Analytical Studies on MHD Flows with Viscous Dissipation and the Boundary Layer Flow over a Vertical Plate

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***Abstract*—In the following paper a thorough analytical study has been made on the boundary layer flow over a vertical plate with viscous dissipation.During this process we have taken into account the simultaneous variations of heat and mass transfers also influenced by MHD and Radiation effects with mixed convection and viscous dissipation along with double diffusion. The coupled nonlinear differential equations are solved numerically using the fourth order Runge-Kutta method with the usual double shooting technique. The velocity, temperature, concentration profiles are presented graphically, against the similarity variable η for different physical parameters. The rates of heat and mass transfer have also been detailed. The results obtained by us are in agreement with those obtained by S. S. Tak [14] and Lai, F. C. and Kulacki F. A [15] without the dissipation terms.**

***Index Terms*—Boundary layer, Mixed convection, Radiation, MHD, Double diffusion, viscous dissipation.**

***MSC* 2010 Codes - 76RXX , 76SXX**

I. INTRODUCTION

FLows in porous media have several applications in geothermal, oil reservoir engineering and astrophysics. Further investigations on boundary layer flow and heat transfer of viscous fluids over a flat sheet are very important for development in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning. Among these studies, Sakiadis [1] initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary-layer equation for two-dimensional and axisymmetric flows.The combined heat and mass transfer in free convection under boundary layer approximations has been studied by Lai and Kulacki [2] and Angirasa et al.[3]. Gorla [4] and Gorla and Pop [5] have investigated the effects of radiation on mixed convection flow over vertical cylinder. Crane [6] was the first to consider the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. The heat transfer aspect of this problem was investigated by Carragher and Crane [7], under the conditions when the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. Magyari and Keller [8] investigated the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Partha et al. [9] studied the effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Chamkha [10] investigated thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink. Ramachandra Prasad and Bhaskar Reddy [11] presented radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate with viscous dissipation. Elbashbeshy, E.M.A [12] examined the effect of surface mass flux on mixed convection along a vertical plate embedded in porous medium. Makinde [13] investigated the similarity of hydromagnetic heat and mass transfer over a vertical plate with a convective surface boundary condition. MHD Mixed Convection Boundary Layer Flow with double diffusion and thermal radiation adjacent to a vertical Permeable surface Embedded in a Porous medium was studied by S.S.Tak et.al [14] in his article he studied the effects of dimensionless velocity, temperature and concentration profiles and he presented graphically against η for various values of the mixed convection parameter. Coupled Heat and Mass Transfer in Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid-saturated porous medium under the effects of radiation and viscous dissipation have also been studied Salem.A.M.[16].Presently we have dealt with the MHD effect on boundary layer flows with viscous dissipation in detail.

II. MATHEMATICAL FORMULATION

We consider the two dimensional mixed convection boundary layer flow in a Darcian porous medium along a vertical permeable plate in the presence of a uniform transverse magnetic field. The plate is maintained at constant temperature T_w and constant concentration C_w. The temperature and mass concentration of the ambient medium are assumed to be T_∞ and C_∞ respectively. The x-coordinate is measured along the plate from its leading edge,and the y coordinate normal to it. Then the governing equations for the boundary layer flow along with the heat and mass transfer from the wall $y = 0$ into the fluid saturated porous medium $x \geq 0$ and $y \geq 0$ are given by

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Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

Momentum Equation:

$$[1+(\frac{K\sigma\mu_e^2 H_0^2}{\mu})]\frac{\partial u}{\partial y} = \pm\frac{gK}{\nu}(\beta_T\frac{\partial T}{\partial y}+\beta_C\frac{\partial C}{\partial y}) \tag{2}$$

Energy Equation:

$$\begin{aligned} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} &= \alpha_m\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}(\frac{\partial u}{\partial y})^2 \\ &\quad -\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \end{aligned} \tag{3}$$

