Robust Guaranteed Cost Control for Uncertain Switched Time-Delay Systems with Sampled-Data State Feedback and Linear Fractional Perturbations

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Abstract—**The robust guaranteed cost control for uncertain switched systems with sampled-data state feedback and time delay is investigated in this paper. Time-varying delay technique is provided to solve the system with sampled-data state feedback control. The upper bound for the sampled time of state is estimated from the proposed LMI optimization approach. A numerical example is illustrated to show the use of the main results.**

Index Terms—**Switched time-delay system, sampled-data state feedback; guaranteed cost control, LMI optimization approach; time-varying delay, linear fractional perturbation.**

MSC 2010 Codes **—93D15, 93C30, 93C57.**

I. Introduction

Switched systems are often encountered in automated highway systems, automotive engine control system, chemical process, constrained robotics, power systems and power electronics, robot manufacture, and stepper motors [1]-[7]. Switched system is composed of a class of subsystems and the switching signal is used to specify which subsystem is activated in each instant of time. Hence many complicated phenomena are studied and proposed in recent years [4]. Time delay is often encountered in various practical systems; such as aircraft stabilization, manual control, models of lasers, neural networks, nuclear reactors, ship stabilization, and systems with lossless transmission lines. Sampled-data input is a practical and useful tool to implement some complicate control schemes; such as parallel distributed control in T-S fuzzy system [8]. Suppose that the states of system are measured by some feasible sensors, then the state values will be held until next measured instant to renew the state [8]-[12]. There are many researchers to consider this important issue for estimating sampled time $T > 0$ to stabilize the systems. In the past, time-varying delay technique had been used to represent the sampled-data input [8]-[12]. This approach provides a useful analytic tool to estimate the upper bound of sampled time $T > 0$ and attain the system performance.

H_∞ control and guaranteed cost control are two stabilization schemes with some respective performance indices [8], [12]. H_∞ control concept was proposed to reduce the effect of the disturbance input on the regulated output within a prescribed level and guarantee that the closed-loop system is stable. In our past results in [8], H_∞ control problem of fuzzy time-delay system with sampled-data input had been considered. Guaranteed cost control which not only makes the closed-loop system asymptotically stable but also guarantees an adequate level of performance. In [12], Lyapunov-Krasovskii functional with Leibniz-Newton formula had been used to find guaranteed control for T-S fuzzy systems by using time-varying delay input approach. In [13], some linear fractional perturbations are considered for T-S fuzzy time-delay systems. In this paper, the guaranteed cost control for switched time-delay systems with linear fractional perturbations is considered. Some additional nonnegative inequalities are introduced to improve the conservativeness of the proposed results in this paper.

Notations. For a matrix A, we denote symmetric nagative definite by $A < 0$. I means the identity matrix. $A \le B$ means that the matrix $B - A$ is symmetric positive semi-definite. $\underline{N}$ is defined by $\{1, 2, \cdots, N\}$.

II. Guaranteed Cost Control for Switched Systems with Nonlinear Perturbation and Sampled-Data State Feedback

Consider a continuous-time switched system with time delay:

$$\dot{x}(t) = A_{1\sigma}x(t) + A_{2\sigma}x(t-\tau) + \Delta f_\sigma(x(t)) + B_\sigma u(t),\ t \ge 0, \tag{1a}$$

$$x(t) = \varphi(t),\ t \in [-\tau, 0], \tag{1b}$$

where $x(t) \in \Re^n$ is the system state, $u(t) \in \Re^p$ is the control input, time delay τ is a nonnegative constant, σ is a switching signal which is a piecewise constant function and depends on t, σ takes its values in the finite set $\underline{N}$, and the initial vector

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$\varphi \in C_0$, where C_0 is the set of continuous functions from $[-\tau, 0]$ to $\Re^n$. Matrices A_{1i}, A_{2i}, and B_i, are given. $\Delta f_i(x(t))$ is a perturbed nonlinear function satisfying

$$\|\Delta f_i(x(t))\| \le \|F_i x(t)\|, \; i \in \{1, 2, \cdots, m\}, \qquad (2)$$

where F_i is a given constant matrix. Define the following functions $\lambda_i(\sigma)$, $\forall\, i \in \underline{N}$, that will be used to represent our system:

$$\lambda_i(\sigma) = \begin{cases} 1, & \sigma = i, \\ 0, & \text{otherwise}, \end{cases} \quad i \in \underline{N}, \qquad (3)$$

where σ is defined in (1). The state equation of switched system is rewritten as

$$\dot{x}(t) = \sum_{i=1}^{m} \lambda_i(\sigma) \cdot [A_{1i}x(t) + A_{2i}x(t-\tau) + \Delta f_i(x(t)) + B_i u(t)], \; t \ge 0, \qquad (4a)$$

$$x(t) = \varphi(t), \; t \in [-\tau, 0], \qquad (4b)$$

where $\lambda_i^2(\sigma) = \lambda_i(\sigma)$, $\lambda_i(\sigma)\lambda_j(\sigma) = 0, i \ne j$, and $\sum_{i=1}^{m} \lambda_i(\sigma) = 1$.

Define the cost function of system (1) with (2) as follows:

$$J = \int_0^{\infty} [x^T(s) \cdot S_1 \cdot x(s) + u^T(s) \cdot S_2 \cdot u(s)] ds, \qquad (5)$$

where $S_1 \in \Re^{n \times n}$ and $S_2 \in \Re^{p \times p}$ are two given positive definite symmetric matrices. We wish to design a sampled-data state feedback control in (3) and find a positive constant J^*, such that the system (1) with (2) is asymptotically stable and $J \le J^*$, where J^* is the guaranteed cost for this sampled-data state feedback control in (3) of switched system (1) with (2).

