Product Cordial Graphs in the Context of Some Graph Operations

S. K. Vaidya and C. M. Barasara

***Abstract*—A product cordial labeling of a graph G is a function $f : V(G) \rightarrow \{0, 1\}$ such that vertices with label 1 & label 0 differ by at most 1 and edges with label 1 & label 0 also differ by at most 1. In this paper we derive product cordial labeling for some graph families.**

***Index Terms*—Cordial labeling, Product cordial labeling, Product cordial graphs.**

***MSC 2010 Codes* - 05C38, 05C76, 05C78.**

I. Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with order p and size q. For all standard terminology and notations we follow West [1]. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1 : If the vertices of the graph are assigned values subject to certain condition(s) then is known as *graph labeling*.

For an extensive survey on graph labeling and bibliographic references we refer to Gallian [2].

Definition 1.2 : A mapping $f : V(G) \rightarrow \{0, 1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the *label* of the vertex v of G under f.

The induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$.

Let us denote

$v_f(0) =$ number of vertices of G having label 0 under f

$v_f(1) =$ number of vertices of G having label 1 under f

$e_f(0) =$ number of edges of G having label 0 under f^*

$e_f(1) =$ number of edges of G having label 1 under f^*

Definition 1.3 : A binary vertex labeling of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *cordial* if it admits cordial labeling.

The concept of cordial labeling was introduced in a seminal paper by Cahit [3] in which he investigated several results on this newly defined concept. Some labelings with variations in cordial theme have been also introduced. Prime cordial labeling, A-cordial labeling, E-cordial labeling, H-cordial labeling, Product cordial labeling, Total product cordial are among mention a few. The present work is aimed to investigate some results on product cordial labeling where unlike in cordial labeling the absolute difference is replaced by product of the vertex labels.

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Definition 1.4 : A binary vertex labeling of graph G with induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ defined by $f^*(e = uv) = f(u)f(v)$ is called a *product cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is called *product cordial* if it admits product cordial labeling.

The product cordial labeling was introduced by Sundaram et al. [4] and they proved that trees, unicyclic graphs of odd order, triangular snakes, dragons, helms and union of two path graphs are product cordial. They have also established that a graph with p vertices and q edges with $p \geq 4$ is product cordial then $q < \frac{p^2 - 1}{4}$. The graphs obtained by joining apex vertices of k copies of stars, shells and wheels to a new vertex are product cordial is proved in Vaidya and Dani [5] while the product cordial labeling for some cycle related graphs is reported in Vaidya and Kanani [6]. In the same paper they have investigated product cordial labeling for shadow graph of cycle C_n. Vaidya and Barasara [7] have proved that the cycle with one chord, the cycle with twin chords, the friendship graph and the middle graph of path admit product cordial labeling while the same authors in [8] have obtained some new results on product cordial labeling. Product cordial labeling in the context of tensor product of some graphs is discussed by Vaidya and Vyas [9].

In this paper we have investigated some new results on product cordial labeling in the context of some graph operations.

Definition 1.5 : *Duplication* of a vertex v_k by a new edge $e = v'v''$ in a graph G produces a new graph G' such that $N(v') = \{v_k, v''\}$ and $N(v'') = \{v_k, v'\}$.

Definition 1.6 : *Duplication* of an edge $e = v_i v_{i+1}$ by a vertex v' in a graph G produces a new graph G' such that $N(v') = \{v_i, v_{i+1}\}$.

Definition 1.7 : For a graph G the *split graph* is obtained by adding to each vertex v, a new vertex v' such that v' is adjacent to each vertex that is adjacent to v in G. The resultant graph is denoted by $spl(G)$.

Definition 1.8 : Let G be a graph with two or more vertices than the *total graph* $T(G)$ of graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.

Definition 1.9 : Consider two copies of graph G(wheel, star, fan & friendship) namely G_1 and G_2. Then the graph $G' = < G_1 \blacktriangle G_2 >$ is the graph obtained by joining the apex vertices of G_1 and G_2 by an edge as well as to a new vertex v'.

II. Main Results

Theorem 2.1 : The graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n is product cordial.
Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_n$ be edges of cycle C_n. Without loss of generality we duplicate the vertex v_n by an edge e_{n+1} with end vertices as v_1' and v_2'. Let the graph so obtained is G. Then $|V(G)| = n+2$ and $|E(G)| = n+3$.

To define $f : V(G) \to \{0,1\}$ we consider following two cases.