Concentration Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u and v are velocities in x and y directions, T is the temperature, K is the permeability constant, C is an empirical constant, ν is the kinematic viscosity, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion , β_C is the coefficient of solutal expansion ρ is the density, c_p is the specific heat at constant pressure, σ is the electrical conductivity of the fluid, M is the magnetic field and q_r is the radiative heat flux.The "$+$" sign corresponds to the case of aiding buoyancy and the "$-$" sign corresponds to the case of opposing buoyancy flow , α_m is the thermal diffusivity and D_m is the mass diffusivity. The Rosseland approximation term is taken as $q_r = -\frac{4\sigma_B}{3k^*}\frac{\partial T^4}{\partial y}$,where σ_B is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Expanding T^4 about T_e by using Taylor series and neglecting higher order terms we get

$$T^4 \cong 4T_e^3 - 3T_e^4$$

The boundary conditions of the problem are

$$y = 0; v = v_w, T = T_w, C = C_w$$

$$y \to \infty, u \to u_\infty, T = T_\infty, C = C_\infty \tag{5}$$

Now the introduced similarity variables are

$$\psi = f(\eta)(\alpha_m u_\infty x)^{\frac{1}{2}}, \eta = \frac{y}{x}(\frac{u_\infty x}{\alpha_m})^{\frac{1}{2}},$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

ψ is the stream function such that

$$u = \frac{\partial\psi}{\partial y}$$

and

$$v = -\frac{\partial\psi}{\partial x}$$

The governing equations (1) to (4) together with the boundary conditions (5) are transformed as

$$(1+M)f^{''} = \pm(\frac{Ra_x}{Pe_x})(\theta' + N\phi') \tag{6}$$

$$(1+\frac{4}{3R})\theta^{''} + \frac{1}{2}f\theta^{'} + D_f\phi'' + PrE_c(f^{''})^2 = 0 \tag{7}$$

$$\phi^{''} + \frac{1}{2}Lef\phi^{'} + LeS_r\theta'' = 0 \tag{8}$$

with the boundary conditions

$$\begin{aligned} \eta = 0: &\quad f = f_w,\ \theta = 0,\ \phi = 0 \\ \eta \to \infty: &\quad f^{'} = 1\ ,\ \ \theta = 0,\ \phi = 0 \end{aligned} \tag{9}$$

(where the prime denotes differentiation with respect to η)

The parameter

$$Ra_x = \frac{Kg\beta_T(T_w - T_\infty)x}{\nu\alpha_m}$$

is a Local Rayleigh number,

$$M = \frac{K\sigma\mu_e^2 H_0^2}{\mu}$$

is the magnetic parameter ,

$$Le = \frac{\alpha_m}{D_m}$$

is the Lewis number ,

$$D_f = \frac{D_m k_T(C_w - C_\infty)}{\alpha_m C_s C_p(T_w - T_\infty}$$

is the Dufour effect ,

$$S_r = \frac{D_m k_T(C_w - C_\infty)}{\alpha_m T_m(T_w - T_\infty)}$$

is the Soret effect ,

$$R = \frac{k^* K_T}{4\sigma_B(T_w - T_\infty)^3}$$

is the radiation parameter,

$$N = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}$$

is the buoyancy ratio ,

$$Pr = \frac{\nu}{\alpha_m}$$

is the Prandtl number ,

$$Pe_x = \frac{u_\infty x}{\alpha_m}$$

is the local Peclet number,

$$f_w = -\frac{2Pe_x^{1/2}v_w}{u_\infty}$$

is the Suction / Injection parameter and

$$E_c = \frac{u_\infty^2}{c_p(T_w - T_\infty)}$$

is the Eckert number.

The fluid flow can be categorized as representing injection of the fluid into the boundary layer through the vertical plate, the fluid suction or withdrawal from the boundary layer and the wall as impermeable respectively for $f_w < 0, f_w > 0, f_w = 0$

The local heat transfer Nu_x and the local Sherwood number Sh_x , which characterize the surface heat transfer rate and surface mass transfer rate are defined as

$$Nu_x = -\frac{x}{(T_w - T_\infty)}(\frac{\partial T}{\partial y})_{y=0}$$

and

$$Sh_x = -\frac{x}{(C_w - C_\infty)}(\frac{\partial C}{\partial y})_{y=0}$$

reduces to

$$\frac{Nu_x}{(Ra_x)^{\frac{1}{2}}} = -\theta^{'}(0)$$

and

$$\frac{Sh_x}{(Ra_x)^{\frac{1}{2}}} = -\phi^{'}(0)$$

III. Method of Solution

The dimensionless equations (6),(7),(8) together with the boundary conditions (9) are solved numerically by means of the fourth order Runge-Kutta method coupled with the shooting technique.The solution, thus, obtained is matched with the given values of $f'(\infty), \theta(\infty), \phi(\infty)$.In addition,the boundary condition $\eta \to \infty$ is approximated by $\eta_{max} = 10$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties.This choice of η_{max} helps to compare the present results with those of the earlier researchers.