The following state feedback control is used to stabilize the switched in this paper:

$$u(t) = -K_i x(kT), \;\; \sigma = i, \;\; kT \le t < (k+1)T,$$

where $K_i \in \Re^{p \times n}$ is designed in this paper. The final feedback control is inferred as

$$u(t) = -\sum_{i=1}^{m} \lambda_i(\sigma) \cdot K_i x(kT), \;\; kT \le t < (k+1)T. \qquad (6)$$

In this paper, we will provide a concept to treat the original system with sampled-data input by time-varying delay. With (6), the sampled-data control input can be described as follows:

$$u(t) = -\sum_{i=1}^{m} \lambda_i(\sigma) \cdot K_i x(t - h(t)), \;\; kT \le t < (k+1)T, \qquad (7a)$$

where $h(t)$ is specified by

$$h(t) = t - kT, \;\; kT \le t < (k+1)T. \qquad (7b)$$

The system (4) can be rewritten as follows:

$$\dot{x}(t) = \sum_{i=1}^{m}\sum_{j=1}^{m} \lambda_i(\sigma) \cdot \lambda_j(\sigma) \cdot [A_{1i}x(t) + A_{2i}x(t-\tau) + \Delta f_i(x(t)) - B_i K_j x(t - h(t))]$$
$$= \sum_{i=1}^{m}\sum_{j=1}^{m} \lambda_i(\sigma) \cdot [A_{1i}x(t) + A_{2i}x(t-\tau) + \Delta f_i(x(t)) - B_i K_i x(t - h(t))], \quad t \ge 0, \qquad (8a)$$

$$x(t) = \varphi(t), \; t \in [-\tau, 0], \; i = 1, 2, \cdots, m. \qquad (8b)$$

From (6b), we have $0 \le h(t) < T$, $t \ge 0$. The following lemma will be used to design the state feedback control.

Lemma 1. (Schur complement of [14]). For a given matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$ with $S_{11} = S_{11}^T$, $S_{22} = S_{22}^T$, then the following conditions are equivalent:

(1) $S < 0$,

(2) $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Now we present a result to design the sampled-data guaranteed cost control (7) for system (8) with (2).

Theorem 1. Suppose for a given constant $\eta > 0$, the following LMIs :

$$\begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} \\ * & \hat{Q}_{22} \end{bmatrix} > 0, \; \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} \\ * & \hat{R}_{22} \end{bmatrix} > 0, \; \hat{R}_{11} > \hat{Q}_{11},$$

$$\hat{P}_2 > \hat{Q}_{22}, \; \hat{P}_2 > \hat{R}_{22}, \qquad (9a)$$

$$T \cdot \hat{P}_2 < \eta^{-1} \cdot \hat{P}_0, \qquad (9b)$$

$$\hat{\hat{\Pi}}_i = \begin{bmatrix}
\hat{\Pi}_{11i} & \hat{\Pi}_{12i} & 0 & \hat{\Pi}_{14i} & \hat{\Pi}_{15i} & 0 & \hat{\Pi}_{17i} & \hat{\Pi}_{18i} & \hat{\Pi}_{19i} \\
* & 0 & 0 & 0 & 0 & \hat{\Pi}_{26i} & \hat{\Pi}_{27i} & 0 & 0 \\
* & * & \hat{\Pi}_{33i} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \hat{\Pi}_{44i} & 0 & 0 & \hat{\Pi}_{47i} & 0 & 0 \\
* & * & * & * & \hat{\Pi}_{55i} & 0 & \hat{\Pi}_{57i} & 0 & 0 \\
* & * & * & * & * & \hat{\Pi}_{66i} & 0 & 0 & 0 \\
* & * & * & * & * & * & \hat{\Pi}_{77i} & 0 & 0 \\
* & * & * & * & * & * & * & \hat{\Pi}_{88i} & 0 \\
* & * & * & * & * & * & * & * & \hat{\Pi}_{99i}
\end{bmatrix}$$

$$+ \begin{bmatrix}
\hat{\Omega} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} < 0, \text{ for all } i \in \{1, \cdots, m\}, \qquad (9c)$$

have a solution with positive constants ε_i, positive definite symmetric matrices $\hat{P}_0$, $\hat{P}_1$, $\hat{P}_2$, $\hat{P}_3$, $\hat{Q}_{22}$, $\hat{R}_{22} \in \Re^{n \times n}$, $\hat{Q}_{11} \in \Re^{3n \times 3n}$, $\hat{R}_{11} \in \Re^{3n \times 3n}$, matrices $\hat{Q}_{12} \in \Re^{3n \times n}$, $\hat{R}_{12} \in \Re^{3n \times n}$, $\hat{K}_i \in \Re^{p \times n}$ where * represents the symmetric form in the matrix and

$\hat{\Pi}_{11i} = A_{1i}\hat{P}_0 + \hat{P}_0 A_{1i}^T + \hat{P}_1 + \hat{P}_3$, $\hat{\Pi}_{12i} = -B_i \hat{K}_i$, $\hat{\Pi}_{14i} = A_{2i}\hat{P}_0$,
$\hat{\Pi}_{15i} = \varepsilon_i \cdot I$, $\hat{\Pi}_{17i} = \hat{P}_0 A_{1i}^T$, $\hat{\Pi}_{18i} = \hat{P}_0 F_i^T$, $\hat{\Sigma}_{19i} = \hat{P}_0$,
$\hat{\Pi}_{26i} = -\hat{K}_i^T$, $\hat{\Pi}_{27i} = -\hat{K}_i^T B_i^T$, $\hat{\Pi}_{33i} = -\hat{P}_1$, $\hat{\Pi}_{44i} = -\hat{P}_3$,
$\hat{\Pi}_{47i} = \hat{P}_0 A_{2i}^T$, $\hat{\Pi}_{55i} = -\varepsilon_i \cdot I$, $\Pi_{57i} = \varepsilon_i \cdot I$, $\hat{\Pi}_{66i} = -S_2^{-1}$,
$\hat{\Pi}_{77i} = -\eta \cdot \hat{P}_0$, $\hat{\Pi}_{88i} = -\varepsilon_i \cdot I$, $\hat{\Pi}_{99i} = -S_1^{-1}$,

$$\hat{\Omega} = T \cdot \hat{R}_{11} + \hat{Q}_{12}[I \quad -I \quad 0] + [I \quad -I \quad 0]^T \hat{Q}_{12}^T + \hat{R}_{12}[0 \quad I \quad -I] + [0 \quad I \quad -I]^T \hat{R}_{12}^T . \tag{9d}$$

Then the system (4) with (2) is stabilizable by sampled-data input (6) and (7) with $K_i = \hat{K}_i \hat{P}_0^{-1}$ and the guaranteed cost is given by

$$J^* = x^T(0)P_0x(0) + \int_{-T}^{0} x^T(s)P_1x(s)ds + \int_{-T}^{0}(s+T)\dot{x}^T(s)P_2\dot{x}(s)ds + \int_{-\tau}^{0} x^T(s)P_3x(s)ds , \tag{9e}$$

where $P_0 = \hat{P}_0^{-1}$, $P_i = \hat{P}_0^{-1}\hat{P}_i\hat{P}_0^{-1}$, $i \in \{1, 2, 3\}$.