Case 1: When n is odd.

$$\begin{aligned} f(v_i) &= 0, \quad 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ f(v_i) &= 1, \quad \text{otherwise} \\ f(v_i') &= 1, \quad i = 1, 2 \end{aligned}$$

In view of the above labeling patten we have

$$\begin{aligned} v_f(0) = v_f(1) - 1 &= \left\lceil \frac{n}{2} \right\rceil \\ e_f(0) = e_f(1) &= \frac{n+3}{2} \end{aligned}$$

Case 2: When n is even.

$$\begin{aligned} f(v_i) &= 0, \quad 1 \leq i \leq \frac{n}{2} + 1 \\ f(v_i) &= 1, \quad \text{otherwise} \\ f(v_i') &= 1, \quad i = 1, 2 \end{aligned}$$

In view of the above labeling patten we have

$$\begin{aligned} v_f(0) = v_f(1) &= \frac{n}{2} + 1 \\ e_f(0) - 1 = e_f(1) &= \frac{n}{2} + 1 \end{aligned}$$

Thus in each cases we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence the graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n is product cordial.

Illustration 2.2 : The graph obtained by duplication of a vertex by an edge in C_7 and its product cordial labeling is shown in Fig. 1.

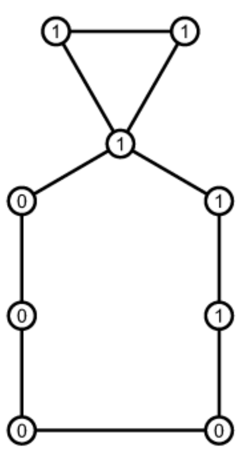

Fig. 1

Theorem 2.3 : The graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n is product cordial except for $n = 3$.
Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_n$ be edges of cycle C_n. Without loss of generality we duplicate the edge $v_{n-1}v_n$ by a vertex v'. Let the graph so obtained is G. Then $|V(G)| = n+1$ and $|E(G)| = n+2$.

To define $f : V(G) \to \{0,1\}$ we consider following three cases.

Case 1: When n is odd $(n \neq 3)$.

$$\begin{aligned} f(v_i) &= 0, \quad 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ f(v_i) &= 1, \quad \text{otherwise} \\ f(v') &= 1 \end{aligned}$$

In view of the above labeling patten we have

$$\begin{aligned} v_f(0) = v_f(1) &= \frac{n+1}{2} \\ e_f(0) - 1 = e_f(1) &= \frac{n+1}{2} \end{aligned}$$

Case 2: When n is even.

$$\begin{aligned} f(v_i) &= 0, \quad 1 \leq i \leq \frac{n}{2} \\ f(v_i) &= 1, \quad \text{otherwise} \\ f(v') &= 1 \end{aligned}$$

In view of the above labeling patten we have

$$\begin{aligned} v_f(0) = v_f(1) - 1 &= \frac{n}{2} \\ e_f(0) = e_f(1) &= \frac{n}{2} + 1 \end{aligned}$$

Thus in case 1 and case 2 we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Case 3: When $n = 3$.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to 2 vertices out of 4 vertices. The vertices with label 0 will give rise at least 4 edges with label 0 and at most 1 edge with label 1 out of total 5 edges. Therefore $|e_f(0) - e_f(1)| = 3$. Thus the edge condition for product cordial graph is violated. Hence G is not a product cordial graph.

Hence we conclude that the graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n is product cordial except for $n = 3$.

Illustration 2.4 : The graph obtained by duplication of an edge by a vertex in C_8 and its product cordial labeling is shown in Fig. 2.

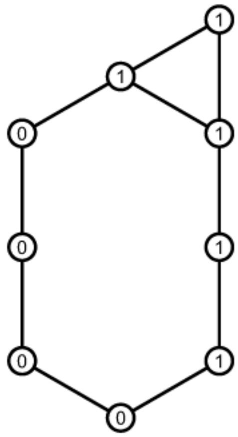

Fig. 2

Theorem 2.5 : The graph obtained by duplicating all the vertices by edges in cycle C_n is not product cordial except for $n = 3$.

Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_n$ be edges of cycle C_n. Let the graph obtained by duplicating all the vertices by edges in cycle C_n is G. Then $|V(G)| = 3n$ and $|E(G)| = 4n$.

We consider following three cases.

Case 1: When $n = 3$.

The graph obtained by duplicating all the vertices by edges in cycle C_3 and its product cordial labeling is shown in Fig. 3.