Table 1 and table 2 shows a comparison of the results obtained through the stated procedure with those obtained in Refs [14,15] for the given conditions.

Table1: Comparison of present results with Lai and Kulacki [15] and S.S.Tak et.al [14], for the $N = 0$, $R = 0$, $M = 0$, $D_f = 0$, $S_r = 0$ and for different value of the parameters viz., mixed convection Ra_x/Pe_x, Suction/ Injection f_w, for *aiding flow*

$\frac{Ra}{Pe}$	$f_w = -1$			$f_w = -0.5$			$f_w = 0.5$			$f_w = 1$		
	Lai & Kulacki [15]	S.S.Tak et.al [14]	Present	Lai and Kulacki [15]	S.S.Tak et.al [14]	Present	Lai & Kulacki [15]	S.S.Tak et.al [14]	Present	Lai and Kulacki [15]	S.S.Tak et.al [14]	Present
0	0.2891	0.2896	0.2891	0.4152	0.4158	0.4155	0.7324	0.7328	0.7328	0.9164	0.9169	0.9169
0.5	0.3720	0.3724	0.3720	0.5000	0.5008	0.4999	0.8120	0.8125	0.8122	0.9916	0.9919	0.9911
1	0.4446	0.4449	0.4446	0.5739	0.5743	0.5737	0.8828	0.8833	0.8827	1.0590	1.0598	1.0586
3	0.6791	0.6788	0.6792	0.8119	0.8123	0.8117	1.1150	1.1157	1.1149	1.2836	1.2842	1.2832
10	1.2334	1.2338	1.2334	1.3708	1.3713	1.3710	1.6694	1.6699	1.6699	1.8298	1.8299	1.8298
20	1.7807	1.7810	1.7808	1.9206	1.9211	1.9208	2.2177	2.2179	2.2178	2.3745	2.3749	2.3746

Table2: Comparison of present results with Lai and Kulacki [15] and S.S.Tak et.al [14], for the $N = 0$, $R = 0$, $M = 0$, $D_f = 0$, $S_r = 0$ and for different value of the parameters viz., mixed convection Ra_x/Pe_x, Suction/ Injection f_w, for *opposing flow*

$\frac{Ra}{Pe}$	$f_w = -1$			$f_w = -0.5$			$f_w = 0.5$			$f_w = 1$		
	Lai & Kulacki [15]	S.S.Tak et.al [14]	Present	Lai and Kulacki [15]	S.S.Tak et.al [14]	Present	Lai & Kulacki [15]	S.S.Tak et.al [14]	Present	Lai and Kulacki [15]	S.S.Tak et.al [14]	Present
0.2	0.2517	0.25196	0.2517	0.3771	0.37729	0.3771	0.6972	0.69788	0.6973	0.8837	0.883319	0.8840
0.4	0.2109	0.21095	0.2107	0.3353	0.33507	0.3353	0.6595	0.65989	0.6599	0.8491	0.849619	0.8493
1	0.038	0.03788	0.0381	0.1677	0.16789	0.1677	0.5229	0.52286	0.5232	0.7289	0.728856	0.7290

IV. Result and Discussion

In problems of heat transfer in porous media under mixed convection and double diffusion due to local flows in the boundary layer flow the flow over a porous medium is significant. In this paper, the effect of double diffusion, i.e., the Soret and Dufour effects due to hydrodynamic flow is incorporated and extended to viscous dissipation by using Tak's linear model in the governing equations and a numerical solution of the problem is obtained using the shooting technique. The scheme is validated against earlier published results [15] without the viscous dissipation effects.The effects of magnetic field M, mixed convection Ra/Pe, buoyancy ratio N and Eckert number E_c on the flow are graphically illustrated.