Proof. Define the Lyapunov functional

$$V(x_t) = x^T(t)P_0x(t) + \int_{t-T}^{t} x^T(s)P_1x(s)ds + \int_{t-T}^{t}(s-(t-T))\dot{x}^T(s)P_2\dot{x}(s)ds + \int_{t-\tau}^{t} x^T(s)P_3x(s)ds , \tag{10a}$$

where $P_0 = \hat{P}_0^{-1}$, $P_i = \hat{P}_0^{-1}\hat{P}_i\hat{P}_0^{-1}$, $i \in \{1, 2, 3\}$, are positive definite symmetric matrices. The time derivatives of $V(x_t)$, along the trajectories of system (8) with (2) satisfy

$$\begin{aligned}\dot{V}(x_t) &= \sum_{i=1}^{m}\lambda_i(\sigma)\cdot\{x^T(t)(P_0A_{1i} + A_{1i}^TP_0)x(t) + 2x^T(t)P_0[A_{2i}x(t-\tau) + \Delta f_i(x(t))]\} \\ &- \sum_{i=1}^{m}\sum_{j=1}^{m}\lambda_i(\sigma)\cdot\lambda_j(\sigma)\cdot\{2x^T(t)P_0B_iK_jx(t-h(t))\} \\ &+ x^T(t)P_1x(t) - x^T(t-T)P_1x(t-T) + T\cdot\dot{x}^T(t)P_2\dot{x}(t) \\ &- \left[\int_{t-T}^{t-h(t)}\dot{x}^T(s)P_2\dot{x}(s)ds + \int_{t-h(t)}^{t}\dot{x}^T(s)P_2\dot{x}(s)ds\right] \\ &+ x^T(t)P_3x(t) - x^T(t-\tau)P_3x(t-\tau).\end{aligned} \tag{10b}$$

From the definition of $\lambda_i(\sigma)$ with the above equation, we have

$$-\sum_{i=1}^{m}\sum_{j=1}^{m}\lambda_i(\sigma)\cdot\lambda_j(\sigma)\cdot\{2x^T(t)P_0B_iK_jx(t-h(t))\} = -\sum_{i=1}^{m}\lambda_i(\sigma)\cdot\{2x^T(t)P_0B_iK_ix(t-h(t))\}. \tag{10c}$$

Now we define a vector by

$$X^T(t) = [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-T)].$$

By Leibniz-Newton formula and LMIs (9a), the following additional nonnegative inequalities can be introduced:

$$\begin{aligned}&\int_{t-h(t)}^{t}\begin{bmatrix}X(t)\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}\begin{bmatrix}X(t)\\ \dot{x}(s)\end{bmatrix}ds \\ &= h(t)\cdot X^T(t)Q_{11}X(t) + 2X^T(t)Q_{12}[x(t) - x(t-h(t))] \\ &\quad + \int_{t-h(t)}^{t}\dot{x}^T(s)Q_{22}\dot{x}(s)ds \ge 0 ,\end{aligned} \tag{10d}$$

$$\begin{aligned}&\int_{t-T}^{t-h(t)}\begin{bmatrix}X\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix}R_{11} & R_{12}\\ * & R_{22}\end{bmatrix}\begin{bmatrix}X\\ \dot{x}(s)\end{bmatrix}ds \\ &= [T - h(t)]\cdot X^T(t)R_{11}X(t) + 2X^T(t)R_{12}[x(t-h(t)) - x(t-T)] \\ &\quad + \int_{t-T}^{t-h(t)}\dot{x}^T(s)R_{22}\dot{x}(s)ds \ge 0 ,\end{aligned} \tag{10e}$$

where

$$\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix} = \begin{bmatrix}\hat{P}_0^{-1} & 0 & 0 & 0\\ 0 & \hat{P}_0^{-1} & 0 & 0\\ 0 & 0 & \hat{P}_0^{-1} & 0\\ 0 & 0 & 0 & \hat{P}_0^{-1}\end{bmatrix}\begin{bmatrix}\hat{Q}_{11} & \hat{Q}_{12}\\ * & \hat{Q}_{22}\end{bmatrix}\begin{bmatrix}\hat{P}_0^{-1} & 0 & 0 & 0\\ 0 & \hat{P}_0^{-1} & 0 & 0\\ 0 & 0 & \hat{P}_0^{-1} & 0\\ 0 & 0 & 0 & \hat{P}_0^{-1}\end{bmatrix} > 0,$$

$$\begin{bmatrix}R_{11} & R_{12}\\ * & R_{22}\end{bmatrix} = \begin{bmatrix}\hat{P}_0^{-1} & 0 & 0 & 0\\ 0 & \hat{P}_0^{-1} & 0 & 0\\ 0 & 0 & \hat{P}_0^{-1} & 0\\ 0 & 0 & 0 & \hat{P}_0^{-1}\end{bmatrix}\begin{bmatrix}\hat{R}_{11} & \hat{R}_{12}\\ * & \hat{R}_{22}\end{bmatrix}\begin{bmatrix}\hat{P}_0^{-1} & 0 & 0 & 0\\ 0 & \hat{P}_0^{-1} & 0 & 0\\ 0 & 0 & \hat{P}_0^{-1} & 0\\ 0 & 0 & 0 & \hat{P}_0^{-1}\end{bmatrix} > 0.$$

From condition (2), we have

$$x^T(t)F_i^TF_ix(t) - \Delta f_i^T(x(t))\Delta f_i(x(t)) \ge 0 , \ i \in \{1, 2, \cdots, m\}. \tag{11}$$