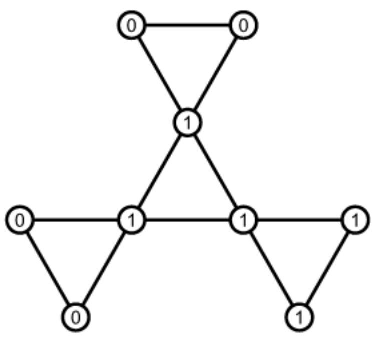

Fig. 3

Case 2: When n is odd $(n \neq 3)$.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to $\left\lfloor \frac{3n}{2} \right\rfloor$ vertices out of $3n$ vertices. The vertices with label 0 will give rise at least $2n+1$ edges with label 0 and at most $2n-1$ edge with label 1 out of total $4n$ edges. Therefore $|e_f(0) - e_f(1)| = 2$. Thus the edge condition for product cordial graph is violated. Hence G is not a product cordial graph.

Case 3: When n is even.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to $\frac{3n}{2}$ vertices out of $3n$ vertices. The vertices with label 0 will give rise at least $2n+1$ edges with label 0 and at most $2n-1$ edge with label 1 out of total $4n$ edges. Therefore $|e_f(0) - e_f(1)| = 2$. Thus the edge condition for product cordial graph is violated. Hence G is not a product cordial graph.

Hence we conclude that duplicating all the vertices by edges in cycle C_n is not product cordial except for $n = 3$.

Theorem 2.6 : The graph obtained by duplicating all the edges by vertices in cycle C_n is not product cordial.

Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_n$ be edges of cycle C_n. Let the graph obtained by duplicating all the edge by vertices in cycle C_n is G. Then $|V(G)| = 2n$ and $|E(G)| = 3n$.

We consider following two cases.

Case 1: When n is odd.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to n vertices out of $2n$ vertices. The vertices with label 0 will give rise at least $\left\lceil \frac{3n}{2} \right\rceil + 1$ edges with label 0 and at most $\left\lfloor \frac{3n}{2} \right\rfloor - 1$ edge with label 1 out of total $3n$ edges. Therefore $|e_f(0) - e_f(1)| = 3$. Thus the edge condition for product cordial graph is violated. Hence G is not a product cordial graph.

Case 2: When n is even.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to n vertices out of $2n$ vertices. The vertices with label 0 will give rise at least $\frac{3n}{2}+2$ edges with label 0 and at most $\frac{3n}{2} - 2$ edge with label 1 out of total $4n$ edges. Therefore $|e_f(0) - e_f(1)| = 4$. Thus the edge condition for product cordial graph is violated. Hence G is not a product cordial graph.

Hence duplicating all the edges by vertices in cycle C_n is not product cordial.

Theorem 2.7 : The graph obtained by duplicating all the vertices by edges in path P_n is product cordial.

Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_{n-1}$ be edges of path P_n. Let the graph obtained by duplicating all the vertices by edges in path P_n is G. Then $|V(G)| = 3n$ and $|E(G)| = 4n - 1$. Let the edge so added corresponding to vertex v_n has end vertices as v'_n and v''_n.

To define $f : V(G) \rightarrow \{0, 1\}$ we consider following two cases.

Case 1: When n is odd.

$$
\begin{aligned}
f(v_i) &= 0, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f(v_i) &= 1, \quad \text{otherwise} \\
f(v'_i) &= 0, \quad 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\
f(v'_i) &= 1, \quad \text{otherwise} \\
f(v''_i) &= 0, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f(v''_i) &= 1, \quad \text{otherwise}
\end{aligned}
$$

In view of the above labeling patten we have

$$
\begin{aligned}
v_f(0) = v_f(1) - 1 &= \left\lfloor \frac{3n}{2} \right\rfloor \\
e_f(0) = e_f(1) + 1 &= 2n
\end{aligned}
$$

Case 2: When n is even.

$$
\begin{aligned}
f(v_i) &= 0, \quad 1 \leq i \leq \frac{n}{2} \\
f(v_i) &= 1, \quad \text{otherwise}
\end{aligned}
$$

$$\begin{array}{ll} f(v_i') = 0, & 1 \le i \le \dfrac{n}{2} \\ f(v_i') = 1, & \text{otherwise} \\ f(v_i'') = 0, & 1 \le i \le \dfrac{n}{2} \\ f(v_i'') = 1, & \text{otherwise} \end{array}$$

In view of the above labeling patten we have

$$v_f(0) = v_f(1) = \frac{3n}{2}$$
$$e_f(0) = e_f(1) + 1 = 2n$$

Thus in case 1 and case 2 we have $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence duplicating all the vertices by edges in path P_n is product cordial.