From the velocity profiles in fig.1, it is observed that velocity decreases with an increase in suction/injection parameter for a fixed value of radiation. With an increase in radiation, the velocity decreases near the wall and increases with distance away from the wall. From fig.2, it is clear that temperature decreases with an increase in suction/ injection parameter for a fixed value of radiation. But in the case of larger values of radiation we could not observe much difference in the variation of temperature. It is clear from the fig.3 that the concentration also decreases near the wall with an increase in suction/ injection parameter for a fixed value of radiation.

Fig.4 shows variation of $f'(\eta)$ with η for different values of N and f_w. It is clear from the figure that the velocity decreases with an increase in suction/ injection parameter for a selected value of the buoyancy ratio. For another increased buoyancy ratio the velocity increases with a fixed value of suction/ injection parameter. From fig.5 it is clear to say that temperature decreases slightly with an increase in buoyancy ratio, ultimately not giving rise to much difference in the observation of profiles. It is shown from the concentration profiles in fig.6 concentration is more when the flow is a way from the wall.

As per fig.7 we can say that much variation is found in velocity near the wall whereas such variation is not observed when we go away from the wall. From figs.8 and 9 we cannot find much variation in the run of profiles of temperature and concentration respectively.

Under the viscous dissipation effect we can have similar observations in velocity (fig.10) and concentration (fig.12) profiles as earlier, as going on decreasing near the wall and not much when away from the wall. But such variation could not be expressed as the profiles are almost all coinciding from the beginning of the curves of temperature (fig.11).

In the case of mixed convection of the opposing flow pattern velocity is found to be increasing near the boundary only with the increase in the suction/injection parameter. It is not the case with aiding flow of mixed convection (fig.13) as the velocity went on decreasing with increase in the suction/injection parameter. The temperature goes on decreasing with the increase of suction/injection parameter for the two types of flows of the mixed convection (fig.14). Concentration profile (fig.15) shows much decrease with the increase of suction/injection parameter even near the boundary in aiding as well as opposing flows. According to fig.16 we observe the velocity decreases with the increase in suction/injection parameter with magnetic effect and much decrease in velocity is shown in the absence of the magnetic field. Fig.17 shows that temperature goes on decreasing with the increase of the suction/injection in the presence of the magnetic field and in its absence. With the magnetic effect, the concentration profile (fig.18) shows that concentration directly varies with the magnetic parameter.

When there is no viscous dissipation the rate of heat transfer is increasing with the increase of suction/injection parameter (from fig.19). With viscous dissipation heat transfer goes down when suction/injection parameter goes up in its value.

Without viscous dissipation with the increase of suction/injection parameter up to some extent only, the mass transfer rate goes on increasing and beyond some value of the suction/injection parameter finally goes on decreasing. When there is viscous dissipation such variation cannot be perceived because the mass transfer goes on increasing with the increase of suction/injection parameter as observed from (fig.20)