From the input in (7), we have

$$\begin{aligned}u^T(t)S_2u(t) &= \sum_{i=1}^{m}\sum_{j=1}^{m}\lambda_i(\sigma)\cdot\lambda_j(\sigma)\cdot[-K_ix(t-h(t))]^TS_2[-K_jx(t-h(t))] \\ &= \sum_{i=1}^{m}\lambda_i(\sigma)[-K_ix(t-h(t))]^TS_2[-K_ix(t-h(t))].\end{aligned} \tag{12a}$$

By the similar derivation of (12), condition (9b), and system (8) with $P_2 = \hat{P}_0^{-1}\hat{P}_2\hat{P}_0^{-1}$, we have

$$\begin{aligned}T\cdot\dot{x}^T(t)P_2\dot{x}(t) &= \sum_{i=1}^{m}\lambda_i(\sigma) \\ &\cdot[A_{1i}x(t) + A_{2i}x(t-\tau) + \Delta f_i(x(t)) - B_iK_ix(t-h(t))]^T\cdot(\eta\cdot\hat{P}_0)^{-1}\cdot \\ &[A_{1i}x(t) + A_{2i}x(t-\tau) + \Delta f_i(x(t)) - B_iK_ix(t-h(t))] .\end{aligned} \tag{12b}$$

From (9a) and (10)-(12), we have

$$\begin{aligned}&\dot{V}(x_t) + x^T(t)S_1x(t) + u^T(t)S_2u(t) \\ &+ \sum_{i=1}^{m}\lambda_i(\sigma)\cdot\varepsilon_i^{-1}\cdot[x^T(t)F_i^TF_ix(t) - \Delta f_i^T(x(t))\Delta f_i(x(t))] \\ &\le \sum_{i=1}^{m}\lambda_i(\sigma)\cdot\{[Y_i^T(t)\cdot\Pi_{ij}\cdot Y_i(t) - h(t)\cdot X^T(t)(R_{11} - Q_{11})X(t)] \\ &- \left[\int_{t-T}^{t-h(t)}\dot{x}^T(s)(P_2 - R_{22})\dot{x}(s)ds + \int_{t-h(t)}^{t}\dot{x}^T(s)(P_2 - Q_{22})\dot{x}(s)ds\right]\} \\ &\le \sum_{i=1}^{m}\lambda_i(\sigma)\cdot[Y_i^T(t)\cdot\Pi_i\cdot Y_i(t)],\end{aligned} \tag{13a}$$

where

$$Y_i^T(t) = [X^T(t) \quad x^T(t-\tau) \quad \Delta f_i^T(x(t))] ,$$

$$\Pi_i = \begin{bmatrix} \Pi_{11i} & \Pi_{12i} & 0 & \Pi_{14i} & \Pi_{15i} \\ * & 0 & 0 & 0 & 0 \\ * & * & \Pi_{33i} & 0 & 0 \\ * & * & * & \Pi_{44i} & 0 \\ * & * & * & * & \Pi_{55i} \end{bmatrix} + \begin{bmatrix} 0 \\ \Pi_{26i} \\ 0 \\ 0 \\ 0 \end{bmatrix} \left(S_2^{-1}\right)^{-1} \begin{bmatrix} 0 \\ \Pi_{26i} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \Pi_{17i} \\ \Pi_{27i} \\ 0 \\ \Pi_{47i} \\ \Pi_{57i} \end{bmatrix} \left(\eta \cdot \hat{P}_0\right)^{-1} \begin{bmatrix} \Pi_{17i} \\ \Pi_{27i} \\ 0 \\ \Pi_{47i} \\ \Pi_{57i} \end{bmatrix}^T + \begin{bmatrix} \Omega & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{13b}$$

$\Pi_{11i} = P_0 A_{1i} + A_{1i}^T P_0 + P_1 + P_3 + \varepsilon_i^{-1} \cdot F_i^T F_i + S_1$, $\Pi_{12i} = -P_0 B_i K_i$,
$\Pi_{14i} = P_0 A_{2i}$, $\Pi_{15i} = P_0$, $\Pi_{17i} = A_{1i}^T$, $\Pi_{26i} = -K_i^T$,
$\Pi_{27i} = -K_i^T B_i^T$, $\Pi_{33i} = -P_1$, $\Pi_{44i} = -P_3$, $\Pi_{47i} = A_{2i}^T$,
$\Pi_{55i} = -\varepsilon_i^{-1} \cdot I$, $\Pi_{57i} = I$,

$$\Omega = T \cdot R_{11} + Q_{12}[I \quad -I \quad 0] + [I \quad -I \quad 0]^T Q_{12}^T + R_{12}[0 \quad I \quad -I] + [0 \quad I \quad -I]^T R_{12}^T.$$

Pre- and post-multiplying the matrix Π_{ij} in (13b) by

$$diag\left[P_0^{-1} \quad P_0^{-1} \quad P_0^{-1} \quad P_0^{-1} \quad \varepsilon_i \cdot I\right] = diag\left[\hat{P}_0 \quad \hat{P}_0 \quad \hat{P}_0 \quad \hat{P}_0 \quad \varepsilon_i \cdot I\right] > 0,$$

with

$$\hat{P}_0 = P_0^{-1},\ \hat{P}_i = P_0^{-1} P_i P_0^{-1},\ \hat{K}_i = K_i P_0^{-1},$$

we have

$$\hat{\Pi}_i = \begin{bmatrix} \hat{\hat{\Pi}}_{11i} & \hat{\Pi}_{12i} & 0 & \hat{\Pi}_{14i} & \hat{\Pi}_{15i} \\ * & 0 & 0 & 0 & 0 \\ * & * & \hat{\Pi}_{33i} & 0 & 0 \\ * & * & * & \hat{\Pi}_{44i} & 0 \\ * & * & * & * & \hat{\Pi}_{55i} \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{\Pi}_{26i} \\ 0 \\ 0 \\ 0 \end{bmatrix} \left(S_2^{-1}\right)^{-1} \begin{bmatrix} 0 \\ \hat{\Pi}_{26i} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \hat{\Pi}_{17i} \\ \hat{\Pi}_{27i} \\ 0 \\ \hat{\Pi}_{47i} \\ 0 \end{bmatrix} \left(\eta \cdot \hat{P}_0\right)^{-1} \begin{bmatrix} \hat{\Pi}_{17i} \\ \hat{\Pi}_{27i} \\ 0 \\ \hat{\Pi}_{47i} \\ 0 \end{bmatrix}^T + \begin{bmatrix} \hat{\Omega} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{14}$$

