

The set of non-inferior solution and the optimal values of the objective functions are listed in the following table by using Lingo 11 package, when the decision maker's has specific choice of λ and also the choices of weights such that sum of the weights are equal to 1.

λ	v_1	v_2	v_3	non-inferior solution	objective values $< m(z), w(z) >$
.3	.4	.4	.2	$x_{13} = 24.43779$, $x_{14} = 1.135092$, $x_{21} = 10.15548$, $x_{24} = 10.64061$, $x_{32} = 18.12110$, $x_{34} = 18.02950$	$<$ 548.4956, 204.6805 $>$, $<$ 488.8165, 121.1358 $>$, $<$ 297.1886, 65.62517 $>$
.5	.5	.3	.2	$x_{13} = 24.43779$, $x_{14} = 1.135092$, $x_{21} = 10.15548$, $x_{24} = 10.64061$, $x_{31} = 18.12110$, $x_{34} = 18.02950$	$<$ 548.4956, 204.6805 $>$, $<$ 488.8165, 121.13598 $>$, $<$ 297.1886, 65.62517 $>$
.6	.4	.3	.3	$x_{13} = 24.43779$, $x_{14} = 1.135092$, $x_{21} = 10.15548$, $x_{22} = 10.64061$, $x_{32} = 7.480488$, $x_{34} = 28.67011$	$<$ 537.8550, 215.3211 $>$, $<$ 515.4180, 115.8155 $>$, $<$ 270.5871, 69.31961 $>$

Table -5

VII. Conclusion

The aim of this paper is to present a solution procedure for multi-objective stochastic unbalanced interval valued transportation problem with log-normal random variables, where the cost coefficients of the objective functions have considered as an interval numbers. The transportation problem is an efficient tool to cope with many real life problems of practical importance. Multi-objective interval valued transportation problem arises in so many cases such as planning of many complex resource allocation systems in the areas of industrial production, in which demand and supply are random variables in nature and the cost coefficients are defined as an interval form.
The important feature of this work in the paper is that it reflects decision maker's pessimistic or optimistic bias in achieving the non-inferior solution. The decision maker's preference may be changed from pessimism to optimism by considering the different values of λ, where $\lambda \in [0, 1]$. Also an another important feature of this paper is the flexibility, where the decision maker to have his desired satisfactory solution by suitable arrangement of objective weights of our specified problem. The last important aspect in this paper is that only a few steps are required to obtained the non-inferior solution of our mentioned problem.

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