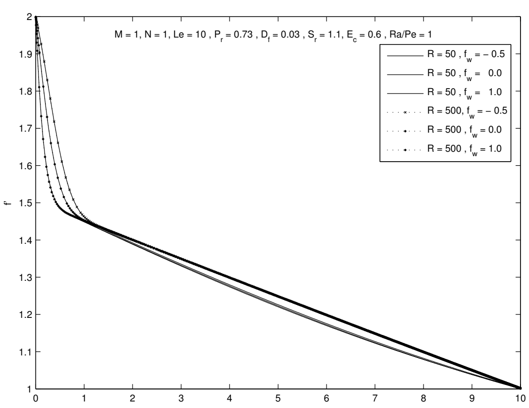

Fig.1. Variation of $f'(\eta)$ with η for different values of R and f_w

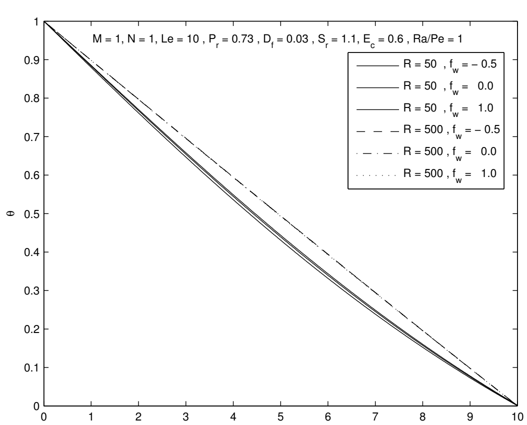

Fig.2. Temperature profile for different values of R and f_w

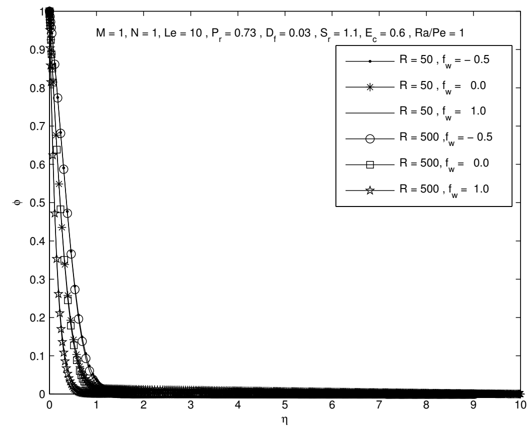

Fig.3. Concentration profile for different values of R and f_w

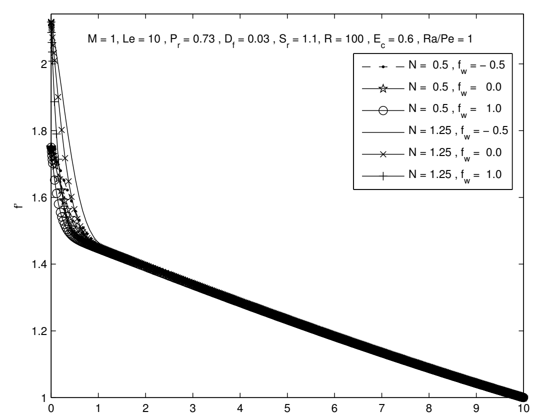

Fig.4. Variation of $f'(\eta)$ with η for different values of N and f_w

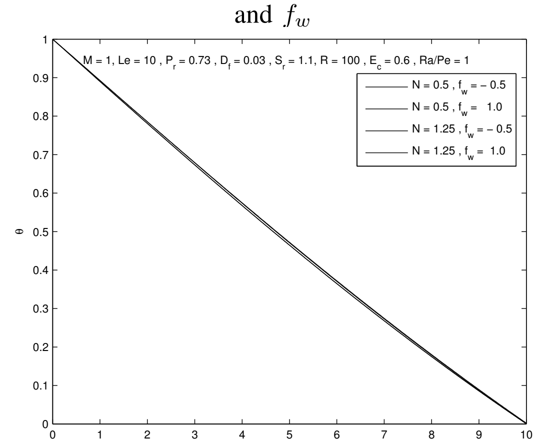

Fig.5. Temperature profile for different values of N and f_w

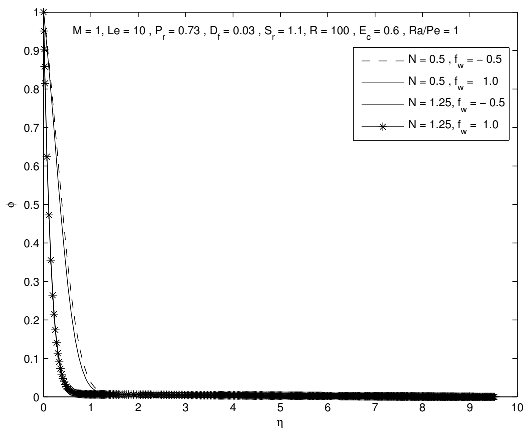