where $\hat{\hat{\Pi}}_{11i} = \hat{\Pi}_{11i} + \varepsilon_i^{-1} \cdot \hat{P}_0 F_i^T F_i \hat{P}_0 + \hat{P}_0 S_1 \hat{P}_0$, $\hat{\Pi}_{kli}$, $k, l \in \{1, 2, \cdots, 7\}$ are defined in (9d). By Lemma 1, LMI $\hat{\hat{\Pi}}_i < 0$ in (9c) is equivalent to $\hat{\Pi}_i < 0$ in (14). The condition $\hat{\Pi}_i < 0$ in (14) is also equivalent to $\Pi_i < 0$ in (13b) for all $i \in \{1, \cdots, m\}$. From (11) and (13a) with the condition $\Pi_i < 0$, there exists a $\rho > 0$ such that

$$\dot{V}(x_t) \le -\rho \cdot \|x(t)\|^2.$$

We conclude that the switched system (4) with (2) is asymptotically stabilizable by sampled-data state input in (6) with (7). Integrating the equation in (13a) from 0 to ∞ with $\Pi_i < 0$, we have

$$V(x_\infty) - V(\phi) + \int_0^\infty \left[x^T(t) S_1 x(t) + u^T(t) S_2 u(t)\right] dt \le 0.$$

With $V(x_\infty) \ge 0$, we have

$$\int_0^\infty \left[x^T(t) S_1 x(t) + u^T(t) S_2 u(t)\right] dt \le V(\phi) = J^*,$$

where J^* is the guaranteed cost and given in (9e). The system (4) with (2) is stabilizable by sampled-data input (6) with (7) and $K_i = \hat{K}_i \hat{P}_0^{-1}$. □

In the next results, the optimal guaranteed cost control for system (4) with (2) is provided. The minimization for the cost function in (9e) is given in the following result.

Corollary 1.

Suppose for a given constant $\eta > 0$, the following optimization problem:

$$\text{Minimize} \quad \alpha + trace\left(W_1^T \Phi_1 W_1 + W_2^T \Phi_2 W_2 + W_3^T \Phi_3 W_3\right), \tag{15a}$$

subject to

(i) (9a)-(9c),

(ii)
$$\begin{bmatrix} -\alpha & x(0)^T \\ x(0) & -\hat{P}_0 \end{bmatrix} < 0,\ \begin{bmatrix} -2\hat{P}_0 + \hat{P}_1 & I \\ I & -\Phi_1 \end{bmatrix} < 0,$$
$$\begin{bmatrix} -2\hat{P}_0 + \hat{P}_2 & I \\ I & -\Phi_2 \end{bmatrix} < 0,\ \begin{bmatrix} -2\hat{P}_0 + \hat{P}_3 & I \\ I & -\Phi_3 \end{bmatrix} < 0, \tag{15b}$$

has a solution with constants $\alpha > 0$, $\varepsilon_i > 0$, positive definite matrices $\hat{P}_0$, $\hat{P}_1$, $\hat{P}_2$, $\hat{P}_3$, $\hat{Q}_{22}$, $\hat{R}_{22}$, Φ_1, Φ_2, $\Phi_3 \in \Re^{n\times n}$, $\hat{Q}_{11} \in \Re^{3n\times 3n}$, $\hat{R}_{11} \in \Re^{3n\times 3n}$, matrices $\hat{Q}_{12} \in \Re^{3n\times n}$, $\hat{R}_{12} \in \Re^{3n\times n}$, $\hat{K}_i \in \Re^{p\times n}$, where

$$\int_{-T}^0 x(s) x^T(s) ds = W_1 W_1^T,\ \int_{-T}^0 (s+T) \dot{x}(s) \dot{x}^T(s) ds = W_2 W_2^T,$$
$$\int_{-\tau}^0 x(s) x^T(s) ds = W_3 W_3^T. \tag{15c}$$

Then the control (6) with (7) and the gain $K_i = \hat{K}_i \hat{P}_0^{-1}$ is the guaranteed cost control of system (4) with (2), and the guaranteed cost is given in (9e) with $P_0 = \hat{P}_0^{-1}$, $P_i = \hat{P}_0^{-1} \hat{P}_i \hat{P}_0^{-1}$.

Proof. By lemma 1, LMIs (15b) are equivalent to

$$x^T(0) P_0 x(0) < \alpha,\ -2\hat{P}_0 + \hat{P}_k + \Phi_k^{-1} < 0,\ k \in \{1, 2, 3\}. \tag{16}$$

Note that

$$\left[\hat{P}_0-\Phi_k^{-1}\right]\Phi_k\left[\hat{P}_0-\Phi_k^{-1}\right]=\hat{P}_0\Phi_k\hat{P}_0-2\hat{P}_0+\Phi_k^{-1}\geq 0\,,\ k\in\{1,2,3\}.$$

The following results are obtained from condition (16):

$$-\hat{P}_0\Phi_k\hat{P}_0+\hat{P}_k<0\,,\ k\in\{1,2,3\}. \qquad (17)$$

Conditions in (17) are equivalent to

$$P_k=\hat{P}_0^{-1}\hat{P}_k\hat{P}_0^{-1}<\Phi_k\,,\ k\in\{1,2,3\}.$$

Hence we have

$$\int_{-T}^{0}x^T(s)P_1x(s)=trace\left(\int_{-T}^{0}x^T(s)P_1x(s)ds\right)=trace\left(P_1\int_{-T}^{0}x(s)x^T(s)ds\right)$$
$$=trace\left(P_1W_1W_1^T\right)=trace\left(W_1^TP_1W_1\right)\leq trace\left(W_1^T\Phi_1W_1\right),$$
$$\int_{-T}^{0}(s+T)\dot{x}^T(s)P_2\dot{x}(s)ds=trace\left(W_2^TQ_2W_2\right)\leq trace\left(W_2^T\Phi_2W_2\right),$$
$$\int_{-\tau}^{0}x^T(s)P_3x(s)ds=trace\left(W_3^TP_3W_3\right)\leq trace\left(W_3^T\Phi_3W_3\right).$$

By the similar formulation of [15], we can complete this proof.