Illustration 2.8 : The graph obtained by duplicating all the vertices by edges in path P_5 and its product cordial labeling is shown in Fig. 4.

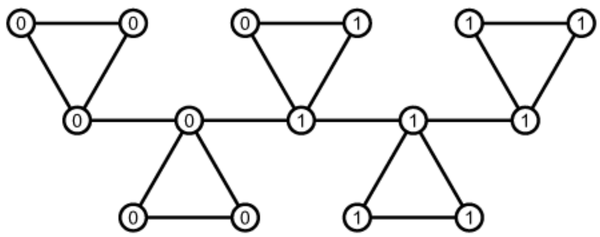

Fig. 4

Theorem 2.9 : The graph obtained by duplication of an arbitrary vertex by a new edge in path P_n is product cordial except for $n = 2$.

Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_{n-1}$ be edges of path P_n. Let the graph obtained by duplication of an arbitrary vertex by a new edge is G. Then $|V(G)| = n + 2$ and $|E(G)| = n + 2$. Let the edge so added has end vertices as v_1' and v_2'.

To define $f : V(G) \to \{0, 1\}$ we consider following three cases.

Case 1: When n is odd.

Subcase 1: When vertex v_k, $k = \left\lceil \frac{n}{2} \right\rceil$ is duplicated.

$$\begin{array}{ll} f(v_i) = 1, & \left\lceil \dfrac{n}{2} \right\rceil \le i \le 2 \left\lfloor \dfrac{n}{2} \right\rfloor \\ f(v_i) = 0, & \text{otherwise} \\ f(v_i') = 1, & i = 1, 2 \end{array}$$

In view of the above labeling patten we have

$$v_f(0) = v_f(1) - 1 = \left\lceil \frac{n}{2} \right\rceil$$
$$e_f(0) = e_f(1) - 1 = \left\lceil \frac{n}{2} \right\rceil$$

Subcase 2: When vertex v_k, $k \ne \left\lceil \frac{n}{2} \right\rceil$ is duplicated.

Without loss of generality we can assume that the duplicating vertex would be v_k, $1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$, then f is define as follows.

$$\begin{array}{ll} f(v_i) = 1, & 1 \le i \le \left\lfloor \dfrac{n}{2} \right\rfloor \\ f(v_i) = 0, & \text{otherwise} \\ f(v_i') = 1, & i = 1, 2 \end{array}$$

In view of the above labeling patten we have

$$v_f(0) = v_f(1) - 1 = \left\lceil \frac{n}{2} \right\rceil$$
$$e_f(0) = e_f(1) - 1 = \left\lceil \frac{n}{2} \right\rceil$$

Case 2: When n is even $(n \ne 2)$.

Without loss of generality we can assume that the duplicating vertex would be v_k, $1 \le k \le \frac{n}{2}$, then f is define as follows.

$$\begin{array}{ll} f(v_i) = 1, & k \le i \le k + \dfrac{n}{2} - 2 \\ f(v_i) = 0, & \text{otherwise} \\ f(v_i') = 1, & i = 1, 2 \end{array}$$

In view of the above labeling patten we have

$$v_f(0) = v_f(1) = \frac{n}{2} + 1$$
$$e_f(0) = e_f(1) = \frac{n}{2} + 1$$

Thus in case 1 and case 2 we have $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Case 3: When $n = 2$.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to 2 vertices out of 4 vertices. The vertices with label 0 will give rise at least 3 edges with label 0 and at most 1 edge with label 1 out of total 4 edges. Therefore $|e_f(0) - e_f(1)| = 2$. Thus the edge condition for product cordial graph is violated. Hence G is not a product cordial graph.

Hence we conclude that the graph obtained by duplication of an arbitrary vertex by a new edge in path P_n is product cordial except for $n = 2$.

Illustration 2.10 : The graph obtained by duplication of a vertex by an edge in P_7 and its product cordial labeling is shown in Fig. 5.

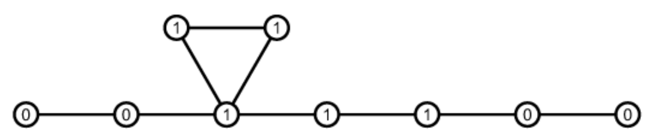

Fig. 5

Theorem 2.11 : The graph obtained by duplication of an arbitrary edge by a new vertex in path P_n is product cordial except for $n = 3$.

Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_{n-1}$ be edges of path P_n. Let the graph obtained by duplication of an arbitrary edge by a new vertex is G. Then $|V(G)| = n + 1$ and $|E(G)| = n + 1$. Let the vertex so added is v'.

To define $f : V(G) \to \{0, 1\}$ we consider following four cases.

Case 1: When $n = 2$.

The graph obtained by duplication of an edge by a vertex in path P_2 and its product cordial labeling is shown in Fig. 6.

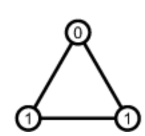

Fig. 6

Case 2: When n is odd $(n \neq 3)$.

Without loss of generality we can assume that the duplicating edge would be e_k, $1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor$, then f is define as follows.

$$\begin{aligned} &f(v_i) = 1, \quad k \leq i \leq k + \left\lfloor \frac{n}{2} \right\rfloor - 1 \\ &f(v_i) = 0, \quad \text{otherwise} \\ &f(v') = 1 \end{aligned}$$

In view of the above labeling patten we have

$$\begin{aligned} v_f(0) = v_f(1) &= \frac{n+1}{2} \\ e_f(0) = e_f(1) &= \frac{n+1}{2} \end{aligned}$$

Case 3: When n is even $(n \neq 2)$.

Without loss of generality we can assume that the duplicating edge would be e_k, $1 \leq k \leq \frac{n}{2}$, then f is define as follows.

$$\begin{aligned} &f(v_i) = 1, \quad k \leq i \leq k + \frac{n}{2} - 1 \\ &f(v_i) = 0, \quad \text{otherwise} \\ &f(v') = 1 \end{aligned}$$

In view of the above labeling patten we have

$$\begin{aligned} v_f(0) = v_f(1) - 1 &= \frac{n}{2} \\ e_f(0) = e_f(1) - 1 &= \frac{n}{2} \end{aligned}$$

Thus in each case we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Case 4: When $n = 3$.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to 2 vertices out of 4 vertices. The vertices with label 0 will give rise at least 3 edges with label 0 and at most 1 edge with label 1 out of total 4 edges. Therefore $|e_f(0) - e_f(1)| = 2$. Thus the edge condition for product cordial graph is violated. Hence G is not a product cordial graph.

Hence we conclude that the graph obtained by duplication of an arbitrary edge by a new vertex in path P_n is product cordial except for $n = 3$.

Illustration 2.12 : The graph obtained by duplication of an edge by a vertex in P_6 and its product cordial labeling is shown in Fig. 7.

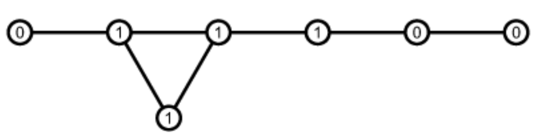

Fig. 7

Theorem 2.13 : $spl(P_n)$ is a product cordial graph except for odd n.

Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_{n-1}$ be edges of path P_n. To obtain $spl(P_n)$ let added vertices are $v'_1, v'_2, \ldots, v'_n$ corresponding to $v_1, v_2, \ldots, v_n$. Let G be the graph $spl(P_n)$. Then $|V(G)| = 2n$ and $|E(G)| = 3n - 3$.

To define $f : V(G) \rightarrow \{0, 1\}$ we consider following three cases.

Case 1: When $n = 2$.

$spl(P_2)$ and its product cordial labeling is shown in Fig. 8.

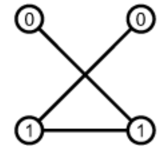

Fig. 8

Case 2: When n is even $(n \neq 2)$.

$$\begin{aligned} &f(v_i) = 1, \quad 1 \leq i \leq \frac{n}{2} + 1 \\ &f(v_i) = 0, \quad \text{otherwise} \\ &f(v'_i) = 1, \quad 2 \leq i \leq \frac{n}{2} \\ &f(v'_i) = 0, \quad \text{otherwise} \end{aligned}$$

In view of the above labeling patten we have

$$\begin{aligned} v_f(0) = v_f(1) &= n \\ e_f(0) = e_f(1) + 1 &= \frac{3n-2}{2} \end{aligned}$$

Thus in case 1 and case 2 we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Case 3: When n is odd.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to n vertices out of $2n$ vertices. The vertices with label 0 will give rise at least $\frac{3n-1}{2}$ edges with label 0 and at most $\frac{3n-5}{2}$ edge with label 1 out of total $3n - 3$ edges. Therefore $|e_f(0) - e_f(1)| = 2$. Thus the edge condition for product cordial graph is violated. Hence G is not a product cordial graph.