Fig.6. Concentration profile for different values of N and f_w

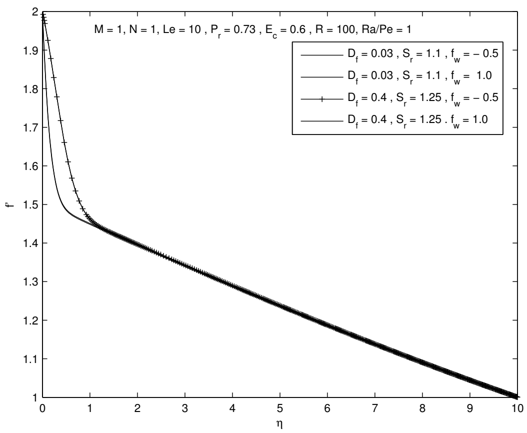

Fig.7. Variation of velocity with η for different values of D_f, S_r and f_w

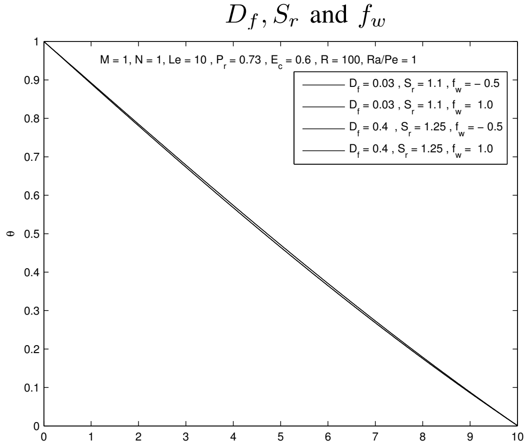

Fig.8. Variation of velocity with η for different values of D_f, S_r and f_w

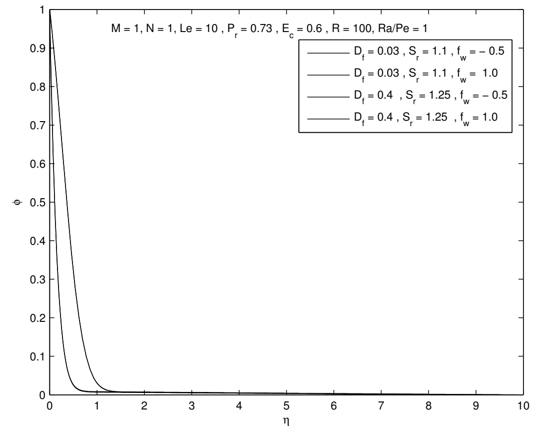

Fig.9. Variation of velocity with η for different values of D_f, S_r and f_w

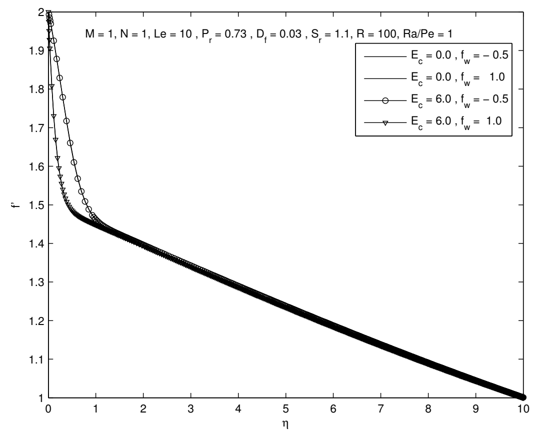

Fig.10. Variation of velocity with η for different values of D_f, S_r and f_w

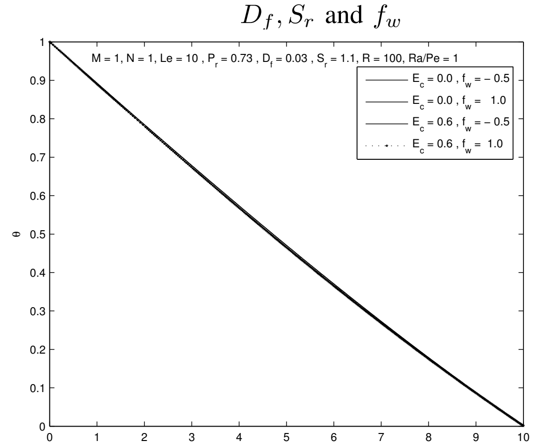