Remark 1. For a given constant $\eta>0$, the LMI optimization problem in (15) can be solved by the LMI Toolbox of Matlab. Simple "for loop" can be used to find the minimization of the guaranteed cost.

III. GUARANTEED COST CONTROL FOR UNCERTAIN SWITCHED SYSTEMS WITH LINEAR FRACTIONAL PERTURBATIONS

Consider the system (1) with linear fractional perturbations in the following form:

$$\dot{x}(t)=\overline{A}_{1\sigma}(t)x(t)+\overline{A}_{2\sigma}(t)x(t-\tau)+\Delta f_\sigma(x(t))+\overline{B}_\sigma(t)u(t)\,,\quad t\geq 0\,, \qquad (18a)$$
$$x(t)=\varphi(t),\ t\in[-\tau,0], \qquad (18b)$$

where $\overline{A}_{1i}(t)$, $\overline{A}_{2i}(t)$, and $\overline{B}_i(t)$ are some system matrices with perturbations and satisfying

$$\left[\overline{A}_{1i}(t)\quad \overline{A}_{2i}(t)\quad \overline{B}_i(t)\right]$$
$$=\left[A_{1i}+\Delta A_{1i}(t)\quad A_{2i}+\Delta A_{2i}(t)\quad B_i+\Delta B_i(t)\right],\ i\in\{1,2,\cdots,m\}, \qquad (19a)$$

where constant matrices A_{1i}, A_{2i}, and B_i are given, $\Delta A_{1i}(t)$, $\Delta A_{2i}(t)$, and $\Delta B_i(t)$ are some perturbed matrices and satisfying

$$\left[\Delta A_{1i}(t)\quad \Delta A_{2i}(t)\quad \Delta B_i(t)\right]=M_i\cdot\Delta_i(t)\cdot\left[N_{1i}\quad N_{2i}\quad N_{3i}\right],\quad i\in\{1,2,\cdots,m\}, \qquad (19b)$$

$$\Delta_i(t)=\left[I-\Gamma_i(t)\Theta_i\right]^{-1}\Gamma_i(t)\,,\ \Theta_i\Theta_i^T<I\,, \qquad (19c)$$

where M_i, N_{ki}, $k\in\{1,2,3\}$, and Θ_i are some given constant matrices with appropriate dimensions. $\Gamma_i(t)$ is an unknown matrix representing the parameter perturbations which satisfies

$$\Gamma_i^T(t)\Gamma_i(t)\leq I\,. \qquad (19d)$$

Remark 2. The perturbations in (19) are the generalization form of the parametric perturbations in [12].

Lemma 2. [13] Suppose that $\Delta_i(t)$ is defined in (19c) and satisfying (19d), then for real matrices U_i, W_i and X_i with $X_i=X_i^T$, the following statements are equivalent:

(I) The inequality is satisfied

$$X_i+U_i\Delta_i(t)W_i+W_i^T\Delta_i^T(t)U_i^T<0\,,$$

(II) There exists a scalar $\mu_i>0$, such that

$$\begin{bmatrix} X_i & \mu_i\cdot U_i & W_i^T \\ * & -\mu_i\cdot I & \mu_i\cdot\Theta_i^T \\ * & * & -\mu_i\cdot I \end{bmatrix}<0\,, \qquad (20)$$

where the matrix Θ_i is defined in (19c).

From Corollary 1 with the switched system in (18) with (2), (19), and Lemma 2, we can obtain the following results.

Theorem 2.

Suppose for a given constant $\eta>0$, the following optimization problem:

$$\text{Minimize}\quad \alpha+trace\left(W_1^T\Phi_1W_1+W_2^T\Phi_2W_2+W_3^T\Phi_3W_3\right), \qquad (21a)$$

subject to

(i) (9a), (9c), (15b),

(ii) $$\tilde{\tilde{\Pi}}_i=\begin{bmatrix}\tilde{\tilde{\Pi}}_{11i} & \tilde{\tilde{\Pi}}_{12i}\\ * & \tilde{\tilde{\Pi}}_{22i}\end{bmatrix}+\begin{bmatrix}\hat{\Omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}<0\,,$$

$$\text{for all } i\in\{1,\cdots,m\}, \qquad (22b)$$

where

$$\tilde{\tilde{\Pi}}_{11i}=\begin{bmatrix}\hat{\Pi}_{11i} & \hat{\Pi}_{12i} & 0 & \hat{\Pi}_{14i} & \hat{\Pi}_{15i} & 0 & \hat{\Pi}_{17i} & \hat{\Pi}_{18i} & \hat{\Pi}_{19i}\\ * & 0 & 0 & 0 & 0 & \hat{\Pi}_{26i} & \hat{\Pi}_{27i} & 0 & 0\\ * & * & \hat{\Pi}_{33i} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \hat{\Pi}_{44i} & 0 & 0 & \hat{\Pi}_{47i} & 0 & 0\\ * & * & * & * & \hat{\Pi}_{55i} & 0 & \hat{\Pi}_{57i} & 0 & 0\\ * & * & * & * & * & \hat{\Pi}_{66i} & 0 & 0 & 0\\ * & * & * & * & * & * & \hat{\Pi}_{77i} & 0 & 0\\ * & * & * & * & * & * & * & \hat{\Pi}_{88i} & 0\\ * & * & * & * & * & * & * & * & \hat{\Pi}_{99i}\end{bmatrix},$$

$$\tilde{\tilde{\Pi}}_{12i}^T=\begin{bmatrix}\hat{\Pi}_{110i}^T & 0 & 0 & 0 & 0 & 0 & \hat{\Pi}_{710i}^T & 0 & 0\\ \hat{\Pi}_{111i}^T & \hat{\Pi}_{211i}^T & 0 & \hat{\Pi}_{411i}^T & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\tilde{\tilde{\Pi}}_{22i}=\begin{bmatrix}-\mu_i\cdot I & \mu_i\cdot\Theta_i^T\\ * & -\mu_i\cdot I\end{bmatrix},$$