Hence we conclude that $spl(P_n)$ is a product cordial graph except for odd n.

Illustration 2.14 : $spl(P_6)$ and its product cordial labeling is shown in Fig. 9.

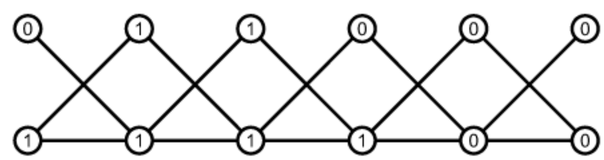

Fig. 9

Theorem 2.15 : $T(P_n)$ is a product cordial graph.

Proof : Let $v_1, v_2, \ldots, v_n$ be vertices and $e_1, e_2, \ldots, e_{n-1}$ be edges of path P_n. Then $v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1}$ are vertices of $T(P_n)$. Let G be the graph $T(P_n)$. Then $|V(G)| = 2n - 1$ and $|E(G)| = 4n - 5$.

To define $f : V(G) \rightarrow \{0, 1\}$ we consider following two cases.

Case 1: When n is odd.

$$\begin{aligned} &f(v_i) = 0, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ &f(v_i) = 1, \quad \text{otherwise} \\ &f(e_i) = 0, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ &f(e_i) = 1, \quad \text{otherwise} \end{aligned}$$

In view of the above labeling patten we have

$$v_f(0)+1=v_f(1)=n$$
$$e_f(0)=e_f(1)+1=2n-2$$

Case 2: When n is even.

$$f(v_i)=0, \quad 1\le i\le \frac{n}{2}$$
$$f(v_i)=1, \quad \text{otherwise}$$
$$f(e_i)=0, \quad 1\le i\le \frac{n}{2}-1$$
$$f(e_i)=1, \quad \text{otherwise}$$

In view of the above labeling patten we have

$$v_f(0)+1=v_f(1)=n$$
$$e_f(0)=e_f(1)+1=2n-2$$

Thus in case 1 and case 2 we have $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.

Hence $T(P_n)$ is a product cordial graph.

Illustration 2.16 : $T(P_6)$ and its product cordial labeling is shown in Fig. 10.

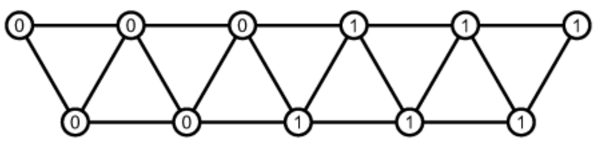

Fig. 10

Theorem 2.17 : Graph $< G_1 \blacktriangle G_2 >$ is product cordial.

Proof : Let $v_1, v_2, \ldots, v_n$ be the vertices of G_1 and $u_1, u_2, \ldots, u_n$ be the vertices of G_2. Let G be the graph $< G_1 \blacktriangle G_2 >$. Then $|V(G)|=2p+1$ and $|E(G)|=2q+3$.

To define $f: V(G)\to\{0,1\}$ we have

$$f(v_i)=1, \quad \text{for all } i$$
$$f(u_i)=0, \quad \text{for all } i$$
$$f(v')=1$$

In view of the above labeling patten we have

$$v_f(0)=v_f(1)-1=p$$
$$e_f(0)-1=e_f(1)=q+1$$

Thus we have $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.

Hence the graph $< G_1 \blacktriangle G_2 >$ is product cordial.

Illustration 2.18 : Graph $< K_{1,5} \blacktriangle K_{1,5} >$ and its product cordial labeling is shown in Fig. 11

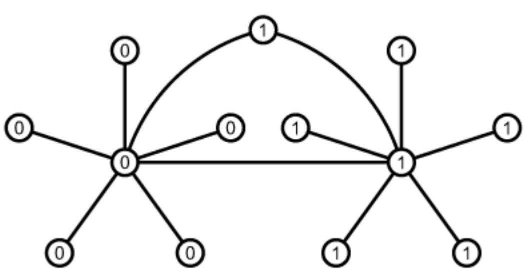

Fig. 11

III. Concluding Remarks

As not every graph admit product cordial labeling it is very interesting to investigate graph or graph families which admits product cordial labeling. We have tried to investigate product cordial labeling for the larger graphs obtained by various graph operations on standard graphs. To investigate analogous results for different graphs as well as in the context of various graph labeling problems is an open area of research.

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