Fig.11. Variation of velocity with η for different values of D_f, S_r and f_w

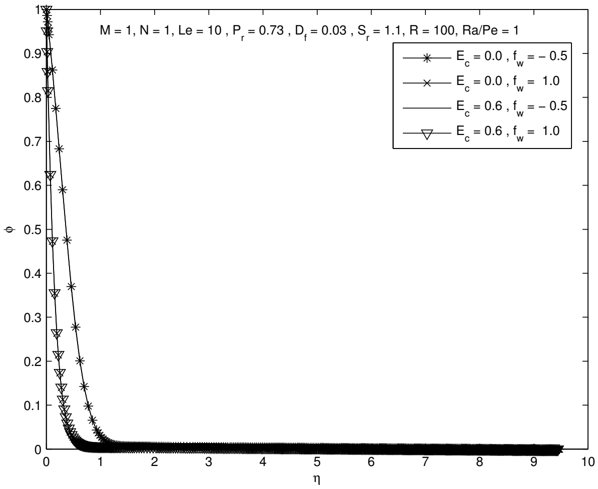

Fig.12. Variation of velocity with η for different values of D_f, S_r and f_w

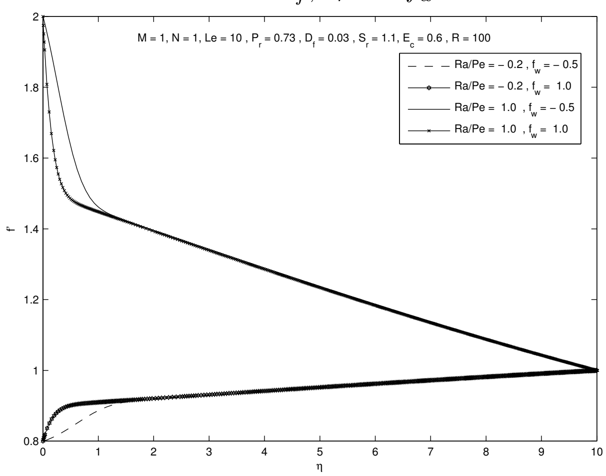

Fig.13. Velocity profile for different values of Ra/Pe and f_w

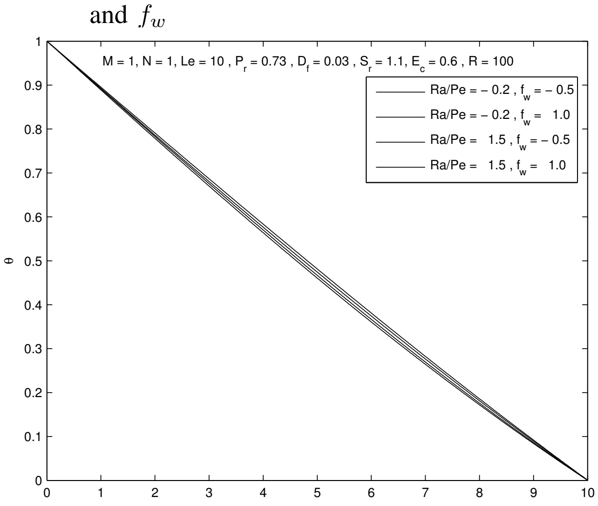

Fig.14. Temperature profile for different values of Ra/Pe and f_w

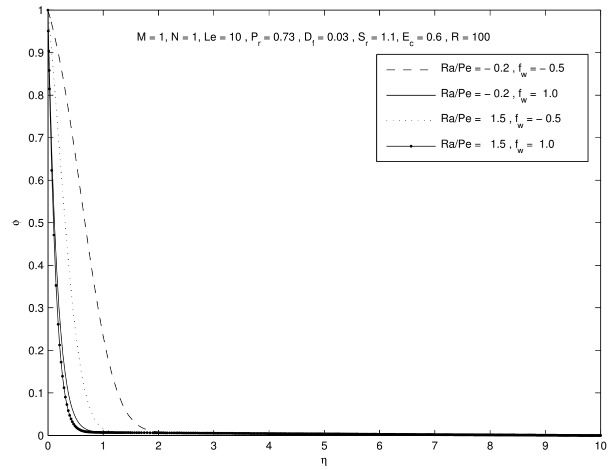

Fig.15. Concentration profile for different values of Ra/Pe and f_w

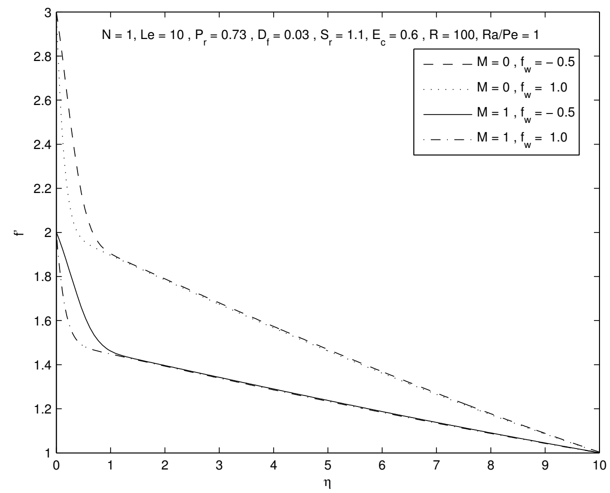