has a solution with constants $\alpha>0$, $\varepsilon_i>0$, $\mu_i>0$, positive definite matrices $\hat{P}_0$, $\hat{P}_1$, $\hat{P}_2$, $\hat{P}_3$, $\hat{Q}_{22}$, $\hat{R}_{22}$, Φ_1, Φ_2, $\Phi_3 \in \Re^{n\times n}$, $\hat{Q}_{11} \in \Re^{3n\times 3n}$, $\hat{R}_{11} \in \Re^{3n\times 3n}$, matrices $\hat{Q}_{12} \in \Re^{3n\times n}$, $\hat{R}_{12} \in \Re^{3n\times n}$, $\hat{K}_i \in \Re^{p\times n}$, where matrices W_1, W_2, and W_3 are given in (15c), $\hat{\Pi}_{kli}$, $k,l \in \{1,2,\cdots,9\}$, and $\hat{\Omega}$ are defined in (9d) and $\hat{\Pi}_{110i} = \hat{\Pi}_{710i} = \mu_i \cdot M_i$, $\hat{\Pi}_{111i} = \hat{P}_0 N_{1i}^T$, $\hat{\Pi}_{211i} = -\hat{K}_i^T N_{3i}^T$, $\hat{\Pi}_{411i} = \hat{P}_0 N_{2i}^T$. Then the control (6) with (7) and the gain $K_i = \hat{K}_i \hat{P}_0^{-1}$ is the guaranteed cost control of the switched system in (18) with (2) and (19), and the guaranteed cost is given in (9e) with $P_0 = \hat{P}_0^{-1}$, $P_i = \hat{P}_0^{-1}\hat{P}_i\hat{P}_0^{-1}$.

Proof. Consider the switched system (18) with (2) and (19), the stability LMI condition in (9b) should be rewritten as follows:

$$\tilde{\Pi}_i = \begin{bmatrix} \tilde{\Pi}_{11i} & \tilde{\Pi}_{12i} & 0 & \tilde{\Pi}_{14i} & \hat{\Pi}_{15i} & 0 & \tilde{\Pi}_{17i} & \hat{\Pi}_{18i} & \hat{\Pi}_{19i} \\ * & 0 & 0 & 0 & 0 & \hat{\Pi}_{26i} & \tilde{\Pi}_{27i} & 0 & 0 \\ * & * & \hat{\Pi}_{33i} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Pi}_{44i} & 0 & 0 & \tilde{\Pi}_{47i} & 0 & 0 \\ * & * & * & * & \hat{\Pi}_{55i} & 0 & \hat{\Pi}_{57i} & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66i} & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{77i} & 0 & 0 \\ * & * & * & * & * & * & * & \hat{\Pi}_{88i} & 0 \\ * & * & * & * & * & * & * & * & \hat{\Pi}_{99i} \end{bmatrix} + \begin{bmatrix} \hat{\Omega} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\Pi}_{11i} & \hat{\Pi}_{12i} & 0 & \hat{\Pi}_{14i} & \hat{\Pi}_{15i} & 0 & \hat{\Pi}_{17i} & \hat{\Pi}_{18i} & \hat{\Pi}_{19i} \\ * & 0 & 0 & 0 & 0 & \hat{\Pi}_{26i} & \hat{\Pi}_{27i} & 0 & 0 \\ * & * & \hat{\Pi}_{33i} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Pi}_{44i} & 0 & 0 & \hat{\Pi}_{47i} & 0 & 0 \\ * & * & * & * & \hat{\Pi}_{55i} & 0 & \hat{\Pi}_{57i} & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66i} & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{77i} & 0 & 0 \\ * & * & * & * & * & * & * & \hat{\Pi}_{88i} & 0 \\ * & * & * & * & * & * & * & * & \hat{\Pi}_{99i} \end{bmatrix}$$

$$+ \begin{bmatrix} M_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M_i \\ 0 \\ 0 \end{bmatrix} \Delta_i(t) \begin{bmatrix} \hat{P}_0 N_{1i}^T \\ -\hat{K}_i^T N_{3i}^T \\ 0 \\ \hat{P}_0 N_{2i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \hat{P}_0 N_{1i}^T \\ -\hat{K}_i^T N_{3i}^T \\ 0 \\ \hat{P}_0 N_{2i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta_i^T(t) \begin{bmatrix} M_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M_i \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \hat{\Omega} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \qquad (22)$$

where $\hat{\Pi}_{kli}$, $k,l \in \{1,2,\cdots,9\}$, and $\hat{\Omega}$ are defined in (9d), and

$\tilde{\Pi}_{11i} = \overline{A}_{1i}(t)\hat{P}_0 + \hat{P}_0\overline{A}_{1i}^T(t) + \hat{P}_1 + \hat{P}_3$, $\tilde{\Pi}_{12i} = -\overline{B}_i(t)\hat{K}_i$,

$\tilde{\Pi}_{14i} = \overline{A}_{2i}(t)\hat{P}_0$, $\tilde{\Pi}_{17i} = \hat{P}_0\overline{A}_{1i}^T(t)$, $\tilde{\Pi}_{27i} = -\hat{K}_i^T\overline{B}_i^T(t)$,

$\hat{\Pi}_{47i} = \hat{P}_0\overline{A}_{2i}^T(t)$.

This proof can be completed in the similar formulation of Theorem 1 and Corollary 1.

IV. Numerical Example

Consider the switched system in (18) with (2), (19), and the following parameters:

$$A_{11} = \begin{bmatrix} -2 & 0.1 \\ 0.4 & -1.1 \end{bmatrix},\ A_{12} = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix},\ A_{21} = \begin{bmatrix} -1 & 0.2 \\ 0 & -1 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -0.8 & 0 \\ -1 & -1 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ -0.3 \end{bmatrix},\ B_2 = \begin{bmatrix} 1.2 \\ -0.4 \end{bmatrix},$$

$$F_1 = F_2 = 0.1 \cdot I,\ \Theta_1 = \Theta_2 = 0.1,\ \tau = 3,$$

$$M_1 = M_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},\ N_{11} = N_{12} = [0.2 \quad 0.1],$$

$$N_{21} = N_{22} = [0.1 \quad 0.05],\ N_{31} = N_{32} = 0.05,$$

$$S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ S_2 = 1,\ x(t) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, t \in [-3, 0]. \qquad (23)$$

(A) If the upper bound for state sampled time is given by $T = 0.1$, the optimization problem in Theorem 2 with $\eta = 80$ has a feasible solution

$$\hat{K}_1 = [0.004 \quad 0.019],\ \hat{K}_2 = [0.0049 \quad 0.0288],$$

$$\hat{P}_0 = \begin{bmatrix} 0.0298 & -0.0366 \\ -0.0366 & 0.0811 \end{bmatrix}.$$

The switched system in (18) with (2), (19), and (23) is stabilizable by sampled-data state input in (6) and (7) with

$$K_1 = \hat{K}_1 \hat{P}_0^{-1} = [0.8711 \quad 0.6015],$$
$$K_2 = \hat{K}_2 \hat{P}_0^{-1} = [1.3435 \quad 0.9604].$$

The guaranteed cost is given by $J^* = 138.1399$. In this case, the guaranteed cost performance can be provided via the stabilizing robust control in (6) and (7) with the sampled time less than 0.1 second.

(B) If the upper bound of state sampled time is given by $T = 0.2$, the optimization problem in Theorem 2 with $\eta = 28$ has a feasible solution

$$\hat{K}_1 = [0.0016 \quad 0.0175],\ \hat{K}_2 = [0.0003 \quad 0.0308],$$
$$\hat{P}_0 = \begin{bmatrix} 0.0296 & -0.0364 \\ -0.0364 & 0.0809 \end{bmatrix}.$$

The switched system in (18) with (2), (19), and (23) is stabilizable by sampled-data state input in (6) and (7) with

$$K_1 = \hat{K}_1 \hat{P}_0^{-1} = [0.7161 \quad 0.5391],$$
$$K_2 = \hat{K}_2 \hat{P}_0^{-1} = [1.0285 \quad 0.8445].$$

The guaranteed cost is given by $J^* = 141.486$. In this case, the guaranteed cost performance can be provided via the stabilizing robust control in (6) and (7) with the sampled time less than 0.2 second.

V. Conclusion

In this paper, the guaranteed cost control problem for a class of uncertain switched time-delay system with sampled-data state feedback has been studied. Based on the LMI optimization approach and time-varying delay transformation technique, some delay-dependent criteria have been proposed to minimize the upper bound of the guaranteed cost for the system with nonlinear and linear fractional perturbations.

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References

[1] X. M. Sun, W. Wang, G. P. Liu, and J. Zhao, " Stability analysis for linear switched systems with time-varying delay," *IEEE Trans. Syst. Man, Cybernetics, Part B*, vol. 38, pp. 528–533, 2008.

[2] Y. G. Sun, L. Wang, and G. Xie, "Delay-dependent robust stability and stabilization for discrete-time switched systems with mode-dependent time-varying delays," *Appl. Math. Comput.*, vol. 180, pp. 428-435, 2006.

[3] Y. G. Sun, L. Wang, and G. Xie, "Delay-dependent robust stability and H_∞ control for uncertain discrete-time switched systems with mode-dependent time delays," *Appl. Math. Comput.*, vol. 187, pp. 1228-1237, 2007.

[4] Z. Sun and S. S. Ge, *Switched linear systems control and design*. London: Spring-Verlag, 2005.

[5] D. Xie, N. Xu, and X. Chen, "Stabilisability and observer-based switched control design for switched linear systems," *IET Control Theory and Applications*, vol. 2, pp. 192-199, 2008.

[6] Zhai, G., Liu, D., Lmae, J. & Kobayashi, T. (2006) Lie algebraic stability analysis for switched systems with continuous-time and discrete-time subsystems. *IEEE Trans. Circuits Syst.*, 53, 152–156.

[7] L. Zhang, P. Shi, and M. Basin, "Robust stability and stabilisation of uncertain switched linear discrete time-delay systems," *IET Control Theory & Applications*, vol. 2, pp. 606-614, 2008.

[8] C. H. Lien, K. W. Yu, C. T. Huang, P. Y. Chou, L. Y. Chung, J. D. Chen, "Robust H_∞ control for uncertain T-S fuzzy time-delay systems with sampled-data input and nonlinear perturbations," Nonlinear Analysis: Hybrid Systems, vol. 4, pp. 550-556, 2010.

[9] H. Gao and T. Chen, "Stabilization of nonlinear systems under variable sampling: a fuzzy control approach," IEEE Trans. Fuzzy Systems, vol. 15, pp. 972-983, 2007.

[10] H. K. Lam, F. H. F. Leung, "Sampled-data fuzzy controller for time-delay nonlinear system: LMI-based and fuzzy-model-based approaches," IEEE Trans. Syst. Man Cybern. Part B: Cybernetics, vol. 37, vol. 617-629, 2007.

[11] H. K. Lam, "Stability analysis of sampled-data fuzzy controller for nonlinear systems based on switching T–S fuzzy model," Nonlinear Analysis: Hybrid Systems, vol. 3, pp. 418-432, 2009.

[12] J. Yoneyama, "Robust guaranteed cost control of uncertain fuzzy systems under time-varying sampling," Applied Soft Computing, vol. 11, pp. 249-255, 2011.

[13] J. Yang, W. Luo, G. Li, and S. Zhong, "Reliable guaranteed cost control for uncertain fuzzy neutral systems, Nonlinear Analysis: Hybrid Systems, vol. 4, pp. 644-658, 2010.

[14] S. P. Boyd, L. E. Ghaoui, E. Feron, V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia: SIAM, 1994.

[15] C. H. Lien, "Delay-dependent and delay-independent guaranteed cost control for uncertain neutral systems with time-varying delays via LMI approach," Chaos, Solitons & Fractals, vol. 33, pp. 1017-1027, 2007.