Fig.16. Velocity profile for different values of M and f_w

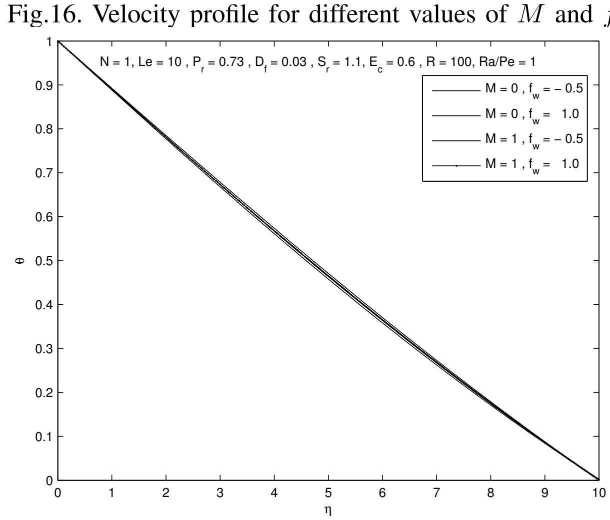

Fig.17. Temperature profile for different values of M and f_w

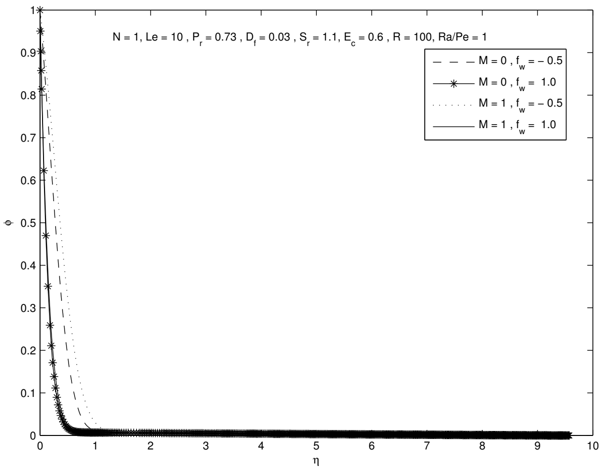

Fig.18. Concentration profile for different values of M and f_w

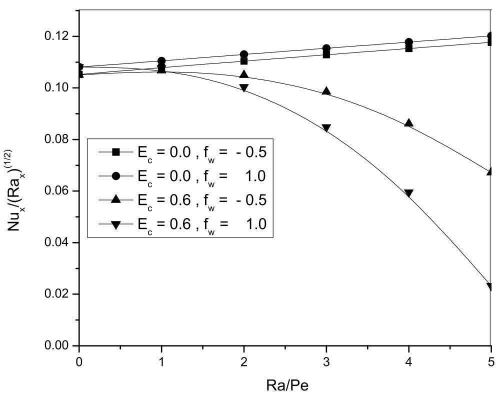

Fig.19. Effect of mixed convection parameter on Nusselt number for various values of E_c and f_w

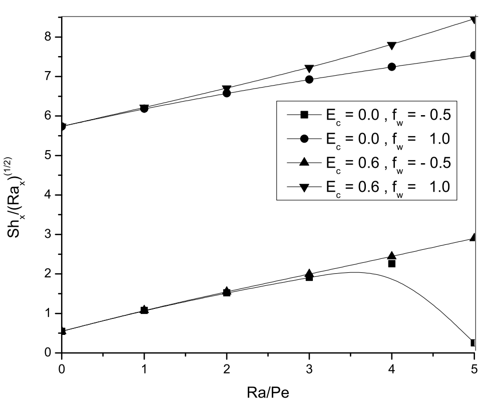

Fig.20. Effect of mixed convection parameter on Sherwood number for various values of E_c and f_